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Statewide Mathematics Professional Development: Teacher Knowledge, Self-Efficacy, and Beliefs

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Abstract

We examined the impact of a state mandated K-12 mathematics professional development course on knowledge, self-efficacy and beliefs of nearly 4,000 teachers and administrators. Participants completed the Mathematical Thinking for Instruction course, emphasizing student thinking, problem-solving, and content knowledge specific to mathematics instruction. Inventories utilizing items from the Learning Mathematics for Teaching project (2005) measured changes in participants' Mathematical Knowledge for Teaching (MKT) and an end-of-course self-evaluation enabled analysis of changes in MKT, self-efficacy and beliefs. Statistically significant changes were found in all three variables. This study adds to our understanding of the potential usefulness of mandating professional development as a policy vehicle for influencing educators' mathematics knowledge and beliefs.

Keywords: Professional Development; Mathematics; Beliefs; Knowledge; Self-Efficacy

Introduction

There have been numerous calls for reform of mathematics education in the United States (Kilpatrick, Swafford, & Findell, 2001; National Math Panel, 2008). Comparisons of students' performance on international assessments (Martin, Mullis, & Chrostowski, 2004; OECD, 2010), research on students' preparedness to enter college or the workforce, and large-scale examination of teachers' instructional practices (Measures of Effective Teaching Project, 2010; Stigler & Hiebert, 1999), all indicate a need for significant changes to mathematics instruction in the United States.

Policy makers at the national and state level have attempted to address the issue of mathematics reform through various means with differing levels of effectiveness (Ball, 1996; Cohen & Hill, 2000; Desimone, Smith, & Phillips, 2007; Hill & Ball, 2004; Swanson & Stevenson, 2002). In 2007, the Idaho State Department of Education formed a joint task force to identify problems and needs within mathematics education statewide. The committee suggested that the mathematical knowledge of many educators is not well developed in terms of either the underlying structure of mathematics or pedagogical approaches to instruction, or both – a stance consistent with much mathematics education research (Ball, Hill, & Bass, 2005; Ma, 1999). Based on the task force's recommendations, the Idaho state legislature mandated in 2008 that all K-12 mathematics educators and administrators take a three-credit professional development course, titled Mathematical Thinking for Instruction (MTI), which aims to significantly shift educators' knowledge and beliefs about mathematics and pedagogy. The course is a requirement for re-certification by September, 2014.

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The MTI course originated as the initial five day workshop of a multi-component, three- year professional development program - the Developing Mathematical Thinking Project - funded by a Mathematics and Science Partnership grant (Brendefur, 2007; Brendefur, Thiede, Strother, Bunning, & Peck, 2013). The Developing Mathematical Thinking project was deemed a success in that it changed teachers' beliefs about the nature of mathematics, increased teachers' pedagogical knowledge, and increased student achievement in both the classroom and on standardized tests (Brendefur, 2007). The state mandate scales up just one component of this successful project – the MTI course – to approximately 12,000 additional K-12 teachers and administrators. Many have suggested the effectiveness of such efforts to scale up professional development for broader implementation need to be evaluated (Adler, Ball, Krainer, Lin, & Novotna, 2005; Borko, 2004; Desimone, 2009).

In addition, the recent adoption by 45 states of the Common Core State Standards for Mathematics (CCSS) has raised questions nationwide about how to best build teachers' pedagogical and content knowledge (Schmidt, 2012). The successful implementation of these standards requires teaching mathematics at a deeper, more conceptual level and with an increased focus on the practice of mathematics. Typical U.S. mathematics instruction does not address the depth of understanding necessary for students to achieve these more rigorous standards (Desimone, Smith, Baker, & Ueno, 2005; Measures of Effective Teaching Project, 2010; Schmidt, 2012; Stigler & Hiebert, 1999). It is thus necessary to understand whether large- scale policy mandates for mathematics professional development can influence participants' knowledge and beliefs related to mathematics and mathematics instruction.

This study examines the framework and content of the MTI course and its value as a large-scale, state-mandated professional development course in influencing nearly 4,000 teachers' knowledge, self-efficacy and beliefs – three years into the project.

Professional Development Model: The MTI Course

Background

Starting in the 1980s with the publication of "A Nation at Risk," teachers, schools and districts have come under increasing levels of scrutiny regarding the content of instruction, their classroom practices and student outcomes (National Commission on Excellence in Education, 1983). K-12 mathematics education in particular has become an area of intensive focus from both policy and education research perspectives. State and national policy movements have tended to focus on mathematics reforms at the state and national level through mandates of content standards, assessments, and curricula (Cohen & Hill, 2000; NGA & CCSSO, 2011). In addition, significant accountability measures have been implemented in order to enforce changes in instructional focus and practice, in order to improve student outcomes ("No Child Left Behind," 2001).

Evidence exists that these reforms may affect surface level features of instruction (e.g., placing students' desks in groups instead of rows or stating the specific standard in lesson plans). However, the nature of classroom practice and therefore student outcomes are seldom significantly affected by high level policy decisions regarding curriculum, standards and assessments (Cohen & Hill, 2000; Leinwand, 2009; Schorr, 2003). Teachers and schools are highly enculturated into the traditional methods of instructional delivery through transmission of knowledge, and thus it is very difficult to achieve deep changes in classroom instructional practices (Hiebert & Stigler, 2000). In order to enact meaningful change at the classroom level, teachers' knowledge and beliefs need to be addressed in conjunction with other policy measures (Palardy & Rumberger, 2008; Swanson & Stevenson, 2002).

Members of the education community have often argued for the use of professional development in order to provide teachers the means to reflect upon and make shifts in their practice based on the recommendations for reforming mathematics education (Schmidt, 2012). The MTI course represents one model of professional development that seeks to fundamentally change teachers' knowledge and beliefs related to the nature of mathematics and mathematics instruction.

The MTI Course

The MTI course addresses the topics of number, number operations, and algebra, and focuses on how students' mathematical ideas and the use of mathematical models develop over time from informal to more formal (Clements & Sarama, 2004; Gravemeijer, 2004; Simon & Tzur, 2004). Three versions of the 45-hour course were developed

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based on educators' grade level: Kindergarten – third grade (K-3), fourth – eighth grade (4-8), and sixth – twelfth grade (6- 12). Although there is broad overlap in content, each course spends more time on a specific grade-band appropriate topic: K-3 on early number, 4-8 on rational number, and 6-12 on algebraic modeling.

The course is built upon social and cognitive learning theories, which hold that students need to learn mathematics by constructing knowledge through meaningful classroom activities and discussions. The teacher's role in the classroom is to facilitate student learning through the meaningful selection of mathematical tasks and high-quality classroom discussion designed to build connections between students' informal knowledge and the formal knowledge of mathematics that has developed over time (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fosnot & Dolk, 2002; Gravemeijer & van Galen, 2003; Hiebert, 1997).

Based on this framework, participants in the MTI course are presented with mathematical problems or situations for which there are multiple solution paths and ways to model the mathematics. The MTI instructor facilitates discussion around these solutions and models, pressing participants to make connections between the solutions and developing a progression from informal to formal methods. Throughout the course, the MTI instructor models five instructional practices that build mathematical understanding: (1) taking student's ideas seriously, (2) pressing students conceptually, (3) encouraging multiple strategies and models, (4) addressing misconceptions, and (5) focusing on the structure of the mathematics (Brendefur et al., 2013). The goals of the course are to model instruction that incorporates the five instructional practices, deepen teachers' mathematics knowledge, and influence teachers' beliefs as an initial effort to improve classroom instruction and student learning outcomes. The role of each practice in classroom instruction and its manifestation in the MTI course are discussed in detail below.

Taking students' ideas seriously. When students solve an unfamiliar yet meaningful math problem, they draw on their prior knowledge and experience. Their solution strategies and notations may seem inefficient or informal to an observer, but by eliciting and valuing students' initial solution strategies, teachers can connect student thinking to more efficient and abstract methods (Freudenthal, 1973, 1991; Gravemeijer & van Galen, 2003; Treffers, 1987). In addition, through such problem solving activities, students gain experience in constructing their own knowledge, rather than absorbing it from a teacher or textbook (Carpenter & Lehrer, 1999). When students' ways of knowing are not valued, they may develop the notion that there is a dictated acceptable solution path. In the MTI course, problems are sometimes presented with the caveat that participants solve them without formal algorithms so educators experience the act of solving a problem based on experiences and conceptual understanding, rather than memorized methods. It is important to emphasize that the MTI course does not suggest that formal algorithms such as long division are "bad" or should not be taught. Instead, the course analyzes the underlying mathematics of formal algorithms and their place in a wider progression of computational models and strategies.

Pressing Students Conceptually. Once students have had the chance to work on their own solution methods, teachers press them to connect and compare between methods, generalize to new situations, and relate to formal mathematical terms and conventions. It is through this process of connection and generalization that students move from their own informal methods to more formal and efficient strategies (Carpenter & Lehrer, 1999; Gravemeijer & van Galen, 2003). During the MTI course, participants experience this practice by identifying differences and similarities between select participants' solutions during facilitated discussions. In addition, instructors press participants to utilize different solution methods in new problems, identifying their respective benefits and limitations.

Encourage multiple strategies and models. Mathematical strategies are the methods by which we solve problems and models are the representations of those strategies. When students generate, evaluate, and utilize different mathematical strategies and models, they recognize there are many ways to solve problems and represent solutions. In addition, different strategies and models highlight different aspects of the mathematics and thus examining the same problem through different lenses deepens students' overall understanding of the topic. In the MTI course, multiple strategies and models are introduced both by the participants themselves and by the instructor who may use specific models to reflect a participants' mathematical thinking. In addition, participants complete challenging tasks that can be represented and solved in many ways, from drawing pictures to using formal algebra. Finding the connections between different representations deepens participants' content knowledge while giving validity to alternative (non-formal) methods.

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Address misconceptions. Making mistakes and learning from them is an integral part of doing mathematics at any level. But mistakes often recur even after teachers demonstrate a correct procedure because they stem from deeper mathematical misconceptions. By being aware of why and how misconceptions develop and taking the time to address misconceptions through models and discussion, teachers can move students to a deeper level of understanding that precludes such mistakes. Additionally, mistakes can be opportunities for students to engage in justification, evaluation, and inquiry (Borasi, 1987). In the MTI course, common student misconceptions, their genesis, and methods of prevention and redress are discussed frequently.

Focus on the structure of mathematics. Many teachers and their students see mathematics as a series of procedures and definitions that build in complexity throughout the K-12 curriculum. But certain fundamental ideas or "structural components" appear continually throughout mathematics, whether one is looking at 2nd or 11th grade. Understanding these structural components can help teachers tie different concepts together both within and across grade levels, rather than teaching topics in isolation. When instruction does not focus on the structure of mathematics, students often rely on memorized tricks or formulas and have difficulty solving complex problems or applying mathematics to new situations. During the MTI course, teachers are introduced to the structural components for Number and Algebra and are asked to identify and connect these foundational ideas in various problems and topics.

Finally, the MTI course was structured in a manner consistent with features of meaningful and effective professional development as identified in the literature (Desimone, 2011). Namely (1) a focus on content knowledge, (2) active teacher engagement in mathematical tasks, (3) integration with broader mandates – in this case the Idaho math initiative and the shift to the CCSS, (4) duration of a minimum of 20 hours – in this case 45 hours, and (5) collective participation of educators from the same grade level, school, and district.

Framework for Evaluating Professional Development

While professional development is widely accepted as a means of influencing teachers' instructional practices and student achievement, large scale examination of the influence of professional development is often not conducted. Clarke and Hollingsworth (2002) developed a framework for teacher professional growth that acknowledges the iterative and idiosyncratic nature of teacher learning across multiple domains. However, quantitative operationalization of such a comprehensive, multidirectional framework to evaluate professional development on a large scale is rarely found in the literature (Goldsmith, Doerr, & Lewis, 2014). Desimone (2009) provided a simpler, unidirectional framework for evaluating professional development that focuses on establishing clear links between the following: (1) the professional development experience, (2) changes in practitioners' knowledge and beliefs, (3) changes in instructional practice, and finally (4) improvements in student achievement.

While Desimone's (2009) model for evaluating teachers' professional growth may not be as comprehensive as Clarke and Hollingsworth (2002), the reality of large-scale professional development that occurs as a result of a policy mandate is that money, time, and resources are typically devoted to delivery. Evaluating the influence of professional development – even from just a unidirectional perspective - on practice and student achievement is difficult due to the cost associated with measuring practice at scale through observations, privacy issues associated with student achievement data, and complications related to numerous factors influencing instructional practice and student achievement. In addition, in the case of the MTI course - given its structure, short time frame and limited resources - the focus was on influencing participants' knowledge and beliefs related specifically to the nature of mathematics and its instruction, and building a common knowledge base across the state from which deeper, more meaningful conversations could occur. Examining factors proximal to the professional development, such as knowledge and beliefs changes, reflect the intention of the MTI course and demonstrate whether this type of whole-group, facilitator directed professional development – often criticized in the literature (Flint, Zisook, & Fisher, 2011) – can be utilized initially to create statewide shifts in knowledge and beliefs related to the nature of mathematics and mathematics instruction that can be further capitalized on at the district and school levels.

Implementation of the CCSS and assessments from the associated consortiums, have increased the rigor of the content and changed the focus of instruction to meaningful application of mathematics (Schmidt, 2012). In order to implement these changes in content and instruction, teachers must be assisted in further developing their mathematics knowledge and shifting their beliefs towards more practice based applications of mathematics. Therefore, the purpose of our analyses is to examine the relationship between the MTI professional development course experience and increases

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in teacher knowledge and changes in their beliefs related to the nature of mathematics and mathematics instruction. The following section briefly articulates the constructs of teachers' mathematics knowledge and beliefs, and their importance related to classroom instructional practice and student achievement.

Mathematical Knowledge for Teaching

Teachers' knowledge, as a research construct, is an extremely broad and varied area of examination (Ben-Peretz, 2011). The mathematics education literature tends to focus on a content knowledge based perspective due to research and information indicating significant issues in this area (Ball, Lubienski, & Mewborn, 2001; CBMS, 2012; Ma, 1999; Schmidt, 2007). This emphasis is not intended to diminish focus on other perspectives but to highlight the need for increased attention to this area. The MTI professional development model responds to this call by focusing on building teachers' knowledge as related to the nature of mathematics, and student learning and instruction in mathematics.

In order to evaluate and make effective decisions regarding the content of professional development, it is important to examine and understand the types of teacher knowledge necessary to teach mathematics effectively. Ball and colleagues at the University of Michigan developed a framework for the specific knowledge needed to effectively instruct in mathematics (Ball, Thames, & Phelps, 2008). The Mathematical Knowledge for Teaching (MKT) framework distinguishes between the domains of subject matter knowledge and pedagogical content knowledge. Within these two domains, additional types of knowledge necessary for effective mathematics instruction are defined. For example, within the subject matter knowledge domain, specialized content knowledge extends beyond common content knowledge (e.g., the knowledge held by engineers) and is specific to the act of teaching. It involves the ability to analyze errors, evaluate and use multiple representations, and provide conceptually and procedurally based explanations of solutions. An example within the pedagogical content knowledge domain is the construct of knowledge of content and students. It relates to the knowledge that allows teachers to predict possible student solution strategies and misconceptions, interpret students' ideas, and predict how students will react to particular tasks.

The MKT framework, while quite intricate, indicates the importance of attending to multiple sub-constructs in the design of professional development activities to improve teachers' mathematical knowledge for teaching. The typical approach of improving teachers' common content knowledge is not sufficient to bring about the necessary teacher understanding and changes in beliefs and instructional practice called for by mathematics policy initiatives and reform advocates. Careful attention must be paid to the development of different types of knowledge through professional development activities.

Measures of MKT have been developed to evaluate the relationship between teachers' knowledge and other variables of interest (Learning Mathematics For Teaching, 2005). Utilizing the LMT measures, several studies have demonstrated the link between teachers' MKT and effective classroom practice (Charalambous, 2010; Hill, Ball, Blunk, Goffney, & Rowan, 2007; Hill et al., 2008), and student achievement (Hill et al., 2007; Hill, Rowan, & Ball, 2005) in mathematics. The strong relationship between knowledge, instructional practice and student achievement highlight the importance of professional development activities supporting increases in these variables. However, numerous studies have also indicated the extreme difficulty in changing teachers' MKT, particularly when measured by the LMT items and assessments. Moyer-Packenham and Westenskow (2012) in an analysis of 22 MSP projects' participants content knowledge following mathematics professional development (n=10,331), found only 31% (n=3220) of the studies utilized the LMT measures, and of those only 24% (n=760) found a significant change in teachers' MKT. As opposed to the 69% (n=7111) of studies that utilized other measures of content knowledge (primarily project developed), and found a significant change for 81% (n=5719) of the studies. Similarly Phelps, Jones, Kisa, and Liu (2013) reported that for users of the online LMT administration system, Teacher Knowledge Assessment System (TKAS), the average change from pre to post across assessments was .15 standard deviations. However, this was accompanied by the caveat that the mathematics professional development intervention that occurred was extremely varied. The research on MKT points to its importance in instructional practice and student achievement but also to the difficulty in effecting meaningful change through professional development.

Teacher Self-Efficacy

The construct of teacher self-efficacy was born out of the results from the RAND corporation survey based on the strong correlation between student achievement and the following two items representing vastly different views of teachers about their role: "when it comes right down to it, a teacher really can't do much (because) most of a student's

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motivation and performance depends on his or her home environment" versus "if I try really hard, I can get through to even the most difficult or unmotivated students" (Armor, Sumner, & Thompson, 1976). Bandura (1997) provided seminal research on understanding the types and sources of self- efficacy and its potential influence on teachers' behaviors.

For the purposes of this article, self-efficacy is examined from the perspective of teachers' perception of their ability to effectively influence student learning. Teachers are the key players in implementing reform-oriented mathematics instruction at the classroom level. Research indicates that teachers' feelings of self-efficacy greatly influence their implementation of and persistence in trying reform-oriented instructional practices (Gabriele & Joram, 2007; Gibson & Dembo, 1984; Guskey, 1988). In addition, teachers' levels of self-efficacy have been linked to student achievement in that teachers who feel they can make significant changes in students' knowledge tend to produce students who do well on outcome measures (Armor et al., 1976; Goddard, Hoy, & Hoy, 2000; Palardy & Rumberger, 2008; Tschannen-Moran, Hoy, & Hoy, 1998). Therefore, it is critical that professional development intentionally influences and improves teachers' self-efficacy in terms of the implementation of reform oriented instructional practices.

Teacher Beliefs

Ernest (1989) examined teachers' beliefs about mathematics in the domains of student learning, classroom instruction and the nature of mathematics. Specifically, three common belief structures about the nature of mathematics are described, (1) problem solving – mathematics as a dynamic and continually changing field of study, (2) Platonist – mathematics as a static, yet interconnected body of knowledge, and (3) instrumentalist – mathematics as a useful collection of rules, procedures, and algorithms. These multiple beliefs structures, in combination with other factors, inform how teachers choose to enact mathematics instruction in their classroom (Bray, 2011; Sherman, 1995; Swan, 2006; Turner, Warzon, & Christensen, 2011; Wilkins, 2008). Teachers' beliefs heavily influence teachers' adoption of new instructional practices, and the depth of their implementation and persistence (Gabriele & Joram, 2007; Prawat & Jennings, 1997; Stipek, Givvin, Salmon, & MacGyvers, 2001), which in turn affects student learning and achievement outcome (Fennema et al., 1996; Staub & Stern, 2002) . While teacher beliefs have been found to have a significant effect on instructional practice and student achievement, the persistence of teachers' education beliefs, and the difficulty of teacher change is well documented (Pajares, 1992; Phillipp, 2007; Richardson, 2001). Therefore, research analyzing the effect of professional development on deeply held teacher beliefs is needed.

Influence of Professional Development on Knowledge, Self-Efficacy and Beliefs

Numerous examples exist in the research literature citing professional developments' positive effect on teacher practice and/or improved student achievement (Boston & Smith, 2009; Cohen & Hill, 2000; Ferrini-Mundy, Burrill, & Schmidt, 2007; Kennedy, 1999; Yoon, Duncan, Wen-Yu Lee, Scarloss, & Shapley, 2007). In addition, several studies have directly examined the influence of professional development participation on knowledge (Fennema & Franke, 1992; Hill, 2011; Hill & Ball, 2004; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Swafford, Jones, & Thornton, 1997), self-efficacy (Gabriele & Joram, 2007; Heck, Banilower, Weiss, & Rosenberg, 2008; Turner et al., 2011), and teachers' beliefs related to mathematics (Fennema et al., 1996; Heck et al., 2008; Swan & Swain, 2010). While multiple professional development models exist that document changes in teacher knowledge and beliefs, these models have typically been implemented on a small to medium-size scale. In addition, the majority of them involve voluntary participation. In order for professional development mandates to be used as an effective policy tool to bring about change in mathematics education, it must be established that these activities can bring about significant changes in teachers knowledge and beliefs.

Research Questions

Did the MTI course significantly influence participants' knowledge, self-efficacy, and beliefs? Does course level or timing of course participation (i.e., cohort) influence the relationship?

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Method

Setting

The MTI courses were typically located at K-12 school buildings across the state of Idaho. The courses were held in over 200 different schools across the entire state during the first three years of the project. The MTI course duration is 45 hours. During summers, the courses were typically held over 5 consecutive days. During the school year, the courses were typically conducted after school with one to two full-day Saturday courses and typically spanned six to ten weeks. The course participants were divided into cohorts, roughly corresponding to the project year; Cohort 1 summer 2008-spring 2009, Cohort 2 summer 2009-spring 2010, and Cohort 3 summer 2010-spring 2011.

Participants and Design

The Mathematical Thinking for Instruction (MTI) course is required by September of 2014 for all individuals to maintain the following Idaho teaching certificates: Early Childhood, Standard Elementary, Standard Secondary with a Basic or Standard Math Endorsement, Exceptional Child, and Administrator. The 3,933 participants in this study completed the MTI professional development in the first three years of the project (Cohorts 1-3). They came from across the state, representing a mix of urban, suburban, and rural districts. While all individuals in the state who fit the certification criteria are mandated to take the course, there is the possibility that the make-up of the participants who registered to take the course during the first three years of the project may not have been representative of the entire population required to take the course, based on potential differences in motivation and interest level. In addition, while many of the course participants were teachers actively instructing in mathematics, the sample population also included (1) administrators, such as principals, district office personnel, and superintendents, (2) special education teachers who may or may not teach mathematics on a regular basis, and (3) all K-8 certified personnel, which included many middle schools teachers of non-mathematics content.

Participants completed one of three MTI courses (i.e., K – 3, 4 – 8, or 6 – 12). It was recommended, but not required, that participants register for the course matching their level of instruction. Participants each completed measures of knowledge, self-efficacy, and beliefs representing these variables before and after participation in the MTI course. Thus, we had a 3 (Cohort: 1 versus 2 versus 3) x 3 (Course: K – 3 versus 4 – 8 versus 6 – 12) x 2 (Time: before versus after) design. All participants had both before and after data. The breakdown of participants by cohort and course is presented in Figure 1.

Insert Figure 1 here

Outcome Measures

Three knowledge inventories were created using items developed by the Learning Mathematics for Teaching (LMT) project at the University of Michigan (Learning Mathematics For Teaching, 2008). The knowledge items correspond to the construct of Mathematical Knowledge for Teaching (MKT) defined earlier in this paper. The specific LMT items selected for the content and pedagogical knowledge inventories align to the typical instructional content related to number, operations and algebra found at the K-3, 4-8, and 6-12 grade levels, creating specific knowledge inventories for each course. However, while the content of the inventories and the MTI courses are similar in terms of their focus on number, operations, and algebra, the inventories were not created to align to the particular content of the MTI courses. Several topics found on the inventories are not specifically addressed in the course materials.

A paper copy of the knowledge inventory specific to the course level was administered by project staff to participants before and after the professional development course to determine changes in content knowledge. The same inventory was utilized for pre/post administration. The time between pre/post administrations ranged from one to fifteen weeks. There was very little resistance from participants regarding completing the measure. Beginning with the summer of 2011 (end of cohort II), the inventories and course evaluation were conducted online instead of in hardcopy form. This

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reduced the completion rate to roughly 79%. The K-3 inventory has 31 items (α = .85), the 4-8 inventory has 27 items (α = .80), and the 6-12 inventory has 29 items (α = .88). A requirement for use of the knowledge items by the LMT project is that the total items correct scores are converted into z-scores for analysis and reporting. A pre-inventory MTI score based on the total items correct was calculated for each participant in each course sample as a standardized variable (i.e., *M*=0, *SD*=1). The post-MKT measure was calculated for each participant in each course sample as a standardized variable using the pre-inventory mean and standard deviation for each course and cohort [(total correct on the post-inventory – mean total correct of the pre-inventory)/SD of the pre-inventory].

We also gathered data on teachers' self-efficacy and beliefs related to the nature of mathematics and student learning (scales provides in Appendix A). These instruments were designed by RMC Research and Math in the Middle project staff at the University of Nebraska, Lincoln in 2005. Items were developed or adapted from existing measures including the *Mosaic II Rand Teacher Survey for Eighth Grade Mathematics* (Rand, 2003), the *Survey of Classroom Practices in Middle School Mathematics* (WCER, undated), and the *TIMSS Teacher Questionnaire for Eighth Grade Mathematics* (IEA, 2003). These instruments used a retrospective pre- and post-test format to collect information about teacher confidence regarding their own knowledge in mathematics, their level of preparedness to teach mathematics using what was learned, and beliefs about the nature of mathematics and student learning. This is the recommended method of evaluating change in circumstances where participants' pre- intervention responses may be biased based on an overestimation or underestimation due to lack of knowledge or understanding regarding the area the intervention is designed to influence (Lam & Bengo, 2003). A paper copy of the course survey was administered by project staff following the conclusion of the professional development course to determine changes in teacher self-efficacy and beliefs based on teacher retrospective analysis.

Teachers' self-efficacy regarding their level of preparedness prior to and following course participation was measured using ten items rated on a three-point scale (1=Limited, 2=Well, 3=Very Well). These items assessed teachers' level of preparedness on topics such as, teaching classes to students of diverse abilities, providing a challenging curriculum to all students, and sequencing mathematics instruction to meet instructional goals. The scale based on these ten items was found to be reliable (α = .90). Scores for this scale were calculated by averaging across the ten items, resulting in self-efficacy scores for both before and after course participation (on the original metric from 1-3).

Teachers' beliefs regarding the nature of mathematics and student learning prior to and following course participation were measured using six items rated on a three-point scale (1=disagree, 2=neither agree nor disagree, 3=agree). These items assessed teachers' beliefs on topics related to the nature of mathematics and student learning (e.g., 'There are different ways to solve most mathematical problems' and 'All students can learn challenging content in mathematics'). Scores for this scale were calculated by averaging across the six items. This resulted in belief scores for both before and after course participation (on the original metric from 1-3).

Results

To address the research questions, we examined change in content knowledge, self- efficacy, and beliefs from before and after the MTI course. The analyses are organized by outcome measure.

<u>Content Knowledge</u>

Content knowledge was analyzed in a 3 (Cohort: 1 versus 2 versus 3) x 3 (Course: K – 3 versus 4 – 8 versus 6 – 12) x 2 (Time: before versus after) analysis of variance (ANOVA). There was a main effect for Cohort, $F(1, 3924) = 14.98$, $MSe = 1.49$, $p < .001$, partial eta squared = .005. There was also a main effect for Course, $F(1, 3924) = 5.96$, $MSe = 1.49$, $p = .003$, partial eta squared = .003. There was also a main effect for Time, $F(1, 3924) = 869.90$, $MSe = .33$, $p < .001$, partial eta squared = .18. These main effects were qualified by significant interactions.

The Cohort x Time interaction was significant, $F(2, 3924) = 9.05$, $MSe = .33$, $p < .001$, partial eta squared = .005; as was the Course x Time interaction, $F(2, 3924) = 52.82$, $MSe = .33$, $p < .001$, partial eta squared = .03. The Cohort x Course interaction was not significant, $F(4, 3924) = 2.04$, $MSe = .33$, $p = .09$; nor was the three-way interaction, $F(4, 3924) = 1.16$, $MSe = .33$, $p = .33$. To better understand the nature of the interactions, follow-up tests of simple effects were conducted. To disentangle the Cohort x Time interaction, we conducted two one-way ANOVAs comparing the three cohorts on knowledge before and then after the MTI course. As seen in Figure 2a, the three cohorts differed in knowledge before the MTI course, $F(2, 3930) = 11.47$, $MSe = .99$, $p < .001$. The cohorts also differed in knowledge after the MTI course, $F(2, 3930) = 5.32$, $MSe = .85$, $p = .005$. More important, knowledge increased significantly from before to after for all three cohorts [Cohort 1: $t(935) = 24.20$, $p < .001$; Cohort 2: $t(1382) = 22.95$, $p < .001$; Cohort 3: $t(1613) = 30.91$, $p < .001$].

Insert Figure 2 here

To disentangle the Course x Time interaction, we conducted two one-way ANOVAs comparing the three cohorts on knowledge before and then after the MTI course. As seen in Figure 2b, the three courses did not differ in knowledge before the MTI course, $F(2, 3930) < 1$. By contrast, the courses differed in knowledge after the MTI course, $F(2, 3930) = 47.05$, $MSe = .83$, $p < .001$. More important, knowledge increased significantly from before to after for all three courses [K – 3 Course: $t(2000) = 37.49$, $p < .001$; 4 – 8 Course: $t(1455) = 25.02$, $p < .001$; 6 – 12 Course: $t(475) = 9.70$, $p < .001$]. Thus, although knowledge increased significantly for all three courses, it increased more for the K – 3 course, than for 4 – 8 course, which in turn increased more than for the 6 – 12 course.

Insert Figure 3 here

Self-Efficacy

Self-efficacy was analyzed in a 3 (Cohort: 1 versus 2 versus 3) x 3 (Course: K – 3 versus 4 – 8 versus 6 – 12) x 2 (Time: before versus after) ANOVA. The was a main effect for Cohort, $F(1, 3610) = 3.96$, $MSe = .32$, $p = .02$, partial eta squared = .002. There was also a main effect for Course, $F(1, 3610) = 25.85$, $MSe = .32$, $p < .001$, partial eta squared = .01. There was also a main effect for Time, $F(1, 3610) = 3408.77$, $MSe = .07$, $p < .001$, partial eta squared = .49. These main effects were qualified by significant interactions.

The Cohort x Time interaction was significant, $F(2, 3610) = 10.72$, $MSe = .07$, $p < .001$, partial eta squared = .006; as was the Course x Time interaction, $F(2, 3610) = 8.02$, $MSe = .07$, $p < .001$, partial eta squared = .004. The Cohort x Course interaction was not significant, $F(4, 3610) < 1$; however, the three-way interaction was significant, $F(4, 3610) = 5.28$ MSe = .07, $p < .001$, partial eta squared = .006. To better understand the nature of the interactions, follow-up tests of simple effects were conducted that paralleled those done with content knowledge.

To disentangle the Cohort x Time interaction, we conducted two one-way ANOVAs comparing the three cohorts on self-efficacy before and then after the MTI course. As seen in Figure 3a, the three cohorts differed in self-efficacy before the MTI course, $F(2, 3616) = 9.93$, $MSe = .22$, $p < .001$; with Cohort 3 having significantly lower self-efficacy than the other cohorts. The cohorts did not differ in self-efficacy after the MTI course, $F(2, 3616) < 1$. More important, self-efficacy increased significantly from before to after for all three cohorts [Cohort 1: $t(563) = 31.23$, $p < .001$; Cohort 2: $t(1441) = 52.15$, $p < .001$; Cohort 3: $t(1612) = 59.67$, $p < .001$].

Insert Figure 4 here

To disentangle the Course x Time interaction, we conducted two one-way ANOVAs comparing the three cohorts on self-efficacy before and then after the MTI course. As seen in Figure 3b, the three courses differed before the MTI course, $F(2, 3616) = 10.20$, $MSe = .22$, $p < .001$; with the K – 3 course having higher self-efficacy than the 4 – 8 course, which in turn had higher self-efficacy than the 6 – 12 course. The courses also differed in knowledge after the

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MTI course, $F(2, 3616) = 50.63$, $MSe = .17$, $p < .001$; with the same pattern of course differences as existed before the courses. More important, self-efficacy increased significantly from before to after for all three courses [K – 3 Course: $t(1868) = 63.47$, $p < .001$; 4 – 8 Course: $t(1311) = 51.32$, $p < .001$; 6 – 12 Course: $t(437) = 24.96$, $p < .001$].

Insert Figure 5 here

<u>Beliefs</u>

Average total score on the beliefs scale was analyzed in a 3 (Cohort: 1 versus 2 versus 3) x 3 (Course: K – 3 versus 4 – 8 versus 6 – 12) x 2 (Time: before versus after) ANOVA. The was a main effect for Cohort, $F(2, 3629) = 24.33$, $MSe = .10$, $p < .001$, partial eta squared = .01. The main effect for Course was not significant, $F(2, 3629) = 2.11$, $MSe = .10$, $p = .12$. There was a main effect for Time, $F(1, 3629) = 2220.23$, $MSe = .08$, $p < .001$, partial eta squared = .38. These main effects were qualified by significant interactions.

The Cohort x Time interaction was significant, $F(2, 3629) = 4.16$, $MSe = .08$, $p = .02$, partial eta squared = .002; as was the Course x Time interaction, $F(2, 3629) = 19.04$, $MSe = .08$, $p < .001$, partial eta squared = .01; as was the Cohort x Course interaction, $F(4, 3629)$ 2.66, $MSe = .10$, $p = .03$, partial eta squared = .003. The three-way interaction was NOT significant, $F(4, 3629) = 1.94$ $MSe = .08$, $p = .10$. To better understand the nature of the interactions, follow-up tests of simple effects were conducted that paralleled those done with content knowledge and self-efficacy.

To disentangle the Cohort x Time interaction, we conducted two one-way ANOVAs comparing the three cohorts on belief scores before and then after the MTI course. As seen in Figure 4a, the three cohorts differed in beliefs before the MTI course, $F(2, 3639) = 13.18$, $MSe = .12$, $p < .001$; with Cohort 3 having significantly lower belief scores than the other cohorts. The cohorts also differed in belief scores after the MTI course, $F(2, 3639) = 22.13$, $MSe = .06$, $p < .001$; belief scores were greater for Cohort 1, than Cohort 2, which had scores greater than Cohort 3. More important, belief scores increased significantly from before to after for all three cohorts [Cohort 1: $t(570) = 28.51$, $p < .001$; Cohort 2: $t(1446) = 42.35$, $p < .001$; Cohort 3: $t(1619) = 47.16$, $p < .001$].

Insert Figure 6 here

To disentangle the Course x Time interaction, we conducted two one-way ANOVAs comparing the three cohorts on belief scores before and then after the MTI course. As seen in Figure 4b, the three courses did not differ before the MTI course, $F(2, 3639) = 1.53$, $MSe = .12$, $p = .22$. By contrast, the courses differed in belief scores after the MTI course, $F(2, 3639) = 35.15$, $MSe = .06$, $p < .001$; belief scores were greater for the K – 3 Course than for 4 – 8 Course, which was greater than for the 6 – 12 Course. More important, belief scores increased significantly from before to after for all three courses [K – 3 Course: $t(1868) = 51.94$, $p < .001$; 4 – 8 Course: $t(1322) = 41.14$, $p < .001$; 6 – 12 Course: $t(445) = 22.08$, $p < .001$].

Insert Figure 7 here

Discussion

The goal of the MTI course from the perspective of policy makers at the state level was to influence teachers' knowledge and beliefs related to mathematics and its instruction, which could in turn - if capitalized upon at the school and district levels - improve classroom practice and student outcomes in mathematics. According to Desimone (2009), in order to connect improved student outcomes to professional development situations, clear links must be established between the following elements: (1) professional development experiences, (2) changes in teachers' knowledge, beliefs and attitudes, (3) changes in instructional practices, and (4) improved student outcomes. This research evaluates the link between (1) and (2) with respect to the success of MTI course in influencing teachers' mathematical knowledge and their beliefs about mathematics and its instruction.

Influence of MTI on MKT, Self-Efficacy, and Beliefs

Please note that we readily acknowledge the inherent difficulties in measuring and quantifying teachers' knowledge and beliefs related to mathematics and instruction. By operationalizing these constructs as a quantifiable variable, we lose important information about the multidimensional and interactive nature of these constructs at the individual level. However, there is also a dearth of quantifiable evidence of the effectiveness of mathematics professional development offered on a large scale (Goldsmith et al., 2014). While we lose detail in quantifying these variables, we begin to gain evidence of the potential impact of high quality professional development offered on a large scale focused on teaching mathematics for understanding, an important area of research given current policy initiatives (Burkhardt, Schoenfeld, Abedi, Hess, & Thurlow, 2012; NGA & CCSSO, 2011; Schmidt, 2012).

Our results provide tentative evidence of the MTI course influence on improvements in participants' mathematics knowledge and self-efficacy, and in influencing their beliefs. While a pre/posttest research design limits the extent to which a substantial claim can be made, the consistent results across courses and cohorts provides initial indication for course effectiveness in influencing these variables. On average, participants' MKT (as measured by LMT assessment items) increased by .59 standard deviations across all cohorts and courses. Given that the LMT items/assessments for each course (i.e., K-3, 4-8, and 6-12) were aligned to the content typically taught in those particular grade bands, more so than the content of the MTI professional development, which spanned K-12 topics, this represents a significant change in knowledge. As discussed previously, Moyer-Packenham and Westenskow (2012) examined pre/posttest changes across multiple MSP projects who utilized LMT items in a very similar manner to our study, including using the same items from pre to post. They found roughly only 24% of the project participants' demonstrated significant gains from pre to post project when using the LMT items. This serves to highlight the difficulty in demonstrating significant changes from pre to post when utilizing the LMT items, even when the same measure is utilized pre and post. In addition, when examined in relationship to other math professional development projects discussed earlier (e.g., Phelps et al., 2013), it indicates a relatively large and consistent effect in comparison.

Changes in self-efficacy related to level of preparedness to teach mathematics and shifts in beliefs towards mathematics as a problem solving activity were both significant. For both cohort and course participants' retrospective analysis of their average level of preparedness prior to participation in the MTI course were all below the cutoff for the category of well-prepared. Following participation in the MTI course, teachers' average level of preparedness was well above the cutoff for the category of well-prepared. Lastly, participants' retrospective analysis of their change in their beliefs towards mathematics as a problem solving activity in which students need to be actively processing and establishing connections between topics indicated consistent significant shifts across all three cohorts and courses.

Our results differ from previous studies on mathematics professional development in several ways. Borko (2004) called for research on the scale-up of professional development. However, we could not find any studies that investigated the effectiveness of mathematics professional development at a statewide level across multiple locations with multiple course instructors. Bell, Wilson, Higgins, and McCoach (2010) investigated the effectiveness of the Developing Mathematical Ideas mathematics professional development program. Across 10 cohorts in multiple locations and with multiple facilitators, they found significant changes in teachers' knowledge (n=234). The MTI course demonstrated similar gains in participants' knowledge with a much larger sample size (n=3,933), however, an additional factor beyond the sheer number of participants differentiates this project from other similar professional development activities. Participants were mandated by state policy to participate in the MTI course. The majority of research in the literature related to inservice teacher professional development involves situations where participants self-select for participation. The consistent changes in outcomes for a policy-mandated inservice mathematics professional development course is an important contribution to our understanding of how policy, in conjunction with high- quality professional development, can be used to influence teachers' MKT, self-efficacy and beliefs.

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Influence of Cohort

Figures 2a, 3a, and 4a present the results of the influence of cohort on knowledge, self- efficacy, and beliefs. The results indicate that timing of course participation, as measured by cohort, was significantly different between cohorts with Cohort 3 consistently lower than cohorts 1 and 2 across all three variables. These results would indicate that by the third year of the project (i.e., Cohort 3), the participants at the outset had consistently lower MKT, self-efficacy and belief scores. One potential explanation for this difference between cohorts is related to participants' initial level of knowledge and interest, with those having higher levels of knowledge and interest in mathematics professional development self-selecting to participate in the course earlier than others. However, the gains are similar across the cohorts, with Cohort 3 as high if not higher than Cohorts 1 and 2 across all three variables. This provides initial evidence that regardless of participants' beginning knowledge and/or motivation level, the MTI course consistently increased their knowledge and self-efficacy, and significantly influenced their beliefs towards a more problem-based and student-oriented perspective. The tentative evidence regarding the ability of a mandated professional development experience to consistently influence these variables across individuals with varying levels of knowledge and motivation has important policy implications.

Influence of Course

Figures 2b, 3b, and 4b present the results of the influence of course on the variables of knowledge, self-efficacy, and beliefs. Figure 2b presents the change in knowledge by course level. Each course had different items on their knowledge assessment. However, all knowledge scores were standardized within each course in order to examine the gains between courses. The results indicate that K-3 participants had the largest gains in knowledge, and 6-12 participants had the smallest gains in knowledge on their respective assessments. Figures 3b and 4b present the change in self-efficacy and beliefs by course level. The relationship between gain in knowledge and course level holds true for changes in belief and self-efficacy. They indicate the MTI course, while significantly influencing all participants' self-efficacy and beliefs, is differentially effective in its ability to influence these variables for K-3 versus 4-8 versus 6-12 participants. It is difficult to parse out the potential factors creating this difference. In the case of knowledge, it may be related to the participants themselves, with elementary teachers possessing lower knowledge levels initially and therefore having more room for gains. This would be consistent with the literature on elementary teachers knowledge (Ma, 1999). It may also be that the variables of knowledge, self-efficacy and beliefs are more difficult to influence in secondary mathematics teachers. However, it could also be a function of the course materials. The results may indicate that the professional development framework is more influential with elementary and middle school teachers. Additional research needs to be conducted in this area in order to parse out the potential reasons for these results.

Limitations

We recognize that while these findings - at this scale - are not commonly found in the research literature there are limitations on both the interpretation and generalizability of our results based on research design, and the difficulty in measuring and quantifying a multi-faceted and iterative process such as teacher learning. Goldsmith et al. (2014) highlight in their review of 106 articles that attempts to quantitatively examine mathematics teachers' learning are limited in number (i.e., 18 or 106 articles reviews) and vary significantly how these constructs are described and measured. The operationalization of constructs (e.g., teachers' beliefs related to mathematics instruction) is extremely difficult due to the varied and nuance perspectives each individual holds related to these ideas and the impact these changes can have based on individual and context differences. Lastly, while the literature has linked the variables we examined to classroom instruction and student learning, our findings need to be further examined (1) over time, (2) in relation to current policy initiatives, such as implementation of the CCSS, (3) when situated in practice, and (4) in relationship to student learning.

Implications

These findings have several implications for teacher professional development, policy, and research. First, the Developing Mathematical Thinking framework utilized throughout the MTI course, (1) taking student's ideas seriously, (2) pressing student's conceptually, (3) encouraging multiple strategies and models, (4) addressing misconceptions, and (5) focusing on the structure of the mathematics, appears useful as a professional development course design model, particularly at the K-8 level. In particular, the consistent use of the framework actively engaged participants in the type of learning experience we wanted them to utilize in their own classrooms and appears to have assisted in the shifts in participants' knowledge, self-efficacy, and beliefs.

Second, in recent years large-scale, facilitator directed professional development has come under fire as being ineffective (Flint et al., 2011). The professional development literature has moved to focus on job-embedded professional development as a more effective means of changing teachers' behavior (Avalos, 2011; McLymont & da Costa, 1998; West & Staub, 2003). However, there is a significant cost differential between these two types of professional development and the ability to scale-up high-quality, job-embedded professional development is seriously limited by resources, such as time, money, and lack of skilled facilitators. While we are not making the claim that facilitator-directed professional development is superior to job- embedded, our results do indicate its usefulness as one of many tools in influencing teachers' knowledge, beliefs, and self-efficacy. We claim that a statewide policy initiative, such as the MTI course, has the potential to provide a common knowledge base and perspective from which to implement educational reforms. For example, in Idaho, the information and knowledge gained from the MTI course now serves as a commonality in conversations around the implementation of the content and practice standards in the CCSS.

Agency is considered an important aspect of influencing teachers change in practice. Our findings indicate similar changes across cohorts, who potentially may have had differences in their feelings of agency based on the course participation mandate. Unfortunately, our data do not allow further investigation into this finding. However, one additional potentially influencing factor was the adoption of the Common Core State Standards for Mathematics by the Idaho legislature in the middle of cohort three. The cohesiveness between the MTI course and the new standards may have assisted later cohorts with potentially lower feelings of agency in maintaining interest (Desimone, 2002; Schwille, 1988).

It should be noted that Idaho's population is relatively small, therefore, the ability to maintain the quality of the MTI course, while difficult, was manageable given the number of courses we had to offer each year. Scaling up this type of project in another state with a significantly larger population could present implementation and quality control issues.

Finally, the focus of this project and its findings brings up an interesting question. While most professional development projects focus on working intensively with a small sample of teachers to produce dramatic shifts in knowledge, self-efficacy and beliefs; for the MTI project, the focus was on slightly shifting these variables for the entire population. This begs the question, what should be the focus of our limited resources – large changes with a small sample or small but consistent changes with the entire population?

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Figure Captions

Figure 1. Sample size by Cohort and Course.
Figure 2a. Before and After Knowledge scores by Cohort.
Figure 2b. Before and After Knowledge scores by Course.
Figure 3a. Before (retrospective) and After Self-efficacy scores by Cohort.
Figure 3b. Before (retrospective) and After Self-efficacy scores by Course.
Figure 4a. Before (retrospective) and After Belief scores by Cohort.
Figure 4b. Before (retrospective) and After Belief scores by Course.

Figure 1.

	Cohort 1	Cohort 2	Cohort 3	Total
K - 3 Course	494	678	829	2001
4 - 8 Course	348	592	516	1456
6 - 12 Course	94	113	269	476
Total	936	1383	1614	3933

Figure 2a.

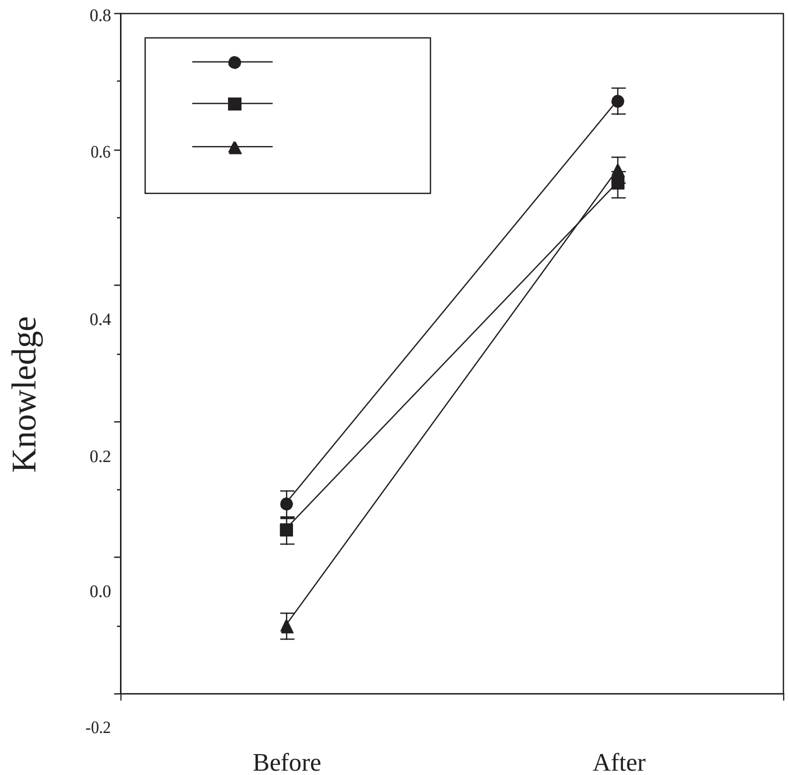


Figure 2b.

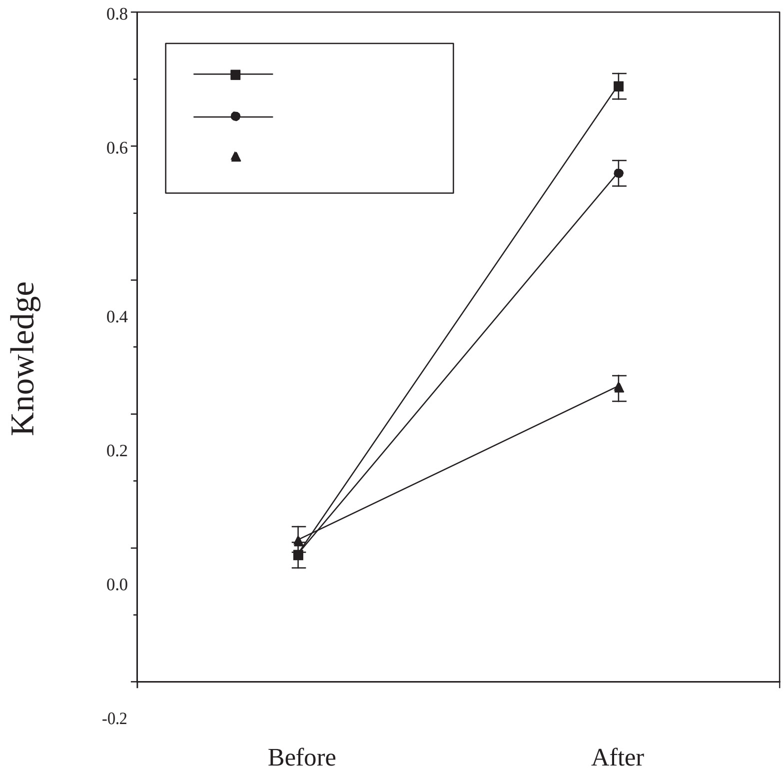


Figure 3a.

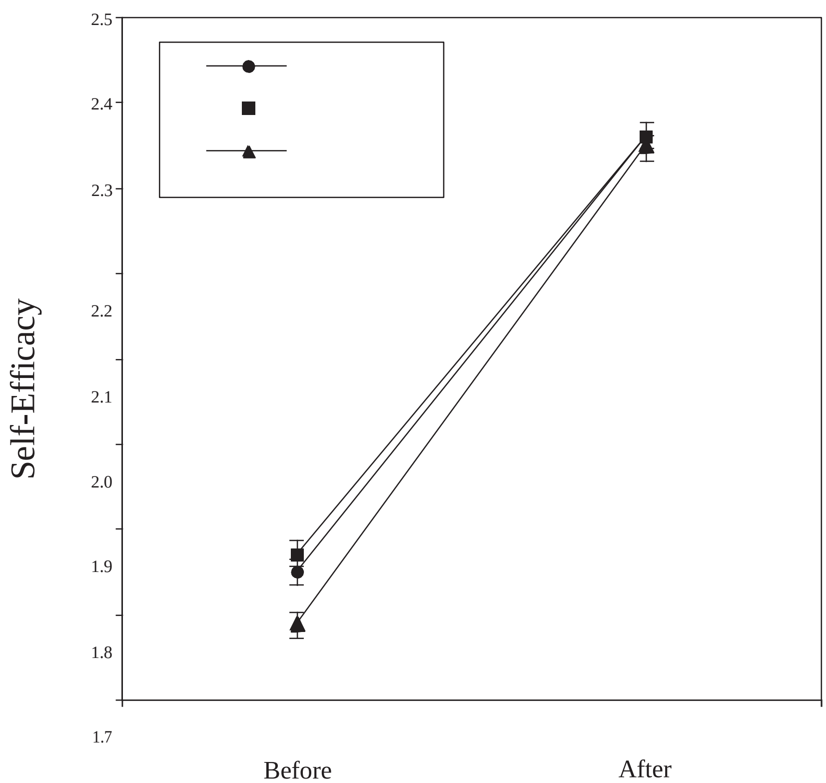


Figure 3b.

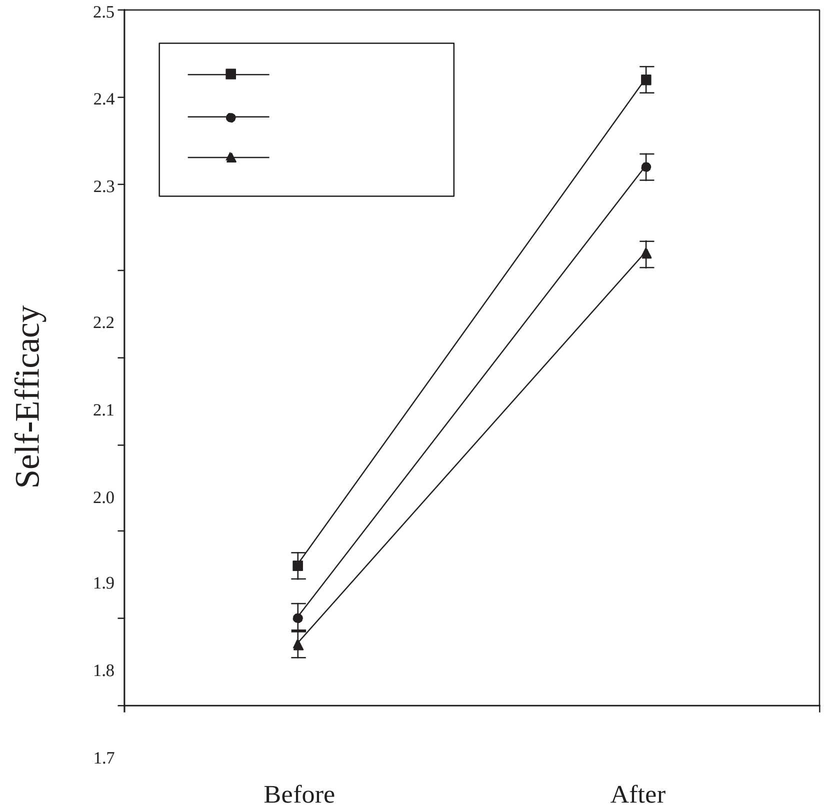


Figure 4a.

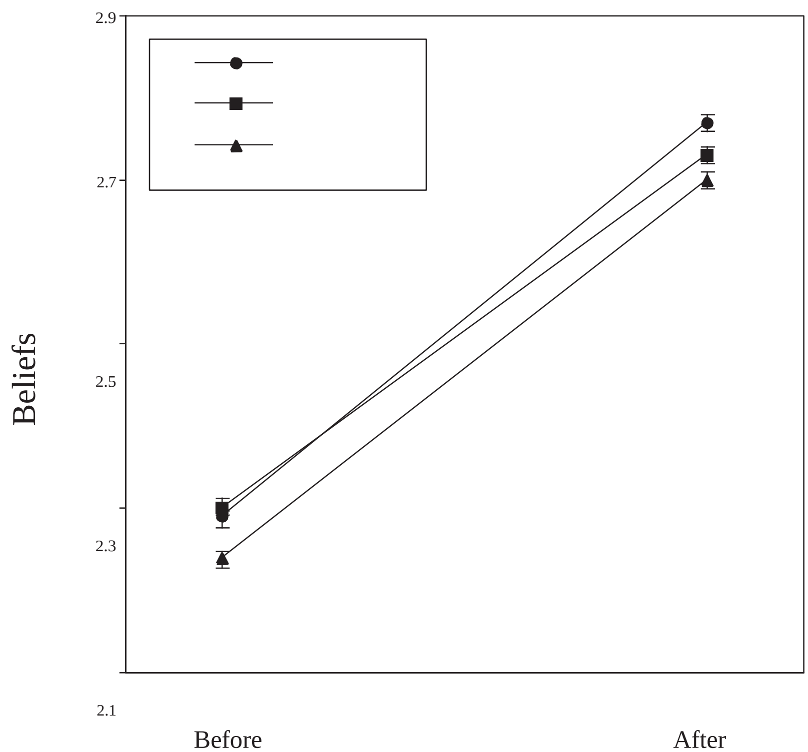
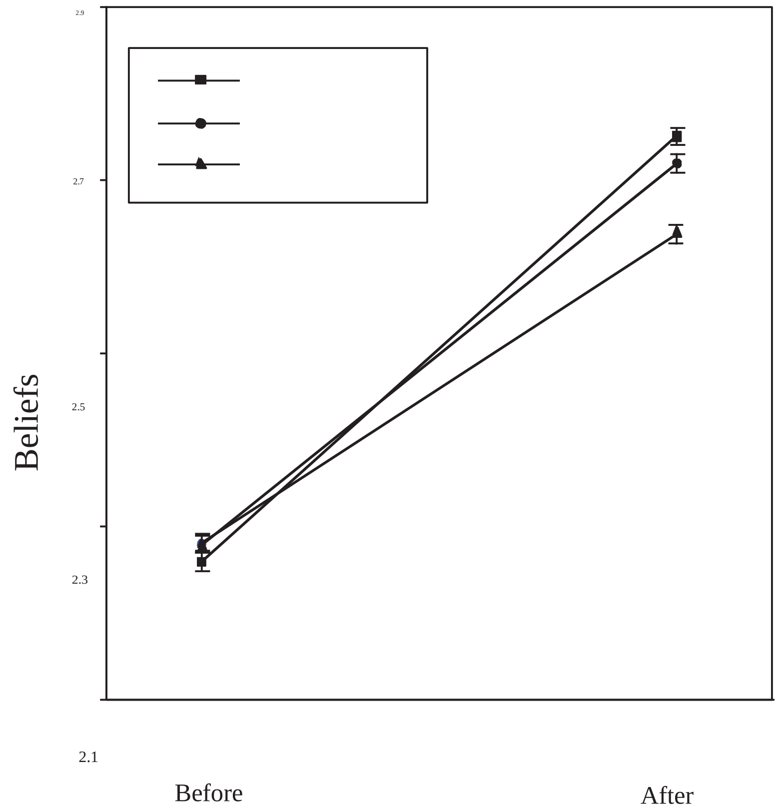


Figure 4b.



Appendix A:

Table 1: Survey items measuring self-efficacy related to level of preparedness. Before participation measured retrospectively following the conclusion of the course. Before and after participation measured on a three-point scale (1=Limited, 2=Well, 3=Very Well).

How would you rate your level of preparedness related to each of the following statements BEFORE PARTICIPATION[1] (*select one*) and AFTER PARTICIPATION (*select one*) in the MTI Institute?
a. Provide mathematics instruction that meets appropriate standards (district, state, national).
b. Teach mathematics with the use of manipulative materials.
c. Teach mathematics with the use of technology tools.
d. Sequence math instruction to meet instructional goals.
e. Select and/or adapt instructional materials to implement your written curriculum.
f. Make connections between math and other subject areas.
g. Teach classes for students with diverse abilities.
h. Teach math to students with diverse abilities and backgrounds.
i. Provide a challenging curriculum for all students you teach.
j. Use a variety of assessment strategies (including objective and open-ended formats).

[1] *Before participation measured retrospectively following the conclusion of the course.*

Table 2: Survey items measuring beliefs related to level of agreement with each statement. Before and after participation measured on a three-point scale (1=Disagree, 2=Neither Agree nor Disagree, 3=Agree).

How would you rate your level of agreement related to each of the following statements BEFORE PARTICIPATION[1] (select one) and AFTER PARTICIPATION (select one) of the MTI Institute and activities?
a. Mathematics should be learned as sets of algorithms or rules that cover all possibilities.
b. Solving mathematics problems often involves making conjectures, testing, and modifying findings.
c. There are different ways to solve most mathematical problems.
d. It is important for students to make connections between mathematics and other subject areas.
e. All students can learn challenging content in mathematics.
f. It is important for students to learn basic mathematics skills before solving problems.

[1] *Before participation measured retrospectively following the conclusion of the course.*