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CROSS-WELL RADAR II: COMPARISON AND EXPERIMENTAL VALIDATION OF MODELING CHANNEL TRANSFER FUNCTION

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ABTRACT

Close agreement between theory and experiment is critical for adequate understanding and implementation of the Cross-Well Radar (CWR, otherwise known as Cross-Borehole Ground Penetrating Radar) technique, mentioned in a previous paper by the authors. Comparison of experimental results to simulation using a half-space dyadic Green's function in the frequency domain requires development of transfer functions to transform the experimental data into a compatible form. A Channel Transfer Function (CTF) was developed to avoid having to model the transmitting and receiving characteristics of the antennas. The CTF considers electromagnetic (EM) wave propagation through the intervening media only (soil in this case), and hence corresponds to the simulation results that assume ideal sources and receivers. The CTF is based on assuming the transmitting antenna, soil, and receiving antenna as a cascade of three two-port microwave junctions between the input and output ports of the Vector Network Analyzer (VNA) used in the experimental measurements. Experimentally determined CTF results are then compared with computational model simulations for cases of relatively dry and saturated sandy soil backgrounds. The results demonstrate a reasonable agreement, supporting both the model and CTF formulation.

CE Database subject headings: Radar; Antennas; Dry Soil; Saturated Soil; Computerized Simulation; CWR; GPR; Cross-Tomography

INTRODUCTION

In order to better understand and implement Cross-Well Radar (CWR), both theoretical and experimental aspects of CWR should be investigated in parallel. While the experimental models provide a large experimental database, the theoretical simulations are used for comparison, assessment, and validation, or as a forward model for future image reconstruction. A good understanding of electromagnetic wave propagation in the soil subsurface is crucial for understanding and implementing CWR. There are several numerical computation methods capable of approximating three-dimensional (3D) wave propagation in the frequency domain in subsurface half-space (Habashy et al. 1993, Peterson 1992, Chew 1995, Weedon and Rappaport 1997, Rappaport et al. 1999, and Dasgupta et al. 1999). However, they are complicated to implement and prohibitively time and storage intensive. A fast forward model that works for subsurface sensing in real time is desired to incorporate into inversion techniques. Zhan et al. (2007) implemented a new model compatible with the CWR system. The model provides simulation of the background and scattered fields of the experimental results using a lossy half-space dyadic Green's function and a Born

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approximation technique in the frequency domain. Both experimental and theoretical results are in the form of frequency-response; however, the two are not compatible in their immediate forms.

Particular difficulties arise in the modeling of antennas that are used in the experimental study. Generally, theoretical simulations may not be able to model the sensor details completely. The Zhan et al. (2007) model simulates the frequency-response of the lossy medium due to an ideal source excitation. The source is assumed to be a vertically polarized electric dipole, which generates the radiated field characteristics of borehole antennas used in the CWR sensing. Real borehole antennas used in the experiment, however, are ferrite-bead-capped PVC-cased monopole antennas, which are far more complicated than ideal dipole antennas. The simulated ideal source radiates waves at every frequency into the surrounding soil with 100% coupling efficiency, while the laboratory borehole antennas radiate more efficiently at frequencies near a particular resonance (1.1 GHz in this experiment).

The objective of this paper is to develop a transfer function that can compare and validate the experimental data with the theoretical results, by accounting for –*and suppressing*— the coupling to the soil of the real antennas used in the experiment. A channel transfer function (CTF) is developed to transform the experimental results to a form directly comparable with the simulations using an ideal dipole. The CTF attempts to factor out the frequency-dependent transmission and reception characteristics of the borehole antennas, leaving only the response due to wave propagation through the soil.

Both experimental and theoretical simulations of this paper model the problem at a smaller scale than real fields within a controlled homogenous soil environment (uniformly dry or fully water-saturated) and with a uniform dielectric property contrast between the background and scatterer. Soils in the field are far more heterogeneous. Scaling down the size to the laboratory scale was accompanied with scaling up the frequency in a linearly proportional manner. This scaling method needs to be extensively studied and validated for future extension of the models to field applications. To extend the outcome to the real field, the scatterer size and the size and separation of the two antennas need to be scaled up to the field size. In contrast, soil grains do not change size. Therefore, the coupling and interaction effects between the soil, water and air that are observed at the laboratory scale may or may not scale up in the field, and may have different unforeseen effects that require extensive study.

THEORETICAL APPROACH

Half-Space Background Field via Dyadic Green's Function in Lossy Media

If the excitation **s**(**r'**, ω) is transmitted from a source at an angular frequency ω, the background electric field $\mathbf{E}_b(\mathbf{r}, \omega)$ (with $e^{-j\omega t}$ time dependence) will satisfy a vector Helmholtz wave equation (Chew 1995):

$$\nabla \times \nabla \times \mathbf{E}_{\mathrm{b}}(\mathbf{r},\omega) - k_b^{\,2}(\mathbf{r},\omega)\mathbf{E}_{\mathrm{b}}(\mathrm{r},\omega) = \mathbf{s}(\mathbf{r}',\omega) \tag{1}$$

where **r** is the position vector of the observation points, and **r'** is the position vector of the source points (Fig. 1). The medium properties are represented by the quantity $k_b = \frac{\omega}{c} n(\omega)$, known as the wave-number of the background, where c is the velocity of light in free space (3 x 10^8 m/s), and $n(\omega)$ is the complex refraction index. The wave number k_b is a complex function with the imaginary part corresponding to the loss (absorption) of the medium and the real part inversely proportional to the wave propagation velocity. In heterogeneous media, $n(\omega)$ is a function of **r** as well as frequency. The associated Green's function is utilized to obtain an analytical solution to Eq. (1). If the source (right hand side of the equation) is a Dirac-delta pulse of the form **s**(**r'**, **r**, k) = δ(**r**, **r**'), the dyadic Green's function will satisfy the following equation (Tai 1971, and Tsang et al. 1985).

$$\nabla \times \nabla \times \overline{\mathbf{G}}_b(\mathbf{r};\mathbf{r}';\omega) - k_b^{\,2}(\mathbf{r},\omega)\overline{\mathbf{G}}_b(\mathbf{r};\mathbf{r}';\omega) = \overline{\delta}(\mathbf{r};\mathbf{r}';\omega) \tag{2}$$

The half-space dyadic Green's function $\overline{\mathbf{G}}_b$ is obtained using plane wave decomposition and Fresnel reflection techniques. Thus, the particular solution to Eq. (1) for a source **s**(**r'**, **r**, k) other than the Dirac-delta pulse δ(**r**, **r**') is the result of the convolution of the Green's function and the source **s**(**r'**, **r**, k) as follows.

$$\mathbf{E}_{\mathrm{b}}(\mathbf{r},\omega) = \int_{\forall_{\mathrm{Source}}} \overline{\mathbf{G}}_{\mathrm{b}}(\mathbf{r},\mathbf{r}';\omega) \bullet \mathbf{s}(\mathbf{r}',\omega)\mathrm{d}\mathbf{r}' \tag{3}$$

where $\forall_{Source}$ is the source volume (for more details about this forward model, please refer to Zhan et al. 2007). In the presence of a scattering object embedded in the background medium, the electric field will be different and referred to as total field ($\mathbf{E}_t$). The difference between the two (so-called scattered field: $\mathbf{E}_s$) is a secondary field due to the scattering effect of the object. The wave number within the scatterer can be called k_s.

Channel Transfer Function (CTF)

In this paper, a channel transfer function (CTF) is developed to transform the experimental data into a form compatible to the computational model. The CTF factors out the frequency-dependent radiation characteristics of the antennas due to their impedance variation. Typically, the impedance of borehole antennas is hard to predict.

Microwave Circuit, Scattering, and Cascade Matrices:

The system of the two antennas and soil can be assumed as an integrated system (Fig. 2) connected to the vector network analyzer (VNA) via its two junctions (ports). Port 1 is the junction between the VNA output port and the transmitting antenna, and Port 2 is the junction between the receiving antenna and the VNA input port. The incident and scattered waves at Port 1 are assumed V_1^+ and V_1^-, and the incident and scattered wave at Port 2 are assumed V_2^+ and V_2^-. These incident and scattered wave pairs are complex values related to each other by

$$[V^-] = [S]\,[V^+] \tag{4}$$

where:

$$[V] = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

[S] is the matrix of scattering parameters, also known as *S*-parameters. *S*-parameters are parameters used in two-port theory that describe the performance of two-port junctions completely. In other words, they describe the *–by default linear—* relation between scattered or reflected waves when a two-port junction is inserted into a transmission line of a certain characteristic impedance Z. The matrix form of Eq. (4) can be written for each junction as follows.

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+ \tag{5}$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+ \tag{6}$$

If Port 2 is terminated with a 50 Ω matched load, V_2^+ will be zero. Then, Eq. (5) will become:

$$V_1^- = S_{11} V_1^+ \tag{7}$$

Eq. (7) supports the definition of S_{11} as the reflection back into Port 1. If there is no input signal into Port 2 (V_2^+ = 0), then Eq. (6) will be simplified to the following form.

$$V_2^- = S_{21} V_1^+ \tag{8}$$

which in turn supports the definition of S_{21} as the transmission from Port 1 to Port 2. The waves transmitted and received at each port of the VNA (V_1^+, V_1^-, V_2^+, and V_2^- in Fig. 2) are physically measured. The total transmitted signal starts from the VNA and proceeds through the transmitting antenna, soil, receiving antenna, and ends back at the VNA. To compute the transmission characteristics of only the soil, the ratio between the wave amplitude induced into the receiving antenna through the soil and the one propagating out of the transmitting antenna into the soil must be computed. To compute these two complex values, the entire system is simulated as a cascade of three two-port junctions, as shown in Fig. 3, to separate the soil and the two antennas.

In Fig. 3, S_{11}, S_{22}, S_{21} and S_{12} of the entire cascade system are measured by the VNA. Eqs. (5) and (6) can be written in the matrix form for the entire system between the ending ports of Fig. 3 (Ports 1 and 6) as follows.

$$\begin{bmatrix} V_{1-} \\ V_{6-} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_{1+} \\ V_{6+} \end{bmatrix} \tag{9}$$

Each two-port junction and the entire system are reciprocal. Therefore:

$$S_{12} = S_{21} \tag{10}$$

$$S_{12\,T} = S_{21\,T} \tag{11}$$

$$S_{12\,R} = S_{21R} \tag{12}$$

$$C_{12} = C_{21} \tag{13}$$

Subscripts T and R respectively stand for the transmitting and receiving antennas, and C_{21} and C_{12} are the transmission S-parameters for the soil channel. For example, S_{11T} is the reflection S-parameter at Port 1 of the transmitting antenna, S_{22T} is the reflection S-parameter at Port 2 of the transmitting antenna, S_{21T} is the transmission S-parameter from Port 1 to Port 2 of the transmitting antenna, S_{12T} is the transmission S-parameter from Port 2 to Port 1 of the transmitting antenna, etc. Considering Eq. (10), Eq. (9) can be simplified to the following cascade form (Collin 1992).

$$\begin{bmatrix} V_{1-} \\ V_{1+} \end{bmatrix} = \begin{bmatrix} 1/S_{12} & -S_{22}/S_{12} \\ S_{11}/S_{12} & S_{12} - S_{11}S_{22}/S_{12} \end{bmatrix} \begin{bmatrix} V_{6-} \\ V_{6+} \end{bmatrix} \tag{14}$$

Eq. (14) for the entire system can be written as:

$$\begin{bmatrix} V_{1-} \\ V_{1+} \end{bmatrix} = \bar{\bar{\mathbf{T}}}\bar{\bar{\mathbf{C}}}\bar{\bar{\mathbf{R}}} \begin{bmatrix} V_{6-} \\ V_{6+} \end{bmatrix} \tag{15}$$

where $\bar{\bar{\mathbf{T}}}, \bar{\bar{\mathbf{C}}}$ and $\bar{\bar{\mathbf{R}}}$ are respectively cascade matrices for the transmitter, soil, and receiver. Similar to Eq. (14) and considering Eqs. (11) through (13), these parameters for the antennas and soil can be written as follows.

$$\bar{\bar{\mathbf{T}}} = \begin{bmatrix} 1/S_{12T} & -S_{22T}/S_{12T} \\ S_{11T}/S_{12T} & S_{12T} - S_{11T}S_{22T}/S_{12T} \end{bmatrix} \tag{16}$$

$$\bar{\bar{\mathbf{R}}} = \begin{bmatrix} 1/S_{12R} & -S_{22R}/S_{12R} \\ S_{11R}/S_{12R} & S_{12R} - S_{11R}S_{22R}/S_{12R} \end{bmatrix} \tag{17}$$

$$\bar{\bar{\mathbf{C}}} = \begin{bmatrix} 1/C_{12} & -C_{22}/C_{12} \\ C_{11}/C_{12} & C_{12} - C_{11}C_{22}/C_{12} \end{bmatrix} \tag{18}$$

To find the unknowns in Eq. (15), the equation can be compared with Eq. (14). In other words, Eq. (19) should be solved.

$$\bar{\bar{\mathbf{S}}} = \bar{\bar{\mathbf{T}}}\bar{\bar{\mathbf{C}}}\bar{\bar{\mathbf{R}}} \tag{19}$$

where:

$$\bar{\bar{\mathbf{S}}} = \begin{bmatrix} 1/S_{12} & -S_{22}/S_{12} \\ S_{11}/S_{12} & S_{12} - S_{11}S_{22}/S_{12} \end{bmatrix} \tag{20}$$

There are nine unknowns (C_{12}, C_{11}, C_{22}, S_{12T}, S_{11T}, S_{22T}, S_{12R}, S_{11R}, and S_{22R}) in matrix Eq. (19), but only four sub-equations available for solution. Therefore, there is a need for at least five assumptions to solve for the soil transmission coefficient C_{21}. Each of the antennas has a connector connected to the VNA, and the other end is placed in the soil. Numbering the ports from the left to right, the receiver is identical to the transmitter, but placed in the reversed direction. This results in the assumption stated in Eq. (21) for the —*obviously reciprocal*— antennas; and if the antennas are placed in a homogenous soil, it results in the assumptions of Eqs. (22) and (23). On the other hand, if the —*obviously reciprocal*— soil is homogenous; the reflection S-parameters at both ends of the soil will be identical. This results in the assumption stated in Eq. (24). The assumptions in Eqs. (22) through (24) were made for the homogeneous soil of this pilot-scale experiment. Most soils behave more similarly at the higher frequencies of this experiment and higher moisture contents. This is due to dominancy of extremely high dielectric constant of water (≈ 80) relative to the one of soil-grains, in determination of the bulk dielectric constant of high moisture content soils, and the relaxation effect at higher frequencies. These assumptions cannot be made easily for non-homogeneous soils in the field, which makes the technique more complicated to apply in the field.

$$S_{11T} = S_{22R} \tag{21}$$

$$S_{22T} = S_{11R} \tag{22}$$

$$S_{12T} = S_{12R} \tag{23}$$

$$C_{11} = C_{22} \tag{24}$$

There is one additional assumption required to solve Eq. (19). This additional assumption is stated in Eq. (25), which assumes virtually no reflection S-parameter for the soil at the junction with the antennas, since the reflection can be accounted for within the antenna S-parameters.

$$C_{11} = 0 \tag{25}$$

This is justified by assuming that the soil channel of interest begins and ends at the points where the waves have fully coupled to the soil. In other words, the imperfect and frequency-dependent coupling between the antennas and soil is assumed to be included in S_{22T} and S_{11R}. The couplings between the antennas and soil at their real boundaries are imperfect. However, the boundaries can be drawn virtually right after the point that the transmitted portion of energy enters the soil. This way, the coupling of antennas and soil are virtually summed up and considered in S_{22T} and S_{11R}. Therefore, S_{11T} and S_{22R} include the real C_{11} and C_{22}, and hence the apparent C_{11} and C_{22} can be assumed to be zero. In other words, C_{11} and C_{22} are implied in the antennas reflection parameters, in order to reduce the number of unknowns by including them in S_{22T} and S_{11R}.

Despite lossy materials such as soil or water, the energy either reflects at each end or transmits through well-designed antennas with no portion absorbed by the antennas. In other words, the amount of energy transmitted through each antenna and the percentage reflected at the two ends of each antenna constitute the entire energy, and there is minimal or no loss. Knowing that both antennas are relatively lossless and for any lossless junction $|S_{21}|^2=1-|S_{11}||S_{22}|$ (Collin 1992), it can be concluded that:

$$|S_{21T}|^2=1-|S_{11T}\,S_{22T}|^2 \tag{22}$$

Besides, S_{12T} has a magnitude and phase, as does any other complex quantity.

$$S_{12T} = \left|S_{12T}\right| \; e^{j\varphi} \tag{23}$$

where $\varphi = \dfrac{\theta_1+\theta_2}{2} + \dfrac{\pi}{2} \mp n\pi$ = phase of S_{12T}, θ_1 = phase of S_{11T}, and θ_2 = phase of S_{22T}. Substituting Eqs. (21) through (27) in Eq. (19), C_{12} (transmission *S*-parameter) of the soil (referred to as the channel transfer function (CTF)), can be found as follows.

$$CTF = C_{12} = \frac{S_{12}(-1+|\,S_{11}\,|^2)}{e^{2j\theta_1} + S_{11}^2(-2+|\,S_{11}\,|^2) - |\,S_{11}\,|^2\; S_{12}^2} \tag{28}$$

Applying the experimental observation that $\left|S_{11}S_{12}\right| << 1-\left|S_{11}\right|^2$ for well-matched antennas, ignoring the existing phase, and applying a phase offset to match the phase of CTF with the phase of the simulated data, the CTF can be further approximated to the following form.

$$CTF(f) = S_{12Soil} = C_{12} = \frac{S_{12}(f)}{1-|\,S_{11}(f)\,|^2}\, e^{j\phi(f)} \tag{29}$$

where the frequency dependence has been emphasized. The phase-offset factor is added, since the assumptions and approximations made in the derivation of the CTF affect the phase. The phase-offset term removes the phase added by the signal passing through the transmitting and receiving antennas and into the soil. The phase offset should be independent of the antennas configuration. A progression analysis was conducted to obtain the phase offset (Zhan et al. 2007).

Eq. (29) presents the CTF or pure transmission characteristics of soil, which can be easily compared to the simulated transmission through the soil due to ideal sources and receivers. Hereafter, the CTFs of the experimental results are calculated based on the S_{11}, S_{22} and S_{21} measured by the VNA and using Eq. (29). The CTFs are then compared to the transmission results simulated by the forward model. In the next section, typical simulated frequency-responses using the forward model (the dyadic Green's function) and the CTFs of the experimental data for the background are compared and discussed.

COMPARISON OF EXPERIMENTATION AND FORWARD MODEL

The experimental CTFs and simulations are both normalized to the square root of the corresponding total energy. A poorly graded sandy soil (representative of a typical soil found in aquifers) was tested at two extreme conditions, one air-dried and the other fully saturated. The moisture content was measured by traditional sampling after air-drying or saturation was performed. The background fields in the sandy soil with measured moisture contents of 3.9% and 16.9%, respectively referred to as drier and saturated soils, were assessed experimentally and theoretically. Fig. 4 shows a schematic of the cross-well antenna installation pattern in the soil. Simulated results were initially computed based on soil properties taken from the table of dielectric properties prepared by von Hippel (1953) for sandy soils, and quadratically interpolated over the frequency range (0.5 – 2.2 GHz). These soil dielectric properties are presented in Table 1. The device and antenna were validated in a previous work by comparing the dielectric permittivity and electric conductivity of a known sandy soil material using different methods (for more information, refer to Zhan et al. 2007 and Kurson 2006). These values of dielectric properties are frequency-dependent and computed over the frequency range. Table 1 shows the end values only (for variations over frequency range, refer to Zhan et al. 2007 and Kurson 2006). Dielectric permittivity values show less variation with frequency, while dielectric conductivity varies more with frequency.

Figs. 5 and 6 show comparisons between the experimental and simulated results in the drier and saturated soil backgrounds at multiple locations and depths, described as follows.

<u>Multiple locations</u>: The soil consists of one soil type (poorly graded soil). Significant time and efforts, and extreme caution were required to uniformly (but loosely and with no compaction) deposit the soil to achieve a homogeneous soil medium. Due to this homogeneity of the soil, background measurements are expected to depend only on the relative separation between the antennas and not on the specific location of each antenna. Two separation cases are presented in Fig. 4. The first case is a set with the antennas located in two boreholes separated by 90^o of arc on the circle of Fig. 4 (e.g., transmission between Boreholes 1 and 3, 2 and 4, 3 and 5, etc.). The second case is for antennas separated by 135^o of arc (e.g., transmission between Boreholes 1 and 4, 2 and 5, 3 and 6, etc.).

<u>Multiple depths</u>: Three depths are selected for cross-tomography data collection: 22.9 cm, 27.9 cm, and 33.0 cm, as described in Part I of these two companion papers. These three antenna depths can be coupled in nine depth-combinations. Only two combinations of the nine possible combinations are presented here. The first combination has the transmitting and receiving antennas both at the depth of 27.9 cm, while the second one has the antennas at the depths of 33.0 cm and 22.9 cm respectively.

As seen in Figs. 5 and 6, for both drier and saturated soil backgrounds, the energy loss of the simulated results is considerably higher than the energy loss of the experimental data, especially at higher frequencies. This may be a result of the discrepancies between the dielectric properties and hence loss-tangent parameters (interpolated for the frequency range of interest from Table 1 by von Hippel (1953)) used to simulate the soil in the model and the actual values for the real soil within the SoilBED facility. As seen in Fig. 5, the energy loss is higher at lower frequencies, which is unexpected. This can be explained by the fact that the figure for the drier soil uses the uncalibrated properties (loss tangent = 0.036 at f = 2.2 GHz, and 0.023 at f = 0.4 GHz). This means the conductivity grows more slowly than the frequency does. In general, the saturated soil simulation of Fig. 5 shows better agreement with the experimental CTFs than the drier soil (Fig. 6), which reflects some limitations on application of the CTF for drier soils. This can be explained by the more continuum type behavior of the saturated soil due to the dominating dielectric property of water. Therefore, the simulated soil parameters should be optimized to fit the simulated results to the experimental CTFs in Figs. 5 and 6. The details of the optimization conducted for calibration do not fit within the scope of this article, and only a brief explanation is given in the following (for more information, refer to Zhan et al. 2007).

Soil Property Calibration

In order to implement the required soil property calibration to improve the agreement between the experimental and simulated results, an optimization technique was conducted (Zhan et al. 2007). The calibrated soil parameters are listed in Table 2. The comparisons between the calibrated forward model and the experimental CTFs for the dry and saturated soils are presented in Figs. 7 and 8.

This method of calibration for soil electrical conductivity adjusts for the energy loss of the soil and hence the magnitude of simulated soil frequency-response, in order to result in a better agreement with the experiment. There is a better agreement for the saturated soil (Fig. 8) than the drier soil (Fig. 7). Figs. 7 and 8 illustrate the same comparison case between the experimental CTFs and theoretical simulation of Figs. 5 and 6 after calibration. Experimental results of the saturated soil are very close to the simulation, indicating the efficiency of CTF in removing the antenna response from the measurements for the saturated soil compared to the drier soil.

Comparing Figs. 5b and 5a, 5d and 5c, 6b and 6a, 6d and 6c, 7b and 7a, 7d and 7c, 8b and 8a, and 8d and 8c, it is unexpectedly observed that, between the two cases with the antennas at two different separations, the cases of wider antenna-separation of 135^o arcs show slightly better agreement between the model and CTFs in both drier and saturated soils than the 90^o arcs. The stronger scattering effect of the neighboring boreholes on the measurements in the case of 90^o arcs may be the factor interfering with the measurements. Higher depth-difference at the same antenna location-combination increases the separation between the antennas, which results in a lower signal to noise ratio and hence, slightly weaker agreement. However, as indicated in these figures, as long as the antennas are placed deeply enough to prevent the ground-surface impedance effect, the relative depth-difference affects the data, while the absolute depths do not.

CONCLUSIONS

The CTF provides the capability to factor out the antenna-resonance and antennas to soil coupling necessary for comparison with the simulated data. The CTF concept was derived in a general form, independent of the type and conditions of the experiment. The CTF is useful for removing sensor dependency in measurements for inversion purposes. There is an acceptable agreement between the simulated data and the experimental CTF results. This supports the forward model for further inverse scattering studies. There are some other important conclusions that are listed in the following:

- The implemented simulation uses ideal dipole sources that propagate EM waves equally over the entire frequency range, and cannot model the frequency response of physical antennas. Therefore, the simulation must be compared to the experimental CTFs.
- The CTF is developed to factor out and suppress the frequency-dependent radiation characteristics of the antennas in order to evaluate the pure transmission characteristics of the medium consisting of soil, air, and/or water.
- CTF is the ratio of the complex intensity of the wave output from the soil to the complex intensity input to the soil. The model measures the electric field at each point in the medium excited by an ideal dipole instead of an actual transmitting antenna. Therefore, both experimental CTFs and simulated electric fields should be simply normalized to the square root of the corresponding total received energy.
- A quadratic interpolation was used to compute the soil properties used in the simulation from the values given by von Hippel (1953) for sandy soils based on the end-values of the frequency range of interest. The mismatch between the simulated results and experimental CTFs may have been due to this inaccurate representation of the soil property. Calibrating and optimizing soil properties by matching the simulated and experimental results helped to achieve the correct soil properties. This calibration process is possible in the field at a much larger size and lower frequency, if the field problem can be theoretically modeled. Sample results were presented for the dry and saturated background (soil only) measurements. This technique can be used for soil characterization as well.

A validated forward model for both background and scattered fields is a requirement for successful inversion. Now that the forward model for the background field is validated based on the experimental CTFs, there is a need to validate the CTF for the scattered field. Afterwards, the validated scattered field forward model can be used for inverse scattering purposes.

As the first step towards a solution to a general problem, a controlled homogenous soil medium was selected for the pilot-scale laboratory experiment and the theoretical simulation. Therefore, an analytical model easily capable of simulating homogenous media was selected as the theoretical model. The assumptions made to find CTF are hence appropriate for the same condition. Besides, most soils behave more similarly at higher water-saturation degrees and higher frequencies. This is also the case of this pilot-scale experiment. Finding the CTF for media made of materials other than soil and as the ultimate goal, heterogeneous soil media are much more challenging and should be extensively studied in the future. Applying the technique to other soils and other frequency ranges to evaluate the sensitivity of the technique to these parameters is another essential study for the future.

The CTF method to transform experimental measurements to a form comparable with theoretical simulations by removing sensor dependence can be extended to the study of real-life problems, such as contaminant detection and monitoring. However, the CTF concept was validated in this paper for a higher frequency and smaller scale and within a controlled laboratory environment of homogenous soils, compared to real-field scale and conditions. There is also a uniform dielectric property contrast between the inclusion and backgrounds simulated by both the theoretical and experimental models of this paper. The basis for this approach has been a linear scaling relation between the size and frequency. The frequency was scaled up from the field, inversely proportional to how the geometry of the problem (antennas size and separation, soil medium extent, and inclusion size) was scaled down. This scaling, however, does not apply to one component, which is the size of soil-grains. Hence, the scaling process needs to be extensively studied and validated, before the outcome of this work is extended to field applications.

NOTATION

c = Velocity of light in free space = 3 x 10^8 m/s;
$\bar{\bar{\mathbf{C}}}$ = Soil cascade scattering matrix;
CTF = Channel Transfer Function;
$\mathbf{E}(\mathbf{r}, \omega)$ = Electric field at each point in space and natural frequency;
f = Frequency;
$\bar{\bar{\mathbf{I}}}$ = Identity operator (unit dyad);
$\bar{\bar{\mathbf{G}}}$ = Dyadic Green's function;
$k(\mathbf{r}, \omega)$ = Wave number at each point and natural frequency;
tan δ= Loss-tangent;
n = Refraction index;
r = Position vector of points, at which the electric field is computed;
r' = Position vector of points inside the source;
$\bar{\bar{\mathbf{R}}}$ = Cascade scattering matrix of receiving antenna;
$\mathbf{s}(\mathbf{r'}, \omega)$ = Source;
S = Scattering parameter;
S_{11} = Reflection parameter at Port 1;
S_{22} = Reflection parameter at Port 2;
S_{21} = Transmission parameter from Port 1 to Port 2;
S_{12} = Transmission parameter from Port 2 to Port 1;
[S]= Scattering matrix;
Subscript *b*: stands for background;
Subscript *s*: stands for scatterer or scattered;
Subscript *p*: stands for perturbation due to scatterer;
Subscript *Ant*: stands for antenna;
$\bar{\bar{\mathbf{T}}}$ = Cascade scattering matrix of transmitting antenna;
V_i^- = Incident wave amplitude;
V_i^+ = Scattered wave amplitude;
$[V^-]$ = Matrix of incident wave amplitude;
$[V^+]$ = Matrix of scattered wave amplitude;
$\forall_{Source}$ = Source volume;
ε = Dielectric permittivity;
$\varepsilon' = \varepsilon_r$ = Relative real dielectric permittivity (real dielectric constant);
φ = Phase of S_{21} of antennas;
σ = Electric conductivity;
θ_1 = Phase of S_{11} of antennas;
θ_2 = Phase of S_{22} of antennas; and
ω = Temporal angular frequency (conjugate to time *t* in the frequency domain);
ϕ = Phase offset in CTF.

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Fig. 1. Schematic of: (a) background field and (b) inverse scattering problem

Fig. 2. A typical two-port microwave junction (Collin 1992), with input (V^+) and output (V^-) voltages at Ports 1 and 2, along with corresponding reflection and transmission *S*-parameters

Fig. 3. SoilBED facility simulated as a cascade of three two-port microwave junctions, with input (V^+) and output (V^-) voltages at Ports 1 through 6 for transmitter, soil and receiver, along with corresponding reflection and transmission *S*-parameters

Fig. 4. Top view of the cross-tomography antenna-installation pattern in the SoilBED facility

Fig. 5. Uncalibrated drier soil background comparisons (results are normalized to the square root of the total received energy) for: (a) 90^o arcs (examples shown with solid lines in Figure 4), transmitter and receiver depths 27.9 cm, (b) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter and receiver depths 27.9 cm, (c) 90^o arcs (examples shown with solid lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm, and (d) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm

Fig. 6. Uncalibrated saturated soil background comparisons (results are normalized to the square root of the total received energy) for: (a) 90^o arcs (examples shown with solid lines in Figure 4), transmitter and receiver depths 27.9 cm, (b) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter and receiver depths 27.9 cm, (c) 90^o arcs (examples shown with solid lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm, and (d) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm.

Fig. 7. Calibrated dry soil background comparisons (results are normalized to the square root of the total received energy) for: (a) 90^o arcs (examples shown with solid lines in Figure 4), transmitter and receiver depths 27.9 cm, (b) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter and receiver depths 27.9 cm, (c) 90^o arcs (examples shown with dashed lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm, and (d) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm.

Fig. 8. Calibrated saturated soil background comparisons (results are normalized to the square root of the total received energy) for: (a) 90^o arcs (examples shown with solid lines in Figure 4), transmitter and receiver depths 27.9 cm, (b) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter and receiver depths 27.9 cm, (c) 90^o arcs (examples shown with dashed lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm, and (d) 135^o arcs (examples shown with dashed lines in Figure 4), transmitter depth 33.0 cm and receiver depth 22.9 cm.