CDS on the Overlap Graph (GCDS)

The overlap graph of a permutation \( \pi \), \( OG(\pi) \), has as vertices the edges of \( BG(\pi) \), each labeled by the pointer supporting the corresponding edge in \( BG(\pi) \). Two vertices of \( OG(\pi) \) are adjacent if the respective edges of \( BG(\pi) \) intersect.

For vertices \( x \) and \( y \), let \( f_{xy}(\pi) \) be 1 if \( x \) and \( y \) are adjacent and 0 otherwise. For any adjacent vertices \( p \) and \( q \), let \( gcds(p,q)(OG(\pi)) \) be the graph \( OG(\pi) \) with the same vertices as \( OG(\pi) \), where for any vertices \( u \) and \( v \), \( OG'(\pi) \) includes the edge \( (u,v) \) if and only if

\[
f_{pu}(\pi)f_{qv}(\pi) + f_{pu}(\pi)v_{q}(\pi) + f_{u}(\pi)v_{q}(\pi) \equiv 1 \pmod{2}.
\]

A graph is \( gcds \)-sortable if some sequence of \( gcds \) operations removes all of its edges.

CDS on the Adjacency Matrix (MCDS)

The adjacency matrix of \( OG(\pi) \) is the matrix \( A_{\pi} \) over \( \mathbb{F}_2 \) such that, for \( 1 \leq i,j \leq (n+1) \), \( A_{\pi}(i,j) = 1 \) if \( (\pi(1),i,\pi(j+1),j) \) is an edge in \( OG(\pi) \) and \( A_{\pi}(i,j) = 0 \) otherwise.

\[
A_{\pi} = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 
\end{pmatrix}
\]

For any entry \( p \) and \( q \), define the cds operation on matrices as

\[
mcds(p,q)(A_{\pi}) = A_{\pi} + A_{\pi}I_{pq},
\]

where \( I_{pq}(i,j) = 1 \) if \( i = p \) and \( j = q \) or \( i = q \) and \( j = p \) and \( I_{pq}(i,j) = 0 \) otherwise.

A matrix is \( mcds \)-sortable if applying some sequence of \( mcds \) operations to it yields the zero matrix.

Future Work

- Further analyze and classify unsortable permutations, graphs, and matrices.
- Extend the overlap graph to include additional relationships.

References

2. H.O. Li et al., Parity Cuts In 2-Rooted Graphs, preprint.

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