<u>Supplementary Material</u>

Developing a Non-Cooperative Optimization Model for Water and Crop Area Allocation Based on Leader-followers Game

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S.1. The Leader – Follower Game

This method which is classified as a non-cooperative game theory method was introduced in 1934. Later in 1952, the method developer; Von-Stackelberg presented a concept to solve the method as the Von-Stackelberg equilibrium. In this game, a Nash equilibrium exists among followers which is a constraint for the leader's objective function. In the game, for each made decision from the leader, the followers calculate their new equilibrium to maximize their individual payoff function (Tharakunnel et al., 2009).

Example S1:

Consider, there is a firm which controls its employment completely. As another player, a monopoly union which is in charge of determining wages controls the wage of firm's employees. The union plays role as the leader which presents its wage request w to the firm, and then the firm, as the follower decides on the number of employees (L) it can employ, based on the wage requirements. The strategy sets of the union is S1=[0,W] with the largest wage W which can be requested, and S2=[0,∞) is the strategy sets of the firm that its payoff function is as follows:

$$\phi_2\left(w,L\right)=R\left(L\right)-wL$$

Where *R(L)* is the amount of revenue produced by L worker, and *wL* is the amount of wages the firm has to pay to the workers and $\phi_1\left(w,L\right)$ is the payoff of the union. As a numerical example select the following numeric and parametric values for the items:

$$W=2$$

$$R\left(L\right)=L$$

$$\phi_1\left(w,L\right)=w^2\left(-w^2+2w+20\right)L$$

Since

$$-w^2+2w+20=-\left(w-1\right)^2+21>0$$

$$\frac{\partial\phi_1}{\partial w}=\left(-4w^3+6w^2+40w\right)L=\left(2w^2\left(3-2w\right)+40w\right)L>0$$

So ϕ_1 strictly increases in *L* and *w*. The best response of the firm is obtained by maximizing

$$\phi_2(w,L)=\sqrt{L}-wL$$

By differentiation

$$\frac{\partial\phi_2}{\partial L}=\frac{1}{2\sqrt{L}}-w=0 \quad \Rightarrow R_2(w)=\frac{1}{4w^2}$$

Therefore the corresponding payoff of the union is:

$$\phi_1(w,R_2(w))=w^2\left(-w^2+2w+20\right)\frac{1}{4w^2}=\frac{1}{4}\left(-w^2+2w+20\right)$$

The maximum value of the union's payoff takes place at w=1 which cause the best response of the firm $R_2(w)$ to be 1/4, so the Stackelberg equilibrium is w^*=1 and L^*=1/4. It is proved that there is at least on Stackelberg equilibrium.

Example S2:

Consider two firms A and B competing in a duopoly market in which the outcomes depend on the strategic assumptions made by the competitors. They can have two level of production yield 60 and 30. In the framework of the Stackelberg model, each firm had two possible strategies to be a "leader" or a "follower". The results of choosing these strategies would address a 2×2 game that its payoff matrix can be shown in Table S1.

Table S1. Payoff Matrix for the Stackelberg Model

		B's Strategies	
		Leader (q_B=60)	Follower (q_B=30)
A's Strategies	Leader (q_A=60)	(0,0)	(1800, 900)
	Follower (q_A=30)	(900, 1800)	(1600, 1600)

The obtained result of leader-leader strategy shows zero profit for each of the two firms that is the worst outcome. A follower-follower strategy has the most profits. But the cheating behavior for grabbing of the leader position causes the unstable profits. Also, in the state of leader-leader solution, there is no Nash equilibrium for the problem. In the last scenario, when each of the leader-follower strategy is chosen, Nash equilibrium definitely exists. In this condition, choosing each of the players as leader depends on the factors such as the history of the industry or the personalities of the firms' managers.

Tharakunnel, K., Bhattacharyya, S., 2009. Single-leader–multiple-follower games with boundedly rational agents. J. Econ. Dynam. Control 33(8), 1593-1603.

Von Stackelberg, H., 1934. Marktform und Gleichgewicht. Vienna: Springer.

von Stackelberg, H., 1952. The Theory of the Market Economy. Oxford University Press, London, UK.

S.2. Non-domination Sorting Genetic Algorithm (NSGA-II) Method

NSGA-II method works based on the elitism-preserving approach, which finds all non-dominated solutions sorted from the initial to last population. The schematic figure of NSGA-II procedure is shown as follows:

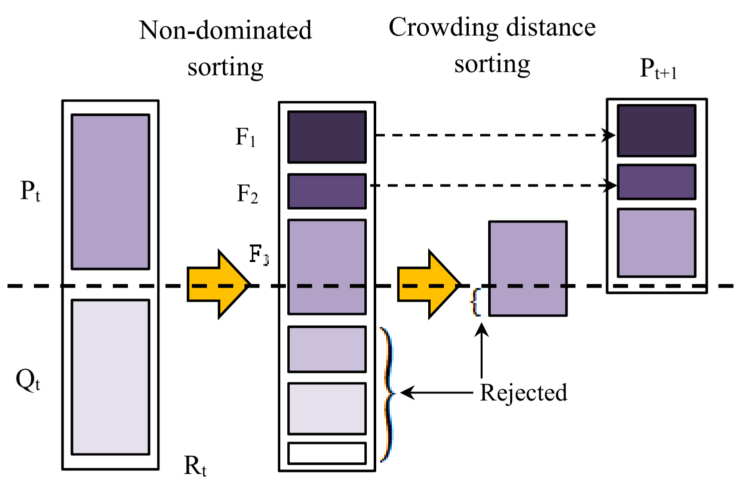

Fig. S1. NSGA-II multi-objective optimization procedure (Adapted from Deb et al. 2002)

Regarding to the Fig. S1, a general NSGA-II procedure can be presented in the following steps:

Step 1: Random creation of the parent population, P_t of size Z.

Step 2: Non-domination sorting of the random parent population.

Step 3: Assigning a fitness rank to its non-domination for each non-dominated solution.

Step 4: Applying binary tournament selection, recombination and mutation operators to create an offspring population, Qt of size Z.

Step 5: Creation of each new generation which is done based on the following steps:

a) Combining the parent population, P_t and the offspring population, Q_t to create population R_t, of size 2Z.

b) Population sorting, according to the fast non-dominated sorting procedure.

c) Identifying all non-dominated fronts (Fr_1 , Fr_2, . . . ,Fr_l).

d) Generating the new parent population, P_{t+1} of size Z.

e) Perform the selection, crossover and mutation operations on the newly generated parent population, P_{t+1} to create the new offspring population, Q_{t+1} of size Z.

Step 6: Repeat step 5 until the stopping criteria is met.

Deb, K., Pratap, A., Agarwal, S., Meyarivan, T., 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans. Evolut. Comput. 6,182-197.