AN INSTRUCTIONAL FRAMEWORK FOR TIER 2 INTERVENTION IN MATHEMATICS

by

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DEDICATION

The DiDio family immigrated to America in 1907 from Italy. I am the first DiDio in three generations to earn a doctoral degree. This dissertation is dedicated to the DiDio family: to those who have gone before me and encouraged me to dream, and to those who will come after me who may be inspired to dream.

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I would like to acknowledge the contributions of many to this dissertation. In my teaching career, I have taught many students who have struggled to learn mathematics. This dissertation is based on what I learned from them and my desire to tell their stories. My advisor, friend, and committee chair, Dr. Jonathan Brendefur, helped me develop the knowledge and skills I needed to articulate all I have learned and share it with others. My committee members, Dr. Margaret Kinzel, Dr. William Parrett, and Dr. Karen Smucker, provided valuable expertise, insight, and guidance on this dissertation. My husband, Jonathan Beals, my mother, Peggy DiDio, and my wonderful children, Carter and Jane Beals, inspired me and encouraged me every step of this journey.

ABSTRACT

This study investigated the effectiveness of an instructional framework for Tier 2 intervention in mathematics. The framework suggests intervention teachers utilize instructional practices shown to be effective for students with disabilities while assessing students strengths and weaknesses in skills that are strong indicators of an MLD. This study used a mixed methods design to determine whether tutoring based on this framework led to changes in understanding of multiplication and division for four low achieving fourth graders. It also evaluated the usefulness of the instructional framework in determining whether a student had patterns of deficit related to a mathematics learning disability. Three of the students made positive changes in their understanding of multiplication and division from the tutoring and one showed little change. Two of the students showed patterns of deficit related to a mathematics learning disability. The framework for instruction as well as the framework for evaluating patterns of strength and struggle was determined to be helpful for instructors of Tier 2 mathematics intervention.

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LIST OF ABBREVIATIONS

IDEA	Individuals with Disabilities Act
LD	Learning Disability
RTI	Response to Intervention
NCLB	No Child Left Behind
SWD	Student with Disability
MLD	Mathematics Learning Disability
LEP	Limited English Proficient
NCSM	National Council Supervisors of Mathematics
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
MD	Mathematics Difficulty
RD	Reading Difficulty
IEP	Individualized Education Plan
NKT	Number Knowledge Test
NUCALC	Number Processing and Calculation in Children
CRA	Concrete Representational Abstract
MTI	Mathematical Thinking for Instruction
ELP	English language proficiency

JM	Judgment of magnitude
MR	Multiple representations
FBC	Fluency with basic calculation
VSS	Visual spatial skills
PM	Procedural memory
PE	Procedural error
PAL	Peer assisted learning
EI	Explicit instruction
SI	Strategy instruction
DIS	Productive disposition
LB	Language barrier
ATT	Attendance
STL	Symbol to language
STM	Symbol to model
LTS	Language to symbol
LTM	Language to model
MTS	Model to symbol
MTL	Model to language

CHAPTER 1: STATEMENT OF PROBLEM

Present practice in special education and general education mathematics instruction is the result of a century of research from a variety of disciplines. While often evidenced in what has been called "The Math Wars" (Van de Walle, 1999), philosophical differences between a behaviorist view of learning and a constructivist view have been debated for many years as educators sought the most effective methods to teach mathematics to children. Within the field of mathematics education, how to help students who struggle to learn mathematics has been an intriguing challenge. Traditionally, mathematics instruction in special education has utilized a direct instruction approach based on a behaviorist view of learning while general education trends have moved toward pedagogy embracing a constructivist approach.

This philosophical gap between special education and general education has taken center stage on a national level with the reauthorization of the Individuals with Disabilities Education Act (IDEA) in 2004. In response to the number of students being diagnosed as having a learning disability (LD), a 300% increase since 1976 (Woodward, 2004), the new legislation calls for states to adopt a Response to Intervention (RTI) model. This model encourages educators to determine whether students actually have an LD or whether there is a gap in achievement between them and their peers for other reasons such as lack of instruction, mobility, poverty, or limited English abilities. The reauthorized IDEA attempted to prevent students from being diagnosed as having an LD who may actually not have a disability. The former discrepancy model of diagnosing students as LD had led to rampant over-qualification of students as learning disabled. The new regulations require schools to provide intervention for students before diagnosing an LD. Additionally, it is the vision of the No Child Left Behind (NCLB) legislation passed in 2002 that students with disabilities (SWD) meet the same state achievement standards as their non-disabled peers. This vision of success for SWD has caused educators to closely examine present practices in educating SWD. Alignment between general education and special education curriculum and instruction is being encouraged by the recent legislation.

Mathematics instruction in special education has traditionally followed the direct instruction model with a heavy emphasis on computational skills and mastery of basic mathematics facts. Reviews of special education mathematics courses have demonstrated the majority of special education mathematics courses consist of procedural instruction and below grade level skills practice (Maccini & Hughes, 1997; Woodward & Montague, 2002; Woodward, 2006). According to the Nation's Report Card, little progress was made between 1996 and 2007 to close the gap in achievement scores in mathematics between SWD and their non-disabled peers (Maccini, Stickland, Gagnon, & Malmgren, 2008). Once a student is removed from the general education setting for mathematics instruction, it is very difficult to get them caught up to their peers. Expectations and content of mathematics classes for SWD must be raised if these students are going to be able to meet high school requirements in mathematics and be prepared for the mathematical demands of a modern day work place. In addition, instructional strategies, grouping practices, and teacher training must also be examined. SWD cannot be expected to meet grade level requirements if they have not participated in general education coursework throughout their school career. "The major question for students with learning disabilities remains. What is the most worthwhile use of limited instructional time for these students?" (Woodward & Montague, 2002, p. 95). Mathematics instruction for SWD needs reexamination to make sure the little instructional time teachers have with these students is spent on what matters most.

Organizing a school system, which provides high quality mathematics instruction for SWD, requires careful consideration and planning. IDEA emphasizes the importance of providing SWD access to the general education curriculum. Technology trends mean SWD cannot just learn a "narrow range of mathematics" to function in a world "filled with computing devices" (Woodward & Montague, 2002, p. 92). Emerging directions in special education are focused on attempts to provide SWD a broader instructional experience in mathematics to help them move toward the type of mathematics proficiency encouraged by reform (Woodward, 2006; Montague & Jitendra, 2006; Bryant & Pedrotty Bryant, 2008).

Proponents of inclusion contend SWD will only master grade level standards by participating in the same general education classes as their non-disabled peers (Bower, 2008). However, teachers certified to teach mathematics typically have not had a lot of training or experiences in working with SWD or on the characteristics of individual disabilities (DeSimone & Parmar, 2006; Maccini et al., 2008). Many teachers do not have a “strong understanding of specific pedagogical strategies to strengthen the mathematical learning of students with LD" (DeSimone & Parmar, 2006, p. 107). Thus, some contend that it is more effective to educate SWD in a more restrictive environment in which students are grouped homogeneously with other SWD and taught by a teacher who is trained to adapt instruction and content specifically for SWD. A problem with this option is that teachers certified in special education often do not have high levels of training in mathematics pedagogy nor strong background knowledge in mathematical content. Furthermore, the newest state regulations for certification aligned to the highly qualified definitions of NCLB require the primary instructor of mathematics at the secondary level to be certified in mathematics. Both special education and general education teachers need to be trained on research-validated teaching methods for SWD (Schumaker et al., 2002, p. 15). "As teachers become increasingly accountable for employing RTI procedures (e.g., evidence-based instructional practices progress monitoring of student performance, tiered instruction) in mathematics instruction, they must be provided the means for doing so" (Bryant & Pedrotty Bryant, 2008, p. 7).

Due to the limited amount of research on mathematics intervention, educators must draw upon research from a variety of disciplines to identify and continue to develop research-based instructional strategies shown to be effective for SWD. Research from cognitive science, mathematics education, and special education can provide collective wisdom on instructional practices to help this population of students learn mathematics.

Future research needs to focus on bridging the gap between general education and special education instructional practices in mathematics.

Teachers need specialized pedagogical content knowledge to provide high quality mathematics intervention (DeSimone & Parmar, 2006; Maccini et al., 2008; Schumaker et al., 2002; Bryant & Pedrotty Bryant, 2008). An instructional framework for mathematics intervention can be a valuable tool to guide teachers in providing high quality instruction for remedial and supplemental mathematics instruction. This study explores the use of an instructional framework created by the researcher, which can be used to educate both general education and special education teachers on effective practices for students who struggle to learn mathematics. The framework integrates research on effective instructional practices for SWD as well as current research on mathematics learning disabilities (MLD). The variety of causes of mathematical difficulties means educators need to be equipped with the ability to recognize the symptoms of MLD. They must also be provided with learning tools to help these students compensate for cognitive deficits related to mathematical processing. Research in cognitive psychology, neuropsychology, mathematics education, and special education must be synthesized and translated into instructional frameworks in order to be useful for practitioners. A better understanding of numerical processing and subtypes of MLD can greatly inform instruction for teachers working with students who struggle with mathematics. Useful tools for data collection and diagnosis of strengths and weaknesses are also needed.

This study attempts to provide such tools in the form of an instructional framework created by the researcher. This instructional framework can be used by intervention teachers to plan instruction, monitor progress, identify strengths and weaknesses, and design accommodations for students who are struggling to learn mathematics. It would also be useful for the education of both preservice and inservice teachers working with students having difficulty learning mathematics. The purpose of this study is to investigate the effectiveness of mathematics instruction based on an instructional framework for Tier 2 mathematics intervention.

CHAPTER 2: REVIEW OF LITERATURE

Mathematics is a complex subject. In order to teach this subject effectively, teachers need knowledge of mathematics as a subject as well as knowledge of pedagogy (Ball, 1993; Hill & Ball, 2009; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Danielson, 2007). Teaching students who struggle to learn mathematics requires even more specialized knowledge about learning disabilities and effective instructional practices for students who have numerical processing problems (DeSimone & Parmar, 2006; Maccini et al., 2008; Schumaker et al., 2002; Bryant & Pedrotty Bryant, 2008). The reauthorization of IDEA mandates students struggling to learn mathematics receive differentiated instruction within the general education program before being qualified as learning disabled. Furthermore, the National Council of Supervisors of Mathematics (NCSM) recommends teachers strategically design accommodations that allow SWD to participate in general education mathematics classes (NCSM, 2008). Mathematics teachers need training on how to differentiate instruction and strategically design accommodations for struggling students (DeSimone & Parmar, 2006; Bryant, Kim, Hartman, & Bryant, 2006; Woodward, 2006; Tomlinson, 1999, 2001). Furthermore, teachers also need specialized knowledge about MLD and the cognitive processes used for numerical concepts in order to understand the cognitive strengths and weaknesses of the students they teach.

This review of literature addresses research on topics related to the pedagogical content knowledge needed to effectively teach mathematics to SWD. This review will begin with an explanation of the RTI framework for intervention and a discussion of issues related to differentiation of instruction. Next, literature on aligning instruction in special education with the pedagogy recommended by reform will be presented. What is known about cognitive processes related to numerical concepts and research on MLD will be reviewed. Lastly, research on effective instructional practices for SWD will be summarized.

3 Tier Model

RTI utilizes a tiered approach to intervention that has been used in reading intervention, called the 3 Tier Model (see Figure 1). This model was sponsored by the U.S. Office of Special Education Programs (OSEP) and was originally designed by Vaughn, Linan-Thompson, and Elbaum (cited in Idaho State Department of Education, 2009) to prevent reading disabilities through early intervention in the primary grades. Using the 3-Tier Model at the middle and high school levels and applying it to mathematics is a relatively new practice (Bryant & Pedrotty Bryant, 2008; Fuchs et al., 2005). The terminology for the tiers differs in the literature. The terms in this explanation are those used by Bryant and Pedrotty Bryant (2008).

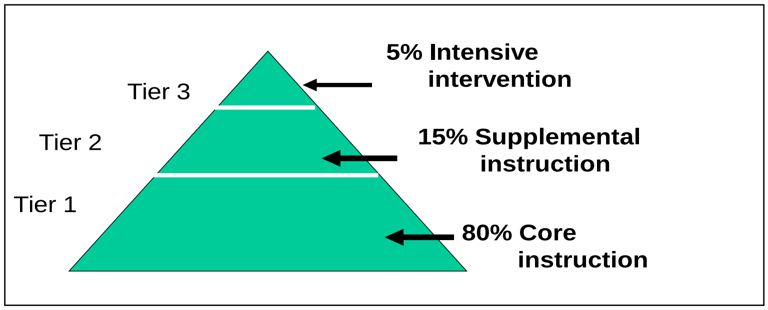


Figure 1: 3 Tier Model

The three tier model recommends 80% of the student body participate in the Core program, which is part of the general education program. An additional 15% more of the student body should participate in the Core program with support labeled as supplemental instruction. A recommended 5% of the students will need an intensive intervention. According to the 3 Tier model, this intensive intervention should be delivered by specialists and can utilize a replacement curriculum. Sometimes, students receiving intensive intervention are removed from the general curriculum and taught a replacement curriculum designed to remediate the missing components of their education and prepare them to reenter the general education program at a later time. Another model is to provide intensive intervention in addition to Core instruction. Special education services can be provided at all three layers of the pyramid, with the most intense individualized instruction being provided as Tier 3 interventions. Many students participating in the top tier of the 3 Tier Model are students who qualify for special education as well as other federal program services such as ESL and Title I. Often what occurs at Tier 3 is an overlap of services provided by these programs.

Differentiation of instruction is a phrase that has become partnered with RTI. As schools face the implementation of RTI programs, what occurs in classrooms at the Core Instruction level becomes central to the effective implementation of the three tier model.

Differentiation of Instruction

IDEA 2004 states teachers must provide differentiation of instruction within the general education setting. However, the lack of clarity in the law of a working definition of differentiation makes implementation of this mandate difficult. "There is a need for differentiating or adapting mathematics instruction to respond to students' needs so that students with mathematical disabilities can benefit from standards-based mathematics instruction" (Bryant et al., 2006, p. 16). These researchers define this differentiation as adaptations. An adaptation is "appropriate adjustments, accommodations, and modifications to instruction or supports that allow students to meet academic requirements and conditions of the curriculum, most often the general education curriculum" (Bryant et al., 2006, p. 16). Curriculum and instruction consistent with the recommendations of the National Council of Teachers of Mathematics (NCTM, 1989, 2000) needs to be adapted for struggling students to provide them with more explicit instruction to help them make connections between topics. Some students need more opportunities to develop and apply strategies, intense practice on meaningful skills and distributed practice over time (Carnine, 1997). SWD need to see mathematics as a "sense making discipline" (Woodward, 2006, p. 47) rather than drill on isolated procedures.

DeSimone and Parmar (2006) conducted a study of middle school mathematics teachers' beliefs about inclusion of students with learning disabilities. Through qualitative interviews that supported quantitative survey data, the researchers concluded teachers in different areas of the country often used the term differentiated instruction, "yet they could not name specific strategies when asked to elaborate more on this term or provide concrete examples of differentiated instruction" (p. 104). Eighteen of twenty-six teachers interviewed admitted they really did not modify instruction specifically for SWD. Ten were able to give examples of curriculum adaptations they had made, which usually involved not requiring SWD to go as far learning a concept as other students. For example, one teacher mentioned when teaching adding fractions, she had her inclusion SWD only do problems with common denominators. The researchers named lack of preparation for general education teachers who teach SWD in inclusion settings as one of three key issues school districts will need to address as they attempt to provide inclusive classrooms that meet the needs of a wide range of learners. Although this study specifically focuses on mathematics, it is probable the lack of clarity on exactly how to differentiate instruction for students who struggle applies to all subjects and all grade levels in general education.

Tomlinson (1999, 2001) has completed extensive writing on differentiation of instruction. She suggests differentiation of instruction is employing a wide range of instructional strategies to plan and deliver engaging instruction, which can be modified to meet the individual learning needs of students. "A differentiated classroom provides different avenues to acquiring content, to processing or making sense of ideas, and to developing products so that each student can learn effectively" (Tomlinson, 2001, p. 6). Because differentiation involves a range of instructional strategies, it is difficult to narrowly define the term specifically. However, teachers need specific names for strategies in order to effectively plan lessons, collaborate, and reflect upon teaching practices.

Alignment of Practice

NCLB created focus on populations who have historically struggled to meet grade level benchmarks in American public education. Schools across the nation have focused upon underachieving subgroups as never before. The implementation of RTI has caused school systems to work toward greater alignment of content and instructional practice between general education and special education so SWD can meet the same state achievement standards as their non-disabled peers.

In reading intervention, this alignment has not been difficult to create. Research on reading disabilities and early intervention for reading difficulties has led to common practices among special educators and general educators in reading instruction. This is not the case in mathematics instruction. Compared to research on reading intervention, there is a minimal amount of comparable research on mathematics intervention (Fuchs et al., 2008; Gersten, Jordan, & Flojo, 2005). "There is a paucity of research on early interventions to prevent MD in struggling students" (Gersten et al., 2005, p. 300). There is even more limited research on adolescents with MLD than young children (Bryant & Pedrotty Bryant, 2008). "There is very little in the special education or at-risk literature that articulates how teachers successfully mediate these challenges and move students toward the kind of mathematical problem solving that appears in the general education reform literature" (Woodward & Montague, 2002, p. 97).

Woodward (2006) elaborates on two concerns regarding applying research on reading intervention to mathematics intervention. First, early intervention in number sense development could become direct instruction drills comparable to phonics drills on sound to phoneme associations. Symbol to meaning association in mathematics is much more complex. For example, the ability to quickly pair the symbolic form of a phoneme to the sound is related to fluency in reading. Research in reading has shown a strong correlation between reading fluency and comprehension of what is being read. It is reasonable that it would be difficult to understand the content of a passage of text if the pace of the decoding is too slow. The same relationship between rapidly pairing a basic number fact with its solution is not necessarily related to greater achievement in mathematics. A student can be very successful in basic fact retrieval and have very little understanding of mathematical concepts. In contrast, a student may struggle with rapid recall of basic facts, but have good conceptual understanding of a mathematical concept. An example of such understanding would be knowing the difference in value between a digit in the tens column versus the hundreds column. Recent research has shown number sense, spatial skills, and the ability to visualize magnitude are greater predictors of mathematical achievement than rapid naming of answers to basic facts (Gersten et al., 2005). This is one of the reasons effective mathematics teachers must understand mathematics as a discipline in order to address this complexity with students (Ball, 1993; Hill & Ball, 2009; Ball et al., 2005; Ball et al., 2008).

Woodward's second area of concern related to adopting the philosophy of early reading intervention to mathematics is it implies the development of number sense is a primary grade phenomenon. Unlike decoding skills in reading, which can be mastered at a young age, number sense develops over students' school career as they engage in new and more advanced mathematical topics. Research on cognitive causes of mathematics disabilities suggests students who have difficulty conceptualizing numbers struggle with it throughout their school careers (Geary, 1993, 2004; Von Aster, 2000; Von Aster & Shalev, 2007). They may need ongoing support and accommodations in order to be successful in mathematics as they progress through the grades.

Special education has traditionally focused on remediating deficits through drill and practice intervention strategies and direct instruction. There has been a heavy emphasis on computational skills and mastery of basic mathematics facts (Maccini & Hughes, 1997; Woodward & Montague, 2002; Woodward, 2006). Warner, Alley, Schumaker, Deshler, and Clark (1980) showed students in special education mathematics classes in grades 7 – 12 only made one grade level year of progress in their six years of middle school and high school. Wagner (1995) reported SWD typically perform about two grade levels behind their peers without disabilities. Cawley and Miller (1989) claimed adolescents with disabilities generally perform at about the fifth-grade level in mathematics, which is also supported by Cawley, Baker-Kroczynski, and Urban (1992). "Far too often students with LD and those in remedial classrooms spend their time completing worksheets or responding to low-level questions in a direct instruction context" (Woodward, 2006, p. 47). These students are not participating in a wide range of mathematical content or developing conceptual understanding.

Much of the mathematics instruction taking place in remedial and special education mathematics courses focuses on below grade level computational skills and utilizes a didactic, rote approach to practicing computation procedures (Warner et al., 1980; Wagner, 1995; Cawley & Miller, 1989; Cawley et al., 1992; Woodward & Montague, 2002; Woodward, 2006; Maccini et al., 2008; Maccini & Hughes, 1997). Experiences with problem solving and exposure to other areas of mathematics such as geometry, measurement, algebra, data analysis, and probability are rare in special education classrooms (Woodward, 2006; Bryant et al., 2006). SWD are performing far below their peers in mathematics achievement and have shown very little progress toward closing the gap in the last decade (Maccini et al., 2008). A new approach to mathematics instruction for this population of students is needed.

The differences in practice between special educators and general educators are based in differing definitions of what constitutes mathematical proficiency (Woodward, 2006; National Research Council, 2001). Special education classrooms show a heavy emphasis on practicing below grade level computational skills such as multiplication facts. General education instruction is moving toward the application of knowledge and skills to solve complex real-world problems that engage students in reasoning and communicating about mathematical ideas and representing them with iconic and symbolic tools. This shift is reflective of the recommendations outlined in the NCTM Standards (NCTM, 1989, 2000). An example of this type of problem might be designing a skateboard ramp that utilizes an understanding of the concept of slope. The National Research Council (NRC) provided a definition of mathematical proficiency in the national report called *Adding It Up* (NRC, 2001). This definition of mathematical proficiency includes the following five components: conceptual reasoning, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. While general educators have adopted this definition, "special educators have perceived competence in mathematics to be fluency in mathematics facts and computational procedures as well as the ability to solve problems quickly and accurately" (Woodward, 2006, p. 30). And, although this might be the case, another issue is how to get students to be proficient with facts or fluent. Often, a problem in special education is a word problem that has a clearly defined arithmetic procedure associated with it. In reform-based pedagogy, a problem consists of a real-world scenario, which does not have an obvious solution path defined. These types of problems allow for the use of multiple representations and solution strategies students can discuss and analyze (Tripathi, 2008). The field of special education needs to reconceptualize instructional practices in mathematics instruction and redefine mathematical proficiency. Instructional strategies should be similar to those called for in reform but adapted for remedial and special education students (NCTM, 1989, 2000; Woodward, 2006; Bryant et al., 2006).

Triple-Code Model

The Triple-Code Model of numerical representation provides a useful theory on which to base mathematics instruction (DeHaene & Cohen, 1995; Feifer & De Fina, 2005; Sousa, 2008; Von Aster, 2000; Von Aster & Shalev, 2007; Chiappe, 2005; Griffin, 2002). This model suggests humans rely on three cognitive processing systems to encode numerical concepts. The brain relies on an interaction between language, mathematical symbols, and the visualization of magnitude to develop conceptual understanding of numbers and execute mathematical procedures. The Triple-Code Model closely aligns to Jerome Bruner's theory of learning which explains learners progress through three stages of conceptual understanding: enactive, iconic, and symbolic (Driscoll, 2005).

Several researchers describe these three mental coding processes used to conduct mathematical tasks (DeHaene & Cohen, 1995; Feifer & De Fina, 2005; Sousa, 2008; Von Aster, 2000; Von Aster & Shalev, 2007; Chiappe, 2005; Griffin, 2002). The Triple-Code Model provides a framework to discuss three different neural coding processes the brain uses to make sense of mathematical symbols and ideas. This hypothetical model has been validated in studies with patients who had suffered from brain injuries or brain surgeries (DeHaene & Cohen, 1995). Neuropsychologists used the Triple-Code Model as a framework for hypothesizing about which mathematical tasks patients would struggle with based on which area of the brain was damaged. Patients performed as predicted, which has contributed greatly to the acceptance of the model as an explanation of how the brain processes numerical information. "The triple-code model assumes that there are essentially three categories of mental representations in which numbers can be manipulated in the brain" (DeHaene & Cohen, 1995, p. 85). A schematization of the Triple-Code Model is illustrated in Figure 2.

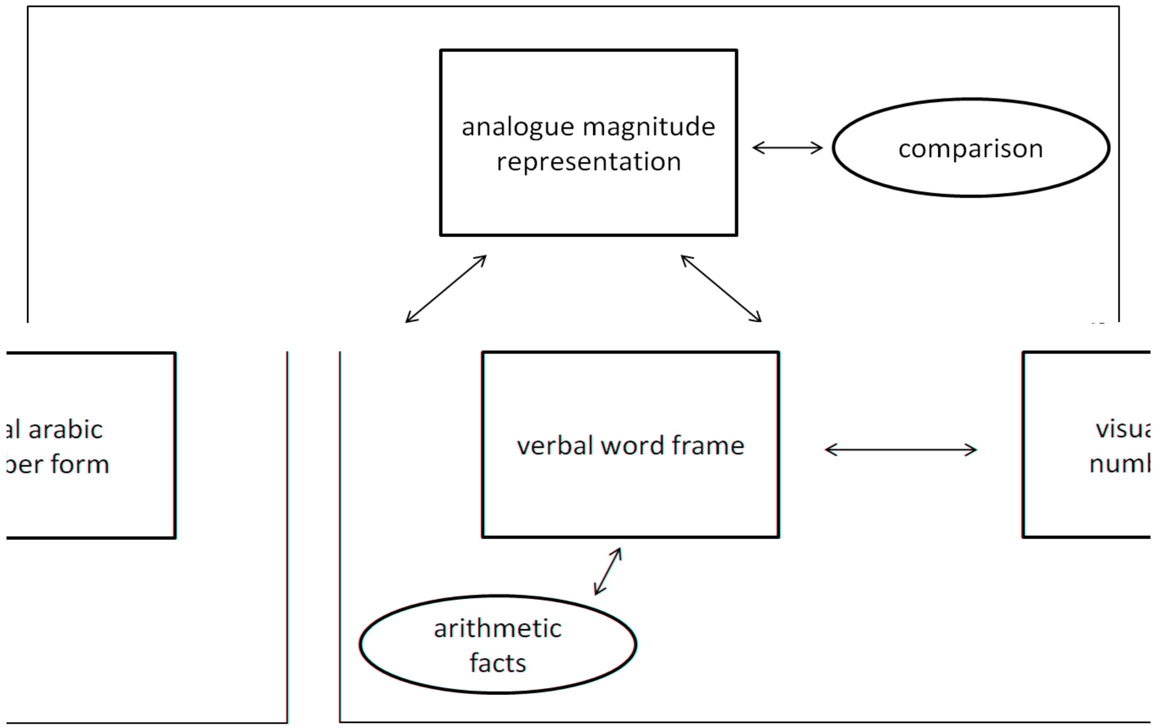


Figure 2: Triple-Code Model (Note: From DeHaene & Cohen, 1995)

The Magnitude Code

The first code, or cognitive function, is to pair a quantity with a symbol or word. To accomplish this, a child must have an intuitive sense of an observed amount such as four crackers and then mentally match that amount to a symbol or word, in this case the symbol 4 or word "four." This ability enables children to represent magnitudes that can be compared as greater than or less than other magnitudes. "At this level, the quantity or magnitude associated with a given number is retrieved and can be put in relation with other numerical quantities" (DeHaene & Cohen, 1995, p. 86). This system allows the brain to make estimations, organize mathematical concepts, utilize mathematics operations, and develop higher level mathematical proofs and problem solving strategies. This representation system contains a variety of "encyclopedic knowledge available about numbers" (DeHaene & Cohen, 1995, p. 97). Such encyclopedic knowledge includes declarative knowledge associated with numbers, such as the fact that 911 is the number to call in an emergency but 9-11 was the date of a significant historical event in America.

The functions of this code take place in the bilateral inferior parietal lobes. Feifer and De Fina (2005) refer to this as the Magnitude code. DeHaene and Cohen call this the analogical magnitude representation, which they describe as an oriented number line. Von Aster and Shalev (2007) also use the idea of a spatially oriented number line to describe this process. One application of this coding system would be to visualize fractional amounts. Coding the magnitude of an amount with the symbol of the amount is comparable to the cognitive skill of matching the sound a letter represents with the symbol. Children who struggle with sound to symbol pairing in reading may also struggle with magnitude to symbol pairing. "Therefore, children who have difficulty with reading fluency and automatically recognizing words in print may also have difficulty with speeded mathematics drills due to a shared inability to recall and retrieve information stored in a language dependent code quickly” (Feifer & De Fina, 2005, p. 37). Their inability to attach symbols to the correct meanings may manifest itself with both letters and numbers.

The Verbal Code

A second coding system in the brain involves encoding numerals as sequences of words, which DeHaene and Cohen (1995) call the “verbal word frame” and Feifer and De Fina (2005) refer to as the “verbal code”. Words are used by the brain to communicate about quantities. By labeling a magnitude or symbol, the brain can then convert the word to either iconic or symbolic form. This coding system holds no semantic meaning of numbers. It operates purely by associating words to symbols or magnitude representations. This system allows the brain to conduct rote mathematical procedures such as counting and retrieving commonly used mathematics facts. For example, with enough practice, the brain will encode the symbols 5 + 2 with the word "seven" without utilizing the magnitude code to visualize the amounts in iconic form. These processes take place in the left perisylvian region of the brain. The verbal word frame differs from the other two codes in that it is not bilateral, which means it does not occur in both hemispheres of the brain. Patients who had severely damaged the left side of the brain lost their ability to recall basic mathematics facts completely because the right side of the brain could not compensate (DeHaene & Cohen, 1995). These processes involve automaticity in matching words with symbols. This coding is comparable to the process the brain uses to pair the name of a letter with its symbol, say the alphabet in order, or memorize the names of the months in order. Children who struggle with such verbal tasks may also struggle with learning the verbal code of mathematics.

The Visual-Verbal Code

A third cognitive function aiding in mathematical processing is the ability to utilize numeric and operational symbols to represent quantities and use the symbols to manipulate numerical amounts. This is the visual-verbal code (Feifer & De Fina, 2005) because it involves the ability to exchange symbols for words and images of quantity. DeHaene and Cohen (1995) call this code the Visual Arabic Number Form. This code uses the brain's ability to replace the visualization of a magnitude with the symbol that represents the magnitude while still holding on to the meaning of the symbol. For example, when a child first learns the concept of 100 they come to understand that it is a big number. The student might visualize a picture of 100 items, such as 100 pennies, and imagine an internal picture, or an icon, of the literal interpretation of the symbol 100. As a learner becomes more familiar and comfortable with the symbol 100 and its corresponding magnitude, the brain can then begin to manipulate the amount 100 using only the symbol. It no longer needs to rely on the iconic visualization of the number, or magnitude code. The brain skips that coding system as it seeks for efficiency in manipulating the amount 100.

In studies conducted on brain damaged patients, participants were able to solve problems by relying on recognition of symbols and their intact magnitude code even when their verbal recognition of numbers was nonexistent (DeHaene & Cohen, 1995). In one case study, a patient could not recall the names of numbers when shown number symbols but could accurately compare single and two-digit numbers. This patient also held onto semantic knowledge about numbers such as the fact that an hour is 60 minutes. He was able to accurately judge unreasonable answers when shown incorrect arithmetic problems. This and other similar case studies showed that although patients with damaged left hemispheres could not utilize their verbal code, they could reason about numbers by relying on their other two intact coding systems. Although they could not pair the word to the symbol, they were able to associate the iconic representation to the symbol. This process involves the bilateral occipital-temporal lobes.

The visual-verbal code allows the brain to visualize a number line, write a story problem as an equation, exchange one operation for an equivalent one and represent a number with a variable. It also allows the brain to manipulate larger quantities that it cannot conceptualize with the magnitude code because this coding system also does not rely on knowledge of meaning to function. "It manipulates Arabic and word symbols ‘blindly,’ without knowledge of their meaning. On the other hand, the paths to and from the magnitude representation are supposed not to be syntactically sophisticated" (DeHaene & Cohen, 1995, p. 87). This explanation fits with Bruner's ideas that humans will utilize language and symbols to manipulate numbers they cannot necessarily represent with images (Driscoll, 2005, p. 229). Thus, the brain's ability to represent a magnitude on an internal number line becomes less precise as numbers get larger.

Although the visual-verbal code integrates words, symbols, and the abstract visualization of magnitude, it is not dependent on language. Thus, it is likely that students who easily manipulate symbols abstractly have cognitive strengths in this area. In contrast, students with cognitive weaknesses in non-verbal tasks may struggle with the association of symbols to magnitude. They may not "see it" internally as other students do. When a student cannot visualize three dots and four dots combined when they see the symbols 3 + 4, they are not able to convert their understanding of magnitude to symbolic representation. Another example of what may be a challenging concept for students with this weakness would be visualizing that when two fractions with the same numerator have different denominators, the fraction with the larger denominator is actually the smaller magnitude. These students rely heavily on language to make sense of mathematics, evidenced by the long-term use of counting strategies and memorized language patterns. They draw on memorized procedures to solve multistep arithmetic problems and can utilize their verbal strengths to recall basic fact combinations. Students with deficits in the visual-verbal code will likely need long-term support in mathematics in order to compensate for their inability to associate magnitude to symbolic and linguistic form. It is interesting to note that students with this area of deficit can appear to be successful in school mathematics because they can use their language abilities to get the right answers to arithmetic problems. Thus, these students' struggles with understanding mathematics may get overlooked when screening measures that emphasize only computation are used.

Triple Code Summary

Table 1 summarizes the three processing codes based on the Feifer and De Fina (2005) labels. It lists the parts of the brain used for each coding system and examples of mathematical tasks associated with each code.

Table 1. Three Coding Processes

Type of Code	*Parts of Brain*	*Function*	*Examples of Task*
Verbal Code	Perisylvan Region Left Hemisphere	Encodes numbers as words	Basic mathematics fact recall Naming digits Counting
Visual-Verbal Code	Bilateral Occipital-Temperal Regions	Attaches symbol to amount or word	Visualizing a number line Number recognition Create an equation from a story problem

Type of Code	*Parts of Brain*	*Function*	*Examples of Task*
Magnitude Code	Bilateral Inferior Parietal Regions	Pairs quantity to a symbol	Problem solving Estimating Visualizing amounts Comparing Fractions

Note. Adapted from Feifer, and De Fina, 2005.

Although various researchers have expanded upon the Triple-Code Model or use different language to describe each of the three processes (Von Aster, 2000; Von Aster & Shalev, 2007; Sousa, 2008), there is consensus among brain researchers regarding these three types of mathematical processing. These coding systems work in tandem to help the brain interpret and create meaning about numbers. It can receive information about numbers as a word, an amount of objects, or an Arabic symbol and convert that input to any of the three forms of the number to work with the information. The brain can then utilize any of the three codes to convert numerical information to whatever form of output is needed for the task at hand.

Triple-Code Model and Theories of Learning

Rather than describing learning as one cognitive process, the Triple-Code Model shows that learning involves many different processes that are interrelated. The Triple-Code Model complements Jerome Bruner's theory of learners progressing through levels of understanding from enactive to iconic to symbolic (Driscoll, 2005). Recognition of the symbol by itself does not allow the brain to create meaning. It is the ongoing exchange between the magnitude code and the visual-verbal code that allows the brain to interpret and use the meaning of the symbol. This implies instruction designed to help students connect symbols to iconic representations of magnitude can help them become more efficient at the abstract manipulation of numbers. This is consistent with Bruner's views of how instruction can aid conceptual development. While Bruner saw the progression from enactive to iconic to symbolic as a sequential process aided by instruction, the Triple-Code Model suggests that understanding a mathematical idea as a symbol, with language and as an iconic representation can happen simultaneously–that these levels of understanding are intertwined and develop together. This theory provides support for the instructional practice of utilizing multiple representations to help students conceptualize mathematical contexts (Tripathi, 2008).

The Triple-Code Model also validates ideas from a behaviorist perspective of learning. The verbal code is capable of forming strong associations between a symbol and a word. With practice, this association can become automatic and rapid. However, this association does not require meaning to be attached. This explains why children can show progress in memorizing arithmetic facts and increase in speed and accuracy on timed mathematics drills, yet still show a lack of understanding about mathematical concepts. Utilizing the verbal code alone in instruction will not help children interpret mathematical notation. Unless the other two codes are integrated with the verbal code, children are memorizing meaningless associations between symbols and words. Mathematical proficiency requires a much higher level of understanding in order for students to become effective problem solvers.

The Triple-Code Model acknowledges language plays a key role in the integration of the symbolic and iconic representations of number. This is consistent with socio-cultural theories of learning, which suggest language, culture, and context are influential factors in learning. Instruction that promotes discourse helps children utilize their language and background knowledge, or schemas, to integrate symbols with iconic representations and language. It helps them interpret and makes sense of numbers. The role of language in numerical processing is discussed in more detail in the next section.

Role of Language

Performing mathematical calculations and solving contextual mathematical problems utilize all three coding systems. Thus, the idea that one part of the brain is used for reading processes and another part is used for mathematical processes is not supported by research. Both language and mathematical processes involve multiple parts of the brain working together. In some studies, participants given the same mathematical tasks activated different parts of the brain depending upon the strategy they used to solve the problem. One patient had a damaged left hemisphere, which is associated with retrieval of memorized basic number combinations. This patient could not retrieve answers for simple addition and multiplication facts using the left hemisphere function of recall and word to symbol association. However, when the number combination was presented orally instead of symbolically, the patient was able to perform the simple calculations by relying on his intact word-to-magnitude pathway, which is mediated in the right hemisphere (DeHaene & Cohen, 1995, p. 107). This patient was able to associate hearing the number word to the representation of magnitude and use the internal iconic image of the numbers to solve the problem. Case studies such as this one provide strong evidence there is an ongoing exchange between numerical representations and the verbal component of mathematical reasoning.

> Mental arithmetic is intimately linked to language and to a verbal representation of numbers. The retrieval of arithmetic facts from memory relies on a subset of left-hemispheric language areas and cannot be performed by the right hemisphere alone. The procedures for multi-digit calculations are more complex and involve the coordination of visuo-spatial and verbal representation of the digits. (DeHaene & Cohen, 1995, p. 89)

This suggests instruction connecting language to symbols is critical in developing mathematical concepts and skills and is consistent with socio-cultural theories of learning, which emphasize the role of discourse and context to aid conceptual understanding. The ongoing interplay between various parts of the brain while engaged in mathematical tasks makes diagnoses and intervention for children who are having difficulty learning mathematics extremely complex.

There is an overlap among the areas of the brain that process understanding of quantity and language (Sousa, 2008; Feifer & De Fina, 2005; DeHaene & Cohen, 1995). In studies that involve pairing a number symbol to an amount, such as a specific number of dots, the brain uses primarily the left parietal lobe of the brain. However, when language number words needed to be paired to the symbol or amount, the brain used Broca's area, the left frontal lobe to retrieve the information. Thus, when the brain needs to pair its intuitive understanding of quantity to the language used in the person's culture to describe that quantity, both parts – language and visual-spatial processing – of the brain were engaged. "Unfortunately, the brain does not draw such dichotomous distinctions between linguistic and mathematic endeavors, but rather relies on elements from multiple cortical regions that subserve both of these tasks. In other words, there is a linguistic element to math, just as there is a spatial element to language." (Feifer & De Fina, 2005, pp. 15-16). Geary (2005) concludes "the language systems are important for certain forms of information representation, as in articulating number words, and information manipulation in working memory, as during the act of counting" (p. 8). These findings call into question the common saying that mathematics is a universal language. They also provide insight into the reasons students with language-related learning disabilities often also struggle to learn mathematics. Current research on MLD complements research on the Triple-Code Model of numerical processing.

Mathematics Learning Disabilities

There are several different types of cognitive disabilities that could contribute to mathematical difficulties. Research on learning disabilities has identified three different types of MLD that reveal cognitive processing deficits related to one or more of the coding systems explained by the Triple-Code Model (Geary, 1993, 2004; Von Aster, 2000; Feifer & De Fina, 2005; Sousa, 2008). An MLD can manifest itself as a deficit in conceptual knowledge or procedural knowledge (Geary, 2004). MLD can be caused by cognitive, neuropsychological, and genetic components (Geary, 1993). If an underlying cognitive deficit is in the visual-spatial system, or the magnitude code, a child will likely struggle with the representations of number magnitudes. If the deficit involves the use of the verbal code, then mathematical fact retrieval and remembering procedures may be difficult. When the interaction between the symbolic and verbal forms of numbers, which utilizes the visual-verbal code, is impaired, a child will typically struggle with conceptual understanding and converting language to symbolic form.

Operational Definition

One of the inconsistencies making interpreting research on mathematics disabilities difficult is the lack of a clear definition of an MLD. Terms such as mathematics difficulties, mathematics disabilities, dyscalculia, and poor mathematics achievement are all used to describe study participants (Mazzocco, 2005). Selection criterion for study samples differ. Some studies have focused on children who have been qualified by their schools for special education services in mathematics. Some studies use a standardized test to select the population sample, but these studies differ in the cut off used to determine whether a student is having difficulty in mathematics. Studies focusing on students with MLD have used the 10th, 25th, 31st, 35th, or 45th percentile as a cutoff (Mazzocco, 2005, p. 319). This lack of a consistent operational definition for MLD has limited the research community's ability to make comparisons among studies and generalize conclusions. Geary (2005) and Gersten et al. (2005) recommend using the 35th percentile on a standardized test as a cut off to identify students as having an MLD. They suggest that this cut off is high enough to make sure students who compensate for cognitive deficits related to mathematics by relying on other cognitive strengths will be in the study sample. Mazzocco (2005) suggests the term *mathematics difficulty* (MD) be used to describe students who have low mathematics achievement but may or may not have an MLD. He proposes the term *mathematics disability* (MLD) be reserved for students who have biologically-based limitations that contribute to their low mathematics achievement. Mazzocco acknowledges it will not always be easy to determine whether a child has mathematics difficulty or a mathematics disability and there is still much that is unknown about biologically-based mathematics disabilities. Gersten et al. (2005) also distinguish between the terms *mathematics disability* and *mathematics difficulty* and use them as suggested by Mazzocco. This study will use these terms as recommended by these researchers.

Domain General Disabilities

Geary (1993) explains the central executive functions of the brain can impact mathematics achievement due to deficits in executive functioning and working memory. Executive functioning deficits can contribute to an inability to generalize mathematical ideas from one situation to another, which impacts problem solving skills. Working memory deficits can display as a weakness with procedural memory. Learning disabilities caused by central executive functions are described as domain-general disabilities (Chiappe, 2005; Geary, 1993; Von Aster & Shalev, 2007). This type of a disability would most likely manifest itself in all subjects in school, not just mathematics. However, domain-general disabilities can have a huge impact on mathematics achievement even though the child's number processing abilities are average or even above average. Some examples of domain-general disabilities include attention deficit disorder, bipolar disorder, slow processing speed, working memory deficits, and auditory processing disorders. Several studies have also shown that reading and language disabilities can contribute to difficulty in mathematics due to the verbal component of conceptual understanding and problem solving (Geary,1993, 2004, 2005; Gersten et al., 2005; Von Aster & Shalev, 2007).

A visual-spatial processing weakness is a domain-general disability that can cause children to have difficulty with mathematics (Feifer & De Fina, 2005; Sousa, 2008). This disability is comparable to a non-verbal learning disability (NLD). It impairs a child's ability to manage and understand non-verbal learning assignments. "Thus, students with NLD will have problems with handwriting, perceiving spatial relationships, drawing and copying geometric forms and designs, and grasping mathematics concepts and skills" (Sousa, 2008, p. 184).

Domain-specific disabilities are those specific to one subject, such as mathematics or reading. The impact on achievement is limited to the specific domain and does not typically impact achievement in other subjects. This means that some students with MLD can be proficient in most areas of school, but struggle in mathematics. Many students with domain-specific reading and language disabilities also have an MLD. However, some students with reading disabilities can do very well in mathematics. It is often difficult to determine whether a student's struggles in mathematics are related to a numerical processing deficit or a language processing deficit (Geary, 2004; Gersten et al., 2005; Von Aster & Shalev, 2007).

Dyscalculia

MLD is also called dyscalculia. This is a cognitive disability affecting a person's ability to process numerical calculations. "Dyscalculia is a difficulty in conceptualizing numbers, number relationships, outcomes of numerical operations, and estimation, that is, what to expect as an outcome of an operation" (Sousa, 2008, p. 179). An estimated 5 to 8 percent of children have this disability (Geary, 2005). Developmental dyscalculia is when the condition is present from birth. Acquired dyscalculia is when the condition develops after birth, such as from a brain injury or stroke. "The neurological basis of developmental dyscalculia is an impairment in the child's innate ability to subitize" (Sousa, 2008, p. 179; see also Von Aster & Shalev, 2007). Subitizing is the ability to quickly pair the number word with an amount of objects (Clements, 1999). Rather than counting a number of objects, most humans can quickly associate amounts up to seven with their corresponding number word. The ability to subitize develops very young. Thus, evidence that a child has developmental dyscalculia can appear as early as first grade.

Children with an MLD "have a poor conceptual understanding of some aspects of counting" (Geary, 2004, p. 6). Because some children with dyscalculia cannot visualize the magnitude of a number, they learn to complete mathematical tasks using different strategies. They rely heavily on sequencing and memorizing. However, when the content gets too immense, their counting strategies become cumbersome and inefficient. This often occurs in third and fourth grade as students encounter multiplication and operations with multi digit numbers (Sousa, 2008).

Several researchers (Sousa, 2008; Feifer & De Fina, 2005; Von Aster, 2000; Geary, 1993, 2004) describe three different types of mathematics disabilities, which have been categorized as subtypes of dyscalculia. Although these different researchers use different labels for the three types, their descriptions of each type are similar. However, the differing and sometimes conflicting use of terms throughout this body of literature makes organizing this information for practitioners challenging. For this discussion, the three types described by Geary (2004) will be used to compare and contrast the views of other researchers.

Procedural Subtype

The first type of dyscalculia is called a procedural deficit. Students with this type of MLD typically struggle with the execution of procedures. They use developmentally immature strategies to solve problems and have a poor understanding of the concepts underlying procedural use. Because these students struggle to comprehend the syntax rules of the numeric symbol system, arranging and executing arithmetic procedures is difficult for them. Sousa (2008) describes this type of mathematics disability as qualitative, which is difficulty in conceptualizing mathematics processes and spatial sense. Feifer and De Fina (2005) also refer to this as the procedural subtype. They elaborate that this type of MLD inhibits a child's ability to link numeric systems into a meaningful language system. Children who have this type of dyscalculia have difficulty reading numbers aloud and may struggle to write numbers from dictation or convert a story problem to its symbolic form. They also have difficulty recalling the sequence of steps to a mathematics algorithm. However, these students can determine magnitude and recall basic mathematics facts.

Von Aster (2000) analyzes the three subtypes of MLD while considering the application of the Triple-Code Model of numerical processing. He labels the procedural subtype of dyscalculia as the Arabic subtype pattern. These students seem to have deficits in the visual-verbal code but can process numbers using the magnitude and visual code. Since this subtype integrates the use of both symbols and language, Von Aster discusses the difficulty in determining whether deficits in this area are the result of language difficulties, lack of instruction or numerical processing weaknesses.

Semantic Subtype

The second subtype of dyscalculia defined by Geary (2004) is the semantic subtype. This subtype affects a child’s ability to count and calculate. This subtype corresponds to a weakness in the verbal code. It impairs a child's ability to retrieve the word associated with symbols. This subtype can become evident when children are delayed in matching the word form of numbers to symbolic form and when they begin working with basic facts.

> . . . The ability to retrieve basic facts does not substantively improve across the elementary school years for most children with MLD/RD and MLD only. When these children do retrieve arithmetic facts from long-term memory, they commit many more errors and often show error and reaction time (RT) patterns that differ from those found with younger, typically achieving children. (Geary, 2004, p. 8)

While some children are only delayed in the development of their verbal code and respond to early intervention, other children struggle with the word-to-symbol matching throughout their school careers. Sousa calls this subtype the quantitative subtype. "Memory deficits do not seem to improve with maturity. Studies indicate that individuals with this problem will continue to have difficulties retrieving basic arithmetic facts throughout life" (Sousa, 2008, p. 184). Students with this type of dyscalculia will likely not improve in basic fact recall even after intervention. Both Von Aster (2000) and Feifer and De Fina (2005) call it the verbal subtype. According to Feifer and De Fina, the verbal subtype of dyscalculia inhibits a child’s ability to use language-based procedures to assist in arithmetic fact retrieval skills. “Verbal dyscalculia does not hinder a student’s ability to appreciate numeric qualities, understand mathematical concepts, or detract from making comparisons between numbers, but does hinder a student’s ability to encode and retrieve mathematics facts stored in a verbal format automatically most notably multiplication and addition" (Feifer & De Fina, 2005, p. 39). Children who have verbal dyscalculia frequently also struggle with the symbol-to-sound pairing required for reading. Thus, "while children with verbal dyscalculia frequently also have difficulty learning language arts skills, children with a procedural error subtype tend to have learning difficulties solely related to math” (Feifer & De Fina, 2005, p. 39).

Visuospatial Subtype

The third subtype of dyscalculia is the visuospatial subtype and involves the magnitude code. Feifer and De Fina’s (2005) description of what they call the semantic subtype of dyscalculia matches Geary's (2004) description of this subtype. Sousa calls this the mixed type, and Von Aster calls it the pervasive subtype pattern. Students who have this subtype of dyscalculia are unable to decipher magnitude representations among numbers. They struggle to spatially represent numerical and other forms of mathematical information. They cannot internally visualize relationships between numerical concepts.

These students frequently misinterpret spatially represented information. Some examples of challenges these students may have would be mentally reversing the order of a clock or the positive and negative directions on a four quadrant Cartesian plane. “Furthermore, the semantic comprehension of numbers also allows for transcoding mathematical operations into more palatable forms of operations, which is also a key executive functioning type of attribute” (Feifer & De Fina, 2005, p. 41). Students with this type of dyscalculia might struggle to convert 9 x 4 to (10 x 4) - 4. They might also write 24 as an answer to 2 x 4. These errors also help illustrate Sousa’s definition of this subtype as an inability to integrate quantity and space. Von Aster (2000) describes this subtype as difficulty with encoding "the semantics of numbers” (p. 49). Children with this subtype of MLD do not develop the internal number line used to make judgments about magnitude of numbers in the same way that typically developing children do. This causes difficulty with judging the reasonableness of answers to problems, estimation, and comparing quantities. Children with difficulty in this area of numerical processing can sometimes be very proficient with basic facts because the verbal code that allows them to pair words to symbols is intact. They can also utilize their visual-verbal code and rely on language and conceptual understanding to solve problems.

Summary of Types of Disabilities

This section has presented different types of disabilities, which can contribute to poor mathematics performance in school. It has also reviewed three different types of dyscalculia. Table 2 provides an overview of the three types of dyscalculia described by each author, a description of the subtype and an example of an error a student with that subtype might make.

Table 2. Types of Dyscalculia

Type of Dyscalculia				Description	Common Student Errors
Geary	Von Aster	Sousa	Feifer/ De Fina		
Procedural	Arabic Subtype Pattern	Qualitative	Procedural	Cannot link language to symbols	-Cannot convert a story problem to numeric form -Have difficulty recalling sequence of steps
Semantic Memory	Verbal Subtype Pattern	Quantitative	Verbal	Cannot use language to match symbols to words	-Struggle to match words to numbers -Cannot recall basic mathematics facts
Visuospatial	Pervasive Subtype Pattern	Mixed	Semantic	Cannot decipher magnitude representation	-Cannot convert 9x4 to (10 x 4) – 4. -May answer 2 x 4 as 24 -Do not recognize unreasonable answers. -May reverse a clock or number line.

The different subtypes of dyscalculia, along with the fact that other learning disabilities can impact mathematics achievement, makes diagnosing an MLD very complex. It is often difficult to rule out cultural and linguistic factors that might contribute to poor performance in mathematics. Furthermore, some children develop slower than others. A child in the primary grades who appears to have a mathematics disability could actually just be delayed in the development of one or all three cognitive processes needed to mentally manipulate numbers. These children can grow out of their difficulties in mathematics with appropriate early intervention. Other children with dyscalculia struggle with mathematics throughout their lives and will need strategically designed instruction in order to be successful in mathematics. The next section of this literature review will focus on the complex issues related to diagnosing mathematics disabilities.

Diagnosis of Mathematics Disabilities

As children fall behind in their mathematics skills when compared to grade level standards, they are often referred to special education. The traditional method presently used in special education to diagnose a student with an MLD is inadequate.

> Most school districts define a learning disability as a significant discrepancy between a student's intelligence quotient and their performance on a standardized test of academic achievement. Therefore, it does not matter how a student is functioning in relation to their own peers, the general curriculum, or certain benchmark standards. Instead, school systems place an undue importance on how the student is functioning with respect to their potential as determined by their performance on an intelligence test measure. (Feifer & De Fina, 2005, p. 24)

This system of diagnosis does not take into account the variety of factors contributing to a child's development of mathematical abilities. Cultural and social factors, pre-school experiences, play opportunities, quality of parenting, teacher skills, values of peer group, curriculum emphasis and emotional attributes are factors that could contribute to a child not developing mathematical skills in comparison to other children at his grade level. Sousa (2008) identifies three main reasons for mathematics difficulties:

inadequate instruction, environmental factors, and cognitive disabilities. Anxiety and other emotional difficulties can also impair the brain's ability to learn. "Low performance on a mathematics test may indicate that a problem exists, but tests do not provide information on the exact source of the poor performance" (Sousa, 2008, p. 164). The complexity of mathematical content can further confound diagnosis of a disability (Geary, 2005). "There is considerable potential to confuse difficulty in learning complex material with an actual cognitive deficit - that is, a disability that impedes learning even with appropriate instruction" (Geary, 2005, p. 306).

Developing Effective Assessments

Presently, assessments that specifically diagnose an MLD are not available (Geary, 2004; Mazzocco, 2005). "Not many measures are available to tap for formal and informal math ability" (Mazzocco, 2005, p. 320). Research on predictive indicators and the development of validated measurement tools to assist with the diagnosis of MLD is a research field "in its infancy" (Gersten et al., 2005). Without the ability to accurately identify the source of a child's difficulty in mathematics, the present system of qualifying students as learning disabled in mathematics is giving them a diagnosis without a cure. Educators need a clear understanding of the cause of the low achievement in order to provide intervention that will help students overcome their difficulties in mathematics. Better assessment tools will lead to improved intervention for students who are performing below grade level in mathematics (Geary, 2004, 2005; Feifer & De Fina, 2005; Gersten, et al., 2005; Mazzocco, 2005; Chiappe, 2005).

Geary (2005) suggests research in cognitive and developmental psychology provide the basis for the development of assessment measures in order to determine whether a child may have a cognitive disability that contributes to their inability to learn mathematics. He contends standardized achievement tests should only be used as initial screening measures to identify students who may have a cognitive disability hindering their ability to learn mathematics. Because standardized achievement tests assess a range of topics and skills, students who have difficulty learning mathematics may have cognitive deficits that interfere with some areas of mathematical proficiency but not others. Thus, high scores in some areas can mask difficulty in other areas of mathematics achievement.

A standardized test score may assist with identifying students who have difficulty learning mathematics, but it will not contribute to determining the cause of the difficulty. "Many children who score low on achievement tests for any single assessment do not appear to have an underlying cognitive deficit and, in fact, often show average test scores on later assessments" (Geary, 2005, p. 305). There is a need for the development of more sensitive measures to diagnose specific areas of deficit (Geary, 2005; Gersten et al., 2005; Mazzocco, 2005; Chiappe, 2005). Geary mentions several areas children with cognitive deficits typically struggle in even after intervention. One area is a persistent deficit in the ability to retrieve number combinations from long-term memory. Another deficit is children's conceptual understanding of counting and numerical relationships. He encourages the development of measures that identify patterns of cognitive strengths and weaknesses of children who may potentially have a learning disability in mathematics.

Using Assessments to Design Accommodations

Feifer and De Fina (2005) also suggest using diagnostic assessments to identify areas of cognitive strengths and weaknesses. These authors suggest specific accommodations be designed based on a student's weaknesses to compensate for cognitive deficits. They suggest examining several key areas of mathematical skills to determine whether a child may have a cognitive deficit that interferes with the child's ability to learn mathematics. They utilize a variety of standardized assessments to examine skills in working memory, executive functioning, mathematical skills, number sense, and mathematics anxiety. They provide samples, suggestions, and resources for developing a wider range of assessment tools to assist with diagnosing mathematical difficulties. Feifer and De Fina recommend using a range of assessments to identify a student's strengths and weaknesses in how they process numerical tasks and building intervention plans that utilize a child’s strengths while accommodating for his weaknesses.

Bryant et al. (2006) have created an Adaptations Framework based on examining a student's strengths and struggles. This framework guides teachers through designing accommodations and adaptations for SWD. The four steps of the process include setting specific demands and identifying what tasks and requisite abilities are needed to complete the demands. Next, teachers examine student strengths and struggles for completing the task. Areas of struggle can be gleaned from formal assessments or from informal assessments and experiences working with the child. Teachers then create adaptations or accommodations that are task specific. The proposed adaptation should allow the student to utilize his areas of strengths and compensate for areas of weakness to help him complete the learning activity. The last step of the process is to evaluate whether the adaptation or accommodation was successful for the student. Over time, successful adaptations and accommodations for that particular student would become more obvious based on this reflective process. This framework helps students with mathematics disabilities participate in the same grade level instruction as their peers. It also has potential as a way of collecting data about an individual student for diagnostic purposes. The effectiveness of this framework needs further validation through empirical research but it seems to epitomize the type of "strategic customization" of instruction encouraged by the NCSM (2008). It also shows promise as a way of involving the classroom teacher in diagnosis and intervention for struggling students.

Identifying the Core Deficit

An important research topic related to diagnostic assessments for MLD is the search for the core deficit contributing to mathematical difficulties (Gersten et al., 2005; Gersten & Chard, 1999; Chiappe, 2005; Dowker, 2005; Mazzocco, 2005). In research on reading disabilities, a major breakthrough in diagnosis and intervention for early reading difficulties occurred once the relationship between phonological awareness and reading achievement was clearly established. This occurred through a converging consensus from a plethora of studies on reading difficulties. By comparison, there is only a minimal amount of research in the field of mathematics difficulties (Gersten et al., 2005). If "robust predictors" (Chiappe, 2005, p. 316) of long term difficulty in mathematics can be established, those predictors can be used for both the development of diagnostic instruments as well as intervention curriculum. "If the deficit that plays a causal role in academic failure is remediated, then improvements should be seen in the relevant academic domain" (Chiappe, 2005, p. 316).

Gersten et al.'s (2005) review of findings of research on mathematics difficulties (MD) discussed several mathematical abilities that might indicate a student has a learning disability in mathematics. One area referred to as "a hallmark of mathematics difficulties" is calculation fluency (Gersten et al., 2005, p. 296), which is efficiency and maturity with counting strategies. Another indicator discussed by these researchers is number sense. A lack of number sense is often indicated by a child's inability to make judgments about magnitude (Gersten & Chard, 1999). Working memory and the relationship between reading difficulties and mathematics difficulties are two other areas identified as potential predictive indicators in this review. Weaknesses in spatial abilities are mentioned by Gersten et al. (2005), but not discussed in detail. This seems to be a significant omission to the work by Gersten et al. considering Geary (2004) suggests that visuospatial skills are related to MLD. These potential indicators of MLD will be discussed in more detail.

Basic Facts

A seminal study by Pellegrino and Goldman (1987) provided a convincing argument there was a strong correlation between automaticity of basic mathematics facts and mathematics achievement (cited by Gersten & Chard, 1999). Because of this argument, a great deal of research and intervention in special education has focused on remediating this weakness. However, correlation does not mean causation nor does it imply importance. This influential work by Pellegrino and Goldman was actually not experimental research but a review of literature and a theoretical framework to guide future research. They were presenting a hypothesis about the relationship between basic facts and the ability to execute mathematical procedures. They proposed students who struggle with basic addition and subtraction facts spend a great deal of cognitive resources on recalling basic facts while trying to make sense of more complicated arithmetic procedures such as subtraction of two digit numbers across zeros.

Pellegrino and Goldman's suggestions were based on information processing theory. They suggested recall of basic facts was stored as declarative memory, and steps for solving more complex problems were stored as procedural memory. They proposed practice, which strengthens the association between a fact and its answer, would shift solving the fact from procedural to declarative memory and decrease the response time, thus making all work in arithmetic more efficient. This theoretical framework presented a topic for further investigation, but was translated into practice in special education. It led to a heavy emphasis on drill and practice of basic facts as the focus of remedial mathematics instruction. A positive correlation between speed of retrieval of basic arithmetic facts and mathematical achievement on a standardized test was not statistically validated in Pellegrino and Goldman's review.

Many years of such instruction has not led to an improvement in the achievement of SWD in mathematics. Thus, using basic facts as a predictor of mathematics achievement may not be effective. Pellegrino and Goldman's theory does not take into account the developmental progression children usually move through as they learn to efficiently execute basic facts (Baroody, Bajwa, & Eiland, 2009; Baroody, 2006; Gersten

& Chard, 1999). Rather than seeing basic fact retrieval as an interrelationship between declarative and procedural memory, Pellegrino and Goldman saw knowledge of number concepts and basic fact retrieval as two separate cognitive processes.

The Triple-Code Model provides an explanation for Pellegrino and Goldman's (1987) observation noting some children seem to recall answers to basic facts effortlessly utilizing their verbal code. However, their theory does not take into account many children quickly, accurately, and flexibly utilize iconic representations of numbers as well as language to quickly calculate answers to basic facts. Pellegrino and Goldman acknowledged that some children who struggle to learn mathematics seem to be very efficient with basic facts but have trouble with conceptual schema for representing quantitative relations. Other children who seem to have strong understanding of numerical concepts and relationships struggle with instant recall of facts. This observation is consistent with the conjecture that there are different types of mathematics disabilities and the Triple-Code Model. Intervention that only focuses on retrieval of basic facts could be the wrong treatment for the wrong problem. In other words, what the research is not saying is basic facts may not be the necessary place to start remediating students in mathematics.

The rationale to work on basic facts, although possibly not an accurate one, was that the lack of automaticity was the cause of the mathematical failure, not a symptom. If intervention could strengthen a child's abilities in basic fact recall, then the hypothesis was achievement in basic skills mathematics would improve. In other words, basic fact recall was seen as the most robust predictive indicator of lower-order mathematics achievement as measured by most state standardized achievement tests – not basic problem solving or higher-level mathematical thinking. This emphasis on a drill and practice intervention was comparable to similar research and intervention on decoding skills in reading. The same strategies that were proven effective to increase automaticity with decoding and fluency were applied to increasing automaticity with basic facts – even though reading and mathematics are clearly not the same. For instance, fluency drills, flash cards, recitation, and chanting are strategies used to increase speed with decoding phonemes. In mathematics, strategies such as timed delay response, repetition, and rote rehearsal were used in intervention programs to improve automaticity (Mastropieri, Scruggs, & Shiah, 1991; Mastropieri, Scruggs, & Chung, 1998; Mastropieri, Scruggs, Davidson, & Rana, 2004; Miller, Butler, & Lee, 1998). These strategies utilized behaviorist techniques to help students memorize basic facts. Counting and reasoning strategies were not encouraged because the emphasis was on instant recall. While these instructional techniques may have strengthened associations and led to memorization and speed in recall of basic facts, these strategies do not lead to greater understanding of the numerical concepts supporting the problem (Gersten & Chard, 1999). Furthermore, children who have a cognitive deficit inhibiting their ability to recall verbal information may always struggle with basic facts (Geary, 2005). For these children, spending a great deal of instructional time on something that will always be a cognitive deficit for them is counterproductive. Instead, these children need to be taught strategies to help them compensate for their inability to recall basic facts and participate in mathematics instruction that engages them in higher level content and problem solving. A greater understanding of the cognitive processes used to derive basic mathematics facts when an answer cannot be recalled is needed. In addition, the growing research base that some students with dyscalculia will always struggle with basic fact recall suggests intervention focusing only on basic fact recall may not be meeting the needs of students with MLD.

Judgment of Magnitude

Difficulty with number sense can be detected by probes that explore judgment of magnitude. Tests assessing children's understanding of magnitude, counting, and differences in quantities show promise as predictive indicators (Chiappe, 2005; Gersten et al., 2005; Sousa, 2008; Feifer & De Fina, 2005). Gersten et al. report on the predictive validity of a variety of measures that predicted subsequent performance in arithmetic. They concluded that a test called the Number Knowledge Test (NKT), developed by Okamoto and Case (1996), was the best predictor of mathematics achievement on the arithmetic tests. This test assessed skills in "understanding magnitude, the concept of 'bigger than,' and the strategies they use in counting" (Gersten et al., 2005, p. 297). For example, one subtest asked children to identify a missing number in a sequence. Another asked them to determine which quantity was bigger. In one study with a sample of 65 kindergarten students, this test had a predictive validity .72 for Total Mathematics on the Stanford Achievement Test 9 (SAT-9) administered in spring of first grade. The predictive validity was also .72 on the NKT given again one year later. In both analyses, $p < .01$ (Gersten et al., 2005, p. 299). These researchers concluded assessments evaluating number sense may be strong indicators of future success in mathematics. In this study, mathematics achievement was defined as performance on a standardized achievement test, specifically SAT-9. The achievement test was given one year after the diagnostic assessment. The two SAT-9 subtests used in this study to determine mathematical achievement were the Procedures subtest and the Problem Solving subtest. Detailed information about the complexity of the problems is not provided by the researchers. Thus, the relationship between number sense and more complex problem solving is not clear from Gersten et al.'s work.

Students with cognitive deficits related to the development of number sense may struggle to visualize the placement of two numbers on a number line and have difficulty comparing magnitude. Because they have difficulty with subitizing and right to left orientation, a child with an MLD caused by the inability to visualize magnitude will rely on counting and sequencing to compare numbers rather than visualization (Sousa, 2008; DeHaene & Cohen, 1995). As numbers get further apart, a child who can easily visualize magnitude usually decreases his response time because it is easier to "see" the larger number. Determining that eight is greater than three is easier than determining that four is greater than three because there is a greater distance. A child who is relying on counting to determine magnitude will often increase his response time as the numbers get further apart because he has further to count between numbers. This simple test can possibly be a predictive indicator of a child's ability to judge magnitude (Feifer & De Fina, 2005; Sousa, 2008).

Children who struggle to visualize magnitude may also have difficulty moving across representational systems. For instance, reading a simple subtraction problem, drawing a representation of the problem and then writing the problem as an equation may be difficult for a child who struggles with visualizing magnitude. Thus, intervention that engages students in representing problems with words, mathematical notation and as visual models could reveal a child's strengths and weaknesses in their ability to move across representational systems. "The role of visual imagery, language and working memory functions have recently been identified as being important in the development of the mental number line" (Von Aster & Shalev, 2007, p. 868).

Working Memory

Difficulties with working memory can be valid predictors of success with mathematical procedures and recall of basic facts (Geary, 2004; Sousa, 2008; Feifer & De Fina, 2005). Difficulty with working memory tasks is considered a domain-general disability that would impact many aspects of school performance. An example of a working memory task related to mathematics would be repeating a string of numbers either forward or backward. A subtest of the NKT is called Digit Span Backward. In a Kindergarten sample of 65 students, the subtest alone had a predictive validity of .47 on the first grade performance on the SAT-9 and a predictive validity of .48 on NKT administered in first grade with a sample of 64 students, $p < .01$ on both measures (Gersten et al., 2005). Rapid automatized naming is another subtest on the NKT that requires children to rapidly name colors and pictures. This test had a predictive validity of .34 ($p < .01$) on first grade performance on the SAT-9 and .31 ($p < .05$) on the NKT with the same sample of Kindergarten children. These results indicate working memory tasks that involve numbers are better predictors of mathematics performance on a standardized test than working memory tasks that do not involve numbers.

It is important to note working memory assessments are subtests in many psychological assessments conducted as part of a special education evaluation. The Weshsler Intelligence Scales for Children-IV (WISC IV) and the Woodcock-Johnson III (WJIII) are standardized assessments commonly used in schools to determine whether a child qualifies for special education. These tests include measures of working memory (Feifer & De Fina, 2005). Often, the focus of a special education evaluation meeting and an individualized education plan (IEP) is on the academic performance of a student and how far it differs from the performance of typical age peers, which is a valid and concerning problem. The strong relationship between working memory and mathematics performance implies IEP teams should pay close attention to scores on subtests that report on working memory tasks, especially those with numbers. These subtest scores could provide a great deal of insight into the cause of the child's difficulty in mathematics as well as other academic subjects. It seems obvious that a student with cognitive deficits in working memory will always struggle with recall of basic facts and remembering procedures for multi-step arithmetic. However, with appropriate tools to accommodate for working memory deficits, students can be very successful with problem solving and understanding high level mathematical concepts.

Reading Difficulties

Another predictor of mathematics achievement is whether or not a child has reading difficulties (RD). RD seems to have a negative impact on mathematics achievement (Gersten et al., 2005). Gersten et al.'s review of research in this area suggests difficulties with reading and language could contribute to a lack of conceptual understanding in mathematics. In studies examining mathematics achievement of students with MD compared to students with MD + RD, students in the latter group showed greater deficits in competence with story problems and accuracy with arithmetic combinations compared to the MD only group (Jordan, Hanich, & Kaplan, 2003; Hanich, Jordan, Kaplan, & Dick, 2001; Fuchs, Fuchs, & Prentice, 2004).

Jordan et al. (2003) examined the growth of 180 second grade students classified into four groups: normal achieving (NA), MD only, RD only, and MD+RD. In this study, MD and RD were described as scoring below the 35th percentile on a standardized test. The researchers analyzed performance on mathematics tasks four times through second and third grade. The tasks involved calculation of arithmetic combinations, story problems, approximate arithmetic, place value, calculation principles, forced retrieval of number facts, and written computation. The researchers concluded the MD only group performed better than the MD+RD group on calculation principles and story problems. They also showed an advantage in exact calculation of arithmetic combinations but those differences disappeared when predictor variables of intelligence quotient (IQ), gender, ethnicity, and income level were considered. These two groups did not show a difference in forced retrieval of number facts, approximate arithmetic, place value, and written computation. These same children had shown significant differences in the calculation of arithmetic combinations and story problems with the MD only group showing an advantage over the MD + RD group when their scores were analyzed midway through second grade (Hanich et al., 2001).

The findings strengthen the claim children with MD + RD tend to have pervasive difficulties with mathematics over time. The researchers discuss students with MD only can rely on strengths in problem solving and interpreting story problems to grasp relationships within and between arithmetic operations. Their verbal comprehension is enhanced by their reading ability. In contrast, students with both MD + RD may struggle to comprehend the words of a story problem and translate them into a symbolic or iconic representation. Students with RD only performed at about the same level as the MD only group on problem solving tasks. Children with RD only and MD only may rely on different cognitive pathways to solve problem. Students with RD only may draw on their mathematical strengths to compensate for weaknesses in language whereas students with MD only rely on their verbal strengths to solve the problems. This hypothesis is consistent with the Triple-Code Model and the idea that learners can utilize more than one code to manipulate numerical information. It also implies intervention utilizing multiple representations will help students connect words to symbols to iconic representations. This is an effective way to help students with MD and RD understand mathematical concepts. Furthermore, this study implies students with MD + RD may struggle with mathematics throughout their school careers and need a great deal of support. In this study, tasks equally as difficult for students with MD only and students with MD + RD were those that involved memory-based retrieval and procedures. Fact retrieval and remembering multi-step procedures may be a defining characteristic of students with MD only but not students with RD only. Students with MD + RD may struggle with memory-based tasks as well as semantic understanding of mathematical concepts due to their weak abilities with verbal comprehension. Jordan et al.'s (2003) work does not address responsiveness to intervention. Their study examined growth patterns in a general education setting similar to what all children experience.

Fuchs et al. (2004) investigated third graders responsiveness to 16 weeks of what the researchers termed as "generally effective" instruction based on their disability risk status. The sample included 201 children who were categorized as at risk for reading disability (RDR), at risk for mathematics disability (MDR), at risk for both reading and mathematics disabilities (RDR/MDR), and not at risk for a disability (NDR). A disability was defined as less than the 25th percentile on the state achievement assessment. This study involved 16 classrooms, 8 control classrooms and 8 treatment classrooms, which were randomly assigned. Instruction took place in general education classrooms using the district's curriculum and provided instruction on four problem types. The problem types were buying multiple quantities of different priced items, problems interpreting half of a number, step-up functions, and summing two addends with one derived from a pictograph. The treatment group combined explicit instruction to transfer and self-regulated monitoring. "The format was explicit instruction, worked examples, and dyadic classroom, individual classroom, and homework practice" (Fuchs et al., 2004, p. 298). The students were evaluated on concept understanding, computation, and labeling using researcher-created tests related to the four problem types. The results revealed different levels of responsiveness to the treatment instruction based on the students' at risk status.

On computation and labeling, students in all three disability categories improved less as a function of treatment than the NDR students. In problem solving, students with MDR and RDR only achieved comparable to their NDR peers, which indicated the treatment was effective for these students in problem solving. Students with MDR/RDR scored significantly lower in problem solving than the other three groups. While Fuchs et al. did not specifically study the effects of intervention outside of the regular classroom, the researchers suggested additional, supplementary instruction in mathematics through intensive tutoring is needed for students with RDR, MDR, and RDR/MDR to perform at comparable levels to their non-disabled peers. This study validates other research in this area, which has concluded reading difficulties contribute to difficulty learning mathematics.

Research similar to Jordan et al. (2003) and Fuchs et al. (2004) highlights the fact that there are different types of disabilities that contribute to low achievement in mathematics. Understanding children with MD only have different needs than children with MD + RD suggests certain instructional approaches may work better for different types of learning difficulties. Children with MD only will need assistance compensating for memory deficits. Children with MD + RD will need assistance with conceptual development, mathematical vocabulary, comprehending context, and translating between symbols and words. Thus, there is a need for differentiated intervention strategies for students having difficulty learning mathematics. Assessments that distinguish these different areas of mathematical understanding and skills are needed.

Promising Trends

The developing research base on predictive indicators of mathematics disabilities demonstrates the need for mathematics assessments that reveal the cause of a child's difficulty with mathematics. The development of the NKT (Okamoto & Case, 1996) is a good example of the types of assessments needed for the field. Another promising example of this type of assessment has been created in a European research network (Von Aster, 2000). The Neuropsychological Test Battery of Number Processing and Calculation in Children (NUCALC) is designed around the Triple-Code Model of numerical processing. The subtests are measures of the following skills:

***Counting* -** enumeration of different sets of dots and counting backwards.

Number transcoding - reading Arabic numbers aloud and writing dictated Arabic numbers.

Magnitude comparison - comparing value of pairs of numbers and stating the largest amount.

***Mental calculation* -** computing simple addition and subtraction problems presented orally using simple number facts and situational contexts. Different types of problems are used: change, combine, and compare.

Placing Arabic numbers on an analogue number line - correctly placing numbers presented visually as symbols on a horizontal line with hatch marks between the symbols of 0 and 100.

Perceptual quantity estimation - estimating the numerosity of two pictures presented visually without counting them.

Contextual estimation - evaluating the relative size of a quantity based in a particular context, such as eight lamps in a room.

Von Aster (2000) concluded, "The NUCALC clearly distinguished children who were indicated by their teachers to have significant problems in coping with school mathematics from those children who had no learning problems according to teacher ratings" (p. 48).

The types of tasks that comprise the NUCALC and the NKT provide examples for both formal and informal assessments. These tests provide guidance on the types of assessments that could be designed to screen students for mathematical difficulty. These tests go beyond measures of simple calculation and word problems. They look more deeply at counting, number sense, and judgment of magnitude. Tests such as this could be very useful for teachers to quickly determine if a child has a pattern of difficulty related to counting, recall of basic facts, mental calculation, number sense, and judgment of magnitude. The NUCALC does not assess visual spatial skills or working memory. The ability to execute multi-step arithmetic procedures, understanding of grade level concepts and mathematical language, the ability to make a representation of a mathematical context and more complex problem solving strategies are also not addressed by the NUCALC. An internet search revealed that this test does not seem to be easily available in America. Greater access to this type of a test for American schools could provide a valuable tool for diagnosis of the cause of a child's difficulties in mathematics. The lack of valid assessment tools to diagnose mathematics disabilities means teachers working with students with MLD may need to rely heavily on curriculum-based measurements, observations, and comparison to typical age peers in class performance. For this reason, it is essential for classroom teachers to have a good understanding of the characteristics of an MLD that students may display in class. Literature on characteristics of MLD will be presented next.

Characteristics of MLD

Bryant and Pedrotty Bryant (2008) provide a list of specific behaviors associated with students who have MLD across the grades. Table 3 categorizes these behaviors into difficulties exhibited with calculation and those exhibited with problem solving.

Table 3. Behaviors Associated with Mathematics Learning Disabilities

Calculation Difficulties	Problem Solving Difficulties
• Identifying the meaning of signs (e.g., +, -, x, <, =, >, %) • Remembering answers to basic arithmetic combinations (e.g., 8 + 9 = ?, 7 x 7 = ?) • Using effective counting strategies to calculate answers to arithmetic problems. • Understanding the commutative property (e.g., 5 + 3 = 8 and 3 + 5 =8) • Solving multi-digit calculations that require regrouping • Misaligning numbers • Ignoring decimal points	• Reading the problem • Understanding the meaning of the sentences • Understanding what the problem is asking • Identifying extraneous information that is not required for solving the problem • Developing and implementing a plan for solving the problem • Solving multiple steps in advanced word problems • Using the correct calculations to solve problems.

Note: Bryant and Pedrotty Bryant (2008)

Many children make the errors described on in Table 4. However, students with MLD are more likely to display "pervasive difficulties" (Bryant & Pedrotty Bryant, 2008, p. 4) with one or two of the listed behaviors. By being aware of these patterns in student errors, teachers can design appropriate accommodations and intervention for students.

Bryant, Bryant, and Hammill (2000) identified 29 characteristics of students with MLD. Teachers of SWD were asked to rank the order of the problems. Table 4 shows these 29 characteristics in decreasing order of frequency. The rank order suggests the first behaviors on the list may be common to many students struggling with mathematics, but the last ones may be related to more severe disabilities.

Table 4. Ranked Mathematics Difficulties Exhibited by Students with Learning Disabilities and Mathematics Weaknesses

- Has difficulty with word problems
- Has difficulty with multi-step problems
- Has difficulty with the language of math
- Fails to verify answers and settles for first answer
- Cannot recall number facts automatically
- Takes a long time to complete calculations
- Makes "borrowing" (i.e. regrouping, renaming) errors
- Counts on fingers
- Reaches "unreasonable" answers
- Calculates poorly when the order of digit presentation is altered
- Orders and spaces numbers inaccurately in multiplication and division
- Misaligns vertical numbers in columns
- Disregards decimals
- Fails to carry (i.e. regroup) numbers when appropriate
- Fails to read accurately the correct value of multi-digit numbers because of their order and spacing.
- Misplaces digits in multi-digit numbers
- Misaligns horizontal numbers in large numbers

- Skips rows or columns when calculating
- Makes errors when reading Arabic numbers aloud
- Experiences difficulties in the spatial arrangement of numbers
- Reverses numbers in problems
- Does not remember number words or digits
- Writes numbers illegibly
- Starts the calculation from the wrong place
- Cannot copy numbers accurately
- Exhibits left-right disorientation of numbers
- Omits digits on the left or right side of a number
- Does not recognize operator signs (e.g. +, -)

Note: Based on Bryant et al. (2000)

Montague (2006) compared the characteristics of good problem solvers with poor problem solvers. She explained students with learning disabilities have "serious perceptual, memory, language, and/or reasoning problems that interfere with mathematical problem solving" (p. 90). Students with learning disabilities may know a variety of strategies they can use to solve problems, but they often do not know which strategy applies best to the context. They do not always know where to begin or how to evaluate whether their strategy for solving the problem is effective. They are often disorganized in their approach to a problem. SWD do not always replace or adapt immature or ineffective strategies for more efficient ones. They also often do not generalize the use of a strategy learned in one context to a new context. Students who have the procedural subtype of MLD may need a great deal of support during problem solving activities.

"The most consistent finding in the literature is that children with MLD/RD or MLD only differ from their typically achieving peers in the ability to use retrieval-based processes to solve simple arithmetic and simple word problems" (Geary, 2005, p. 7). The ability to retrieve basic mathematics facts can be impacted by all three subtypes of MLD. A child with the visuospatial subtype will not be able to visualize the magnitude of the numbers in a problem or picture a number line. Thus, when recall fails, they cannot rely on visualization of the problem as other children do. Children with the procedural subtype of MLD may not be able to translate a basic fact to its linguistic form. They may have difficulty creating a word story for a fact or realize they can use repeated addition as a multiplication problem. When recall fails, these children do not generalize the patterns of numbers and rely on them to solve unknown problems. Students with the semantic subtype of MLD will most likely have long term difficulty with memorizing basic facts. This is a manifestation of their disability.

An awareness of the behaviors exhibited by students with MLD can help teachers evaluate the level of support a student may need to be successful in grade level mathematics curriculum. Observing long-term, recurring patterns, and communicating those observations to future teachers who work with the student can be a way to move toward diagnosis as well as the strategic customization of instruction for students with MLD.

Effective Instructional Practices

A great deal of research has been conducted within the field of special education on specific instructional strategies shown to be effective for teaching mathematics to students with learning disabilities. Several reviews of literature describe the wide range of specific instructional strategies researched and provide recommendations for educators

(Mastropieri et al., 1991, 1998, 2004; Miller et al., 1998; Maccini & Hughes, 1997; Maccini & Gagnon, 2000; Maccini, Mulcahy, & Wilson, 2007; Maccini et al., 2008; Kroesbergern & Van Luit, 2003; Steele, 2002; Carnine, 1997; Jintendra & Xin, 1997; Fuchs & Fuchs, 2001; Fuchs et al., 2008; Woodward, 2006; Koontz, 2005; Wendling & Mather, 2009). While this research tradition in special education has provided "a wealth of research-validated instructional practices for students with LD" (Rivera, 1997, p. 4), there are some significant limitations within this body of research that minimize the usefulness of this research for practitioners shifting toward reform-based mathematics instruction and the implementation of the RTI model of intervention.

First, these literature reviews presented empirical evidence supporting the effectiveness of a wide range of strategies. While there are hundreds of studies, often there were only a few studies supporting one specific strategy, and sometimes studies on the same strategy gave conflicting results. For example, Miller et al. (1998) reviewed 54 studies, which took place between 1988 and 1998. These 54 studies looked at 7 different instructional strategies: constant time delay, use of manipulatives, devices and drawings, direct instruction, strategy instruction, lecture pause, use of goal structure, and use of self-regulation. Differing results, vague use of terms, and a broad definition of effectiveness made it difficult to make conclusive judgments about the effectiveness of any one strategy. These limitations appear consistently in this genre of research literature.

Another significant limitation to be considered when evaluating research studies on effective instructional strategies for SWD is to recognize many of these studies are designed based on a behaviorist view of learning. They usually defined mathematical proficiency as mastery of basic computational procedures and utilized behaviorist instructional techniques. There was a heavy emphasis on rote rehearsal. Although a specific strategy may have shown statistically significant improvement in achievement on a post-test, the definition of mathematical achievement and the alignment of the instructional practice to the pedagogy of reform should be kept in mind. Educators seeking to improve children's mathematical understanding should carefully evaluate this research based on the recommendations of cognitive and social cultural theories of learning, which form the foundation of the NCTM Standards and modern day mathematics instruction. Many, but not all, of the studies in special education research are based on an outdated definition of mathematical proficiency (Woodward & Montague, 2002; Woodward, 2004, 2006; Bryant et al., 2006; Maccini & Hughes, 1997). Very little research in special education has focused on teaching strategies promoting conceptual understanding (Woodward, 2004; Baroody & Hume, 1991).

While the RTI model of intervention encourages inclusion and alignment between special education support and learning activities in the general education classroom, many of the studies on instructional strategies for special education students took place in pull out settings. Much of the research was a case study design or small group instruction. The content was usually below grade level content, which differs from the present day vision of SWD participating in the same rigorous curriculum as their non-disabled peers and experiencing a wide range of mathematical topics.

In spite of these limitations, this body of literature makes significant contributions to improving mathematics instruction for SWD. There were many recurring themes in the literature. Collectively, this body of literature provides validation for five key instructional practices, which are compatible with the pedagogy encouraged by reform documents. These five key practices are as follows:

1. Strategy instruction
2. Concrete - representational - abstract (CRA) instructional sequence
3. Peer-assisted learning
4. Real-world problems
5. Progress monitoring

All of these practices have been validated by research in special education but are also consistent with the recommendations of the NCTM Standards and a constructivist approach to mathematics instruction.

Strategy Instruction

Strategy instruction helps SWD become aware of the metacognitive processes used for problem solving (Woodward, 2006; Woodward, Monroe, & Baxter, 2001; Xin & Jintendra, 2006; Xin, Jintendra, & Deatline-Buchman, 2005; Xin, 2003; Jintendra & Hoff, 1996; van Garderen, 2006; van Garderen & Montague, 2003; Montague, 1992, 2006; Montague & Applegate, 1993; Montague, Applegate, & Marquard, 1993; Carnine, 1997). While students without disabilities may develop strategies for problem solving more intuitively, SWD may need these processes made explicit due to their weaknesses in executive functioning abilities (Goldman, 1989; Woodward et al., 2001; Woodward, Baxter, & Robinson, 1999; Woodward & Baxter, 1997). Strategy instruction also helps students who have difficulty translating language to the symbolic code of mathematics. The use of self-regulation strategies can help SWD be aware of their thinking and help them sequence their thinking as they solve problems. Schema diagrams, student-created drawings and diagrams, mnemonics, and prompt cards can all be used as a part of strategy instruction. Strategy instruction during problem solving also helps students conceptualize the big ideas of mathematics and apply computational skills in a context. It allows for rich student discourse, connecting mathematics to background knowledge, prior learning in mathematics, and other topics within mathematics

CRA Instructional Sequence

Using manipulatives, drawings, diagrams, or graphic organizers before teaching abstract level procedures helps student progress through different levels of understanding of a mathematical concept (Fuchs & Fuchs, 2001; Witzel, Riccomini, & Schneider, 2008; Butler et al., 2003; Bryant et al., 2006). Many SWD have difficulty visualizing magnitude. Teaching in a CRA sequence allows students to access more informal strategies for solving problems. Comparing and contrasting multiple representations of mathematical concepts at all three levels helps students see the patterns of mathematics and grounds the use of abstract computational procedures in conceptual understanding (Tripathi, 2008). Utilizing a CRA sequence also allows for communication, connections, and representation as recommended by the NCTM Standards. This instructional practice is consistent with Jerome Bruner's progression of developing understanding through enactive, iconic, and symbolic representations (Driscoll, 2005).

Peer Assisted Learning

Fuchs et al. (1997) explored the effectiveness of peer-mediated strategies. This study found that explicitly teaching effective helping behavior during collaborative learning activities had a significant positive impact on the learning of SWD as well as students without disabilities. This study supports the work of Kazemi (1998) and Sims (2008), who recommended that strategies promoting student discourse increase the achievement of all students. Kroger and Kouch (2006) also fostered success in mathematics for SWD through the use of peer-assisted learning strategies.

Peer-assisted learning strategies can take a variety of forms. Reciprocal teaching, role reversal with the teacher, cross age tutoring, think pair share, and cooperative learning are all teaching techniques that allow students to teach each other. These strategies change a classroom from a teacher-directed learning environment to a student directed one. Students engage in mathematical discourse and the language of mathematics is translated into the language of children. These strategies align with a social cognitive theory of learning as a result of engagement in a community. Research has shown SWD struggle with the pace of the instruction and the complexity of the material (Baxter, Woodward, & Olson, 2001; Woodward & Baxter, 1997). SWD often need support and explicit modeling of social skills to enhance their abilities to work with other students. They may also suffer from learned helplessness and lack of confidence when interacting with non-disabled peers.

Preteaching using advanced organizers to prepare SWD to engage in problem solving with peers in a general education setting may be a strategy that helps them be more successful in whole class activities (Carter & Dean, 2006; Githua & Nyabwa, 2008). A synthesis study by Stone (1983, cited in Marzano, Pickering & Pollock, 2001) reports an effect size of .80 on the impact of the use of expository advance organizers on student achievement in the general education setting. Another strategy that may be helpful would be teaching non-disabled peers skills for involving SWD in group work. These are strategies worthy of additional research that could increase the chances of success for SWD participating in cooperative group work activities with learning disabled peers

Real-World Problems

The previously mentioned three key strategies all allow for embedding computational skills, conceptual development, and big ideas of mathematics within real-world problems. Real-world problems help students see mathematics as relevant and important, and engage them in complex mathematical tasks. It helps them connect to background knowledge, especially when the contextual problems are written specifically with the culture and interests of the students in mind (Tate, 1995). Real-world contextual problems and tasks allow for the integration of vocabulary and writing in mathematics class (Murray, 2004). Utilizing children's literature to situate problems is another way to create a context that connects language and mathematics (Lewis, Long, & Mackay, 1993). For students who struggle to interpret the symbolic code of mathematics, understanding mathematical context in a linguistic way can be very helpful for them. Using strategy instruction, CRA sequence, and peer-assisted learning to solve real-world problems will help them utilize their linguistic abilities to develop conceptual understanding in mathematics. Teaching mathematics to SWD in this way will minimize the rote drill and practice common to many special education classrooms and allow students to experience more challenging contextual problems in a broader range of content. In addition, this approach will give students a purpose for learning more advanced computational skills. The use of real-world problems is supported by research on situated cognition (Gersten and Baker, 1998) and anchored instruction, which embeds systematic skills practice in authentic problem solving (Jintendra & Hoff, 1996; Jintendra, Hoff, & Beck, 1999; Montague, 1992, 1995; Bottge & Hasselbring, 1993; Goldman et al., 1997).

Progress Monitoring

The purpose of any intervention program is to close the gap in achievement between students who are low achievers and their typical age peers. In order to effectively do this, teachers must define clear learning targets and monitor progress toward those targets (Fuchs et al., 2008). Progress monitoring does not always need to utilize a standardized test. Formative assessment is an integral part of good instructional practice (Marzano, 2000; Marzano, Pickering, & Pollock, 2001; Wiggins & McTighe, 2006; Stiggins, 2008; Chappuis & Chappuis, 2002; Danielson, 2007). Curriculum-based assessments, student work samples, observation notes, and student portfolios are all methods of monitoring progress. It is desirable for students to be fully aware of learning targets, and to be involved in monitoring their progress and the setting of individualized goals. While many advocates of direct instruction suggest reinforcers for positive growth and good behavior for SWD, many students are motivated by seeing measureable progress toward a clear learning goal. For many SWD who have experienced years of failure in mathematics, seeing a graph or folder of work samples that demonstrate success could be rewarding even without extrinsic rewards. Progress monitoring through varied assessments can also be helpful to analyze patterns in student errors and processing problems. These patterns can then be used to strategically design accommodations for individual students. The assessment principle in the NCTM Standards (2000) encourages the use of assessment to enhance student learning and to make instructional decisions.

Summary

This literature review has described current issues related to teaching mathematics to SWD. The challenges of aligning practice between special education and general education in an RTI model have been presented. The Triple-Code Model of numerical representation can provide a theoretical foundation on which to base differentiation of instruction for students struggling to learn mathematics as well as intervention efforts. It is only recently that researchers in mathematics education, special education, cognitive psychology, and neuropsychology are beginning to gain insight on the various types of MLD. This has led to a greater understanding and work on assessments that help educators look beyond a child's inability to recall basic facts and determine the cause of poor mathematical performance in school. This chapter has highlighted the development of promising assessments and some of the core deficits that could be considered as indicators of future mathematical achievement. It has also reviewed the characteristics of students with MLD, which can contribute to appropriate diagnosis and design of intervention. Lastly, effective instructional strategies supported by research in special education were summarized as five key instructional practices for SWD.

This emerging field of research connects to many instructional questions. How can educators develop effective assessments that informally screen students to identify those who may have an MLD? How can assessments be used to design strategic customization of instruction? What is the core deficit causing a child's difficulty in mathematics? What are the characteristics of students with MLD of which classroom teachers should be aware? What instructional practices should be used for students with MLD? This study is designed to address some of these questions and proposes an instructional framework to guide assessment and instruction for students experiencing difficulty learning mathematics.

CHAPTER 3: A PROPOSED FRAMEWORK

This study is based on an instructional framework for Tier 2 Mathematics Intervention developed by the researcher. The framework is shown in Figure 3. This framework is based on the review of literature presented in Chapter 2 of this study. It synthesizes the recommendations of many researchers in the fields of special education, mathematics education, cognitive psychology, and neuropsychology. It provides practitioners specific recommendations for mathematics intervention at the Tier 2 level in an RTI model.

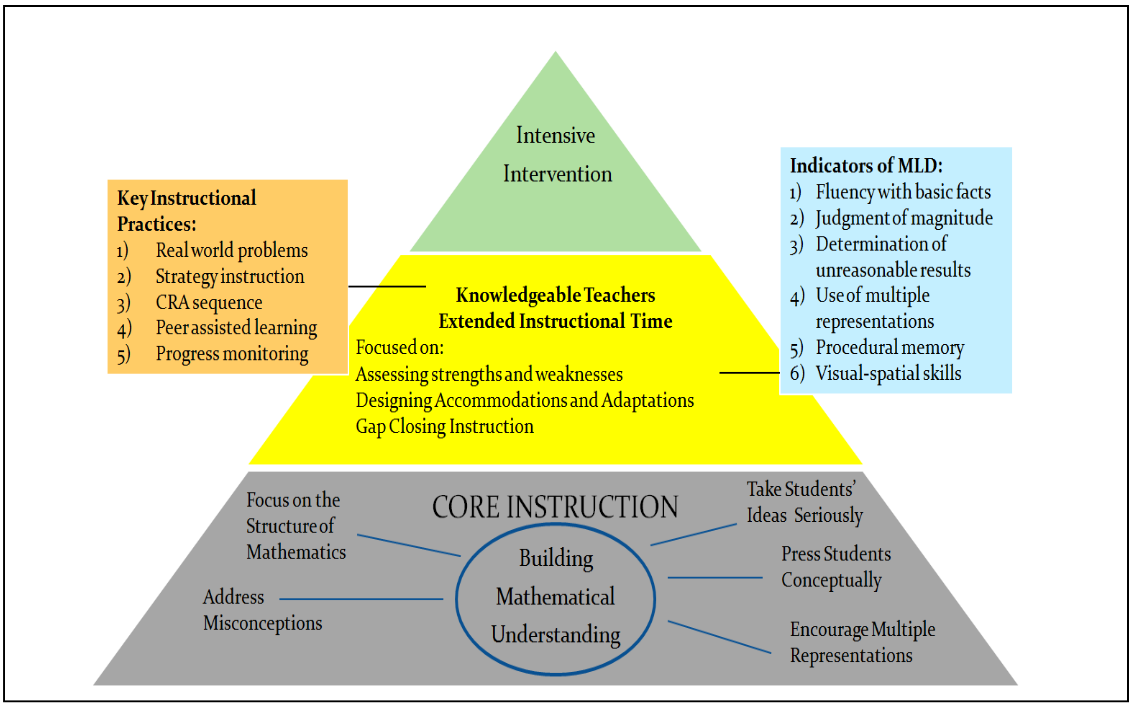


Figure 3. Instructional Framework for Tier 2 Mathematics Intervention

Note: Tier 2 Framework (Beals, 2011); Core Instruction Five Big Ideas (Brendefur, 2008)

The Tier 2 framework builds upon an instructional framework for Core level instruction developed by Brendefur (2007, 2008). This framework for Core instruction is referred to as the Five Big Ideas of Teaching Mathematics. This framework is shown in the Core Instruction level in Figure 3. The Five Big Ideas helps teachers focus on these five key instructional practices for general education mathematics instruction to build mathematical understanding. The five key practices are as follows:

1. Focus on the structure of mathematics
2. Address misconceptions
3. Take students' ideas seriously
4. Press students conceptually
5. Encourage multiple representations

The Tier 2 framework is based on instruction for struggling students being provided by qualified teachers who have knowledge of the content pedagogy needed to teach this population of students. This will require professional development beyond that most elementary teachers and secondary mathematics teachers learn in their preservice teacher education programs. In addition, this model is based on intervention which does not remove the student from their grade level mathematics lessons. The intervention should be provided at a time that keeps the student exposed to general education content and the same mathematics curriculum as their grade level peers. This framework supplements Core mathematics instruction and provides extended instructional time for struggling students. The purpose of the intervention is clearly defined. It is not remedial tutoring of below grade level skills, which throughout the literature has not been shown to increase student achievement over the long term. This intervention focuses on assessing strengths and weaknesses, designing accommodations and adaptations to help the student be successful in the general education curriculum, and providing gap closing instruction. The intervention is based on grade level learning targets defined by state standards. It is not based on below grade level content and targets.

Intervention learning activities should engage students in the five key instructional practices identified though the review of literature on effective instructional practices for teaching mathematics to SWD. These five key practices are as follows:

1. Strategy instruction
2. CRA instructional sequence
3. Peer-assisted learning
4. Real-world problems
5. Progress monitoring

Lessons should utilize real-world problems and provide instruction on strategies for thinking. They should work with concepts in a progression of concrete, representational, and abstract models - called the CRA sequence. This sequence is comparable to the enactive, iconic, and symbolic levels of understanding developed by Bruner (Driscoll, 2005). The term *CRA sequence* is used for the framework because it is familiar to special educators through special education research literature. Instruction should heavily emphasize the representational level and teach students visual models for thinking about numbers. Some examples of these models are arrow math, tree diagrams, area models, pictures, and open number lines. The use of multiple representations at all three levels is encouraged during problem solving experiences and mathematical discussions. Peer-assisted learning encourages a great deal of discourse about the mathematics and vocabulary development. Progress monitoring, both formal and informal, can be used to evaluate progress and individual student strengths and weaknesses.

As students are participating in learning activities, teachers should make observations and utilize assessment data and student work samples to evaluate student strengths and weaknesses in six areas the research literature suggests are possible indicators of an MLD. These six indicator areas are as follows:

1. Fluency with basic calculation
2. Judgment of magnitude
3. Determination of unreasonable results
4. Use of multiple representations
5. Procedural memory
6. Visual-spatial skills

The sixth indicator, visual-spatial skills, is not specifically related to mathematics. This is a cognitive deficit that can appear in mathematics, but is not specifically related to numerical processing. Procedural memory is also an indicator that can be related to other domain-general disabilities such as working memory deficits, attention deficit disorder, and emotional disorders. However, students with the procedural subtype MLD have difficulties with multi-step procedures and problem solving that are specific to tasks involving numerical reasoning.

Subskills identified for each indicator of the framework are listed in Table 5. In addition, the corresponding processing code from the Triple-Code Model is also noted for each indicator. These six indicators can be used to look for evidence of ability or difficulty within each area.

Table 5. Indicators of Mathematics Disability

Indicator	Subskills	Processing Code
Fluency with Basic Calculation	Recall Counting Relational Thinking Composing/Decomposing Inhibit irrelevant associations Automaticity Processing speed	Symbols to Language
Judgment of Magnitude	Spatial visualization Non-verbal abilities Quantity to symbol code Symbol to quantity code	Magnitude
Determination of Unreasonable Results	Conceptual Understanding Judgment of magnitude	Magnitude Language to Symbols

Indicator	Subskills	Processing Code
Procedural Memory	Language to symbols code Working memory Conceptual understanding	Language to Symbols
Visual-spatial Skills	Non-verbal cognitive skills Executive functioning	Not a numerical processing weakness

Work samples, assessment information, and notes from class or tutoring sessions can be used to collect evidence regarding ability or difficulty within each area of the framework. Using this method, a teacher would be able to identify consistent patterns of strengths and weaknesses in a student's mathematical skills. By organizing assessment data, student work samples, and observations around these indicators of an MLD, teachers can communicate specifically about a child with an evaluation team. This collection of data will provide a working hypothesis of the type of cognitive deficit a child may have prior to an extensive evaluation process. This framework can also contribute to the strategic design of adaptations and accommodations that may help an individual student. Teachers can provide intervention and try some accommodations to test the working hypothesis. This approach is comparable to the Adaptations Framework developed by Bryant et al. (2006). Students may show a strong pattern of weakness in just one indicator areas or they may struggle in several, which indicates difficulty with the integration of two or even all three of the cognitive processing systems of the Triple-Code Model.

As a test of this framework, the 29 ranked difficulties identified by Bryant et al. (2000) and Bryant and Pedrotty Bryant (2008) are categorized according to the indicator they best correspond with in Table 6. Some of the difficulties could be a manifestation of more than one area of struggle. Those were placed in more than one indicator area. The fact that all of the identified characteristics seemed to fit with at least one of the indicators provides some validation the suggested six indicators will address most of the difficulties presented by students who struggle to learn mathematics. This table might be helpful for teachers to refer to in order to know which behaviors and characteristics are noteworthy when diagnosing MLD and could be recorded in anecdotal observations.

Table 6. Characteristics of MLD Matched to Indicators from Framework

	Characteristics from Bryant et al. (2000)	**Characteristics from Bryant and Pedrotty Bryant (2008)**
Fluency with Basic Calculation	• Cannot recall number facts automatically • Takes a long time to complete calculations • Counts on fingers • Makes errors when reading Arabic numbers aloud	• Identifying the meaning of signs (e.g., +, -, x, <, =, >, %) • Remembering answers to basic arithmetic combinations (e.g., 8 + 9 = ?, 7 x 7 = ?) • Using effective counting strategies to calculate answers to arithmetic problems

	Characteristics from Bryant et al. (2000)	**Characteristics from Bryant and Pedrotty Bryant (2008)**
Judgment of Magnitude	• Disregards decimals • Experiences difficulties in the spatial arrangement of numbers • Fails to read accurately the correct value of multidigit numbers because of their order and spacing. • Misaligns horizontal numbers in large numbers	
Determination of Unreasonable Results	• Fails to verify answers and settles for first answer • Reaches "unreasonable" answers • Calculates poorly when the order of digit presentation is altered • Disregards decimals • Fails to read accurately the correct value of multi-digit numbers because of their order and spacing.	• Ignoring decimal points • Understanding the commutative property (e.g., 5 + 3 = 8 and 3 + 5 = 8) • Identifying the meaning of signs (eg., +, -, x, <, =, >, %)
Use of Multiple Representations	• Difficulty with language of mathematics • Difficulty with word problems • Does not remember number words or digits • Does not recognize operator signs (e.g. +, -)	• Identifying the meaning of signs (e.g., +, -, x, <, =, >, %) • Reading the problem • Understanding the meaning of the sentences • Understanding what the problem is asking • Identifying extraneous information that is not required for solving the problem

	Characteristics from Bryant et al. (2000)	**Characteristics from Bryant and Pedrotty Bryant (2008)**
Use of Multiple Representations (Cont'd.)		• Developing and implementing a plan for solving the problem
Procedural Memory	• Difficulty with word problems • Difficulty with multi-step problems • Makes "borrowing" (i.e., regrouping, renaming) errors • Fails to carry (i.e., regroup) numbers when appropriate • Starts the calculation from the wrong place	• Understanding the commutative property (e.g., 5 + 3 = 8 and 3 + 5 = 8) • Solving multi-digit calculations that require regrouping • Developing and implementing a plan for solving the problem • Solving multiple steps in advanced word problems • Using the correct calculations to solve problems.
Visual-spatial Skills	• Orders and spaces numbers inaccurately in multiplication and division • Misaligns vertical numbers in columns • Disregards decimals • Misplaces digits in multidigit numbers • Misaligns horizontal numbers in large numbers • Skips rows or columns when calculating • Reverses numbers in problems • Writes numbers illegibly • Cannot copy numbers accurately • Exhibits left-right disorientation of numbers • Omits digits on the left or right side of a number	• Misaligning numbers

If a child continues to struggle long term with a consistent pattern of weakness that presents as one of the subtypes of MLD, an evaluation team should consider a more extensive psychological evaluation to determine specific cognitive deficits. If a child does not have a recurring pattern in one of the indicator areas and responds to intervention, an evaluation team could confidently assume the child does not have a domain-specific learning disability in mathematics. This framework might be especially helpful for Tier 2 intervention in an RTI model as it could help teachers determine whether the difficulty is due to a cognitive processing problem or related to instructional, cultural, or linguistic factors.

This suggested instructional framework could be a starting place to educate teachers about how to effectively help students with MLD. Knowing a wide variety of factors can contribute to a child falling behind in mathematics achievement, this suggested framework can help teachers examine the cause of a student's difficulty in mathematics. That knowledge can then be used to design effective intervention in mathematics helping a child close the gap between them and their peers in mathematical abilities. It can also be used to design accommodations and adaptations that will help a student compensate for an MLD throughout their lifetime. It could be useful to communicate with students and parents to help them understand a student's unique strengths and weaknesses in learning mathematics. This framework provides a suggested method of diagnoses and intervention for students who have difficulty learning mathematics that does not rely on extensive standardized testing to determine strengths and weaknesses. It can draw upon student work samples, informal curriculum-based assessments, observations, and teacher judgment to collect evidence of strengths and weaknesses. Once weaknesses are identified, accommodations can be strategically chosen to address deficits while keeping the student in grade level instruction. This is "strategic customization" of instruction (NCSM, 2008). This type of instruction can effectively supplement a general education mathematics class and provide support for students with MLD from knowledgeable special educators and general education mathematics educators. This study provides a pilot of this framework and a model for designing intervention curriculum based on it.

CHAPTER 4: METHODS

This study addresses two research questions. The first question is does low-achieving fourth graders' participation in tutoring based on five key instructional practices for students with mathematics disabilities increase their understanding of multiplication and division? The five key instructional practices are real-world problems, strategy instruction, CRA sequence, peer-assisted learning, and progress monitoring. The second research question is what are the patterns of deficits low achieving fourth graders have in relation to the six indicators of MLD? These six indicators are fluency with basic facts, judgment of magnitude, determination of unreasonable results, use of multiple representations, procedural memory, and visual-spatial skills. This chapter provides an overview of the methodology used in this study.

Context of Study

For the last several years, the state of Idaho has focused resources on a statewide effort to improve mathematics instruction with the passage of the Idaho Math Initiative in 2007. This initiative has required every teacher of mathematics of students in grades Kindergarten through 12 to take a state-sponsored class called Mathematical Thinking for Instruction (MTI). This class has encouraged teachers in the state to shift toward mathematics instruction that develops conceptual understanding and embeds skill development in meaningful contexts. The course is based on an instructional framework for Core level instruction at all grade levels. This instructional framework was developed by Brendefur (2007, 2008) and is referred to as the Five Big Ideas of Teaching Mathematics. The Five Big Ideas helps teachers focus on these five key instructional practices for general education mathematics instruction to build mathematical understanding. The five key practices are as follows:

1. Focus on the structure of mathematics
2. Address misconceptions
3. Take students' ideas seriously
4. Press students conceptually
5. Encourage multiple representations

The implementation of this instructional framework across the state and in school-wide improvement efforts has led to greater student achievement as measured by the Idaho Standards for Achievement and project evaluation data (Brendefur, Pittman, & Thiede, in review; Brendefur, Thiede, Strother, Jesse, & Sutton, in review).

Although the MTI professional development efforts have shown success in improving general education mathematics instruction, there is still much work to be accomplished in Idaho to improve mathematics instruction at the Tier 2 and Tier 1 levels in an RTI model of intervention. The framework developed for this study is designed to complement the framework for Core Instruction being taught in the MTI classes while still keeping in mind the unique needs of students with MLD and the need for diagnostic assessments.

Setting

This study took place in an elementary school in a rural school district in southwestern Idaho. The school has grades kindergarten through fifth grade and was chosen based on its diverse student population and convenience of location for the researcher. Based on the 2010 - 2011 School Report Card, the school has a student population of 510. The ethnicity of the students is comprised of 46.47% Caucasian, 42.35% Hispanic, 11.18% other ethnicities. Students who qualify for free and reduced lunch comprise 79.22% of the student population. English Language Learners are 19.80% of the student population, and 9.41% participate in special education. During the 2011-2012 school year when the study was conducted, the school was in Year 5 of school improvement for reading but had 85% of the students meeting the state reading benchmark in 2010 and had met 18 of 20 goals. Missed reading goals included proficiency levels for Hispanic and Economically Disadvantaged students. In mathematics, the school was in year 1 of school improvement due to missing goals in the past. They met all 20 goals in this subject in 2010 and had 87% of the students reach the state math benchmark.

Student Sample

The sample population for this study was all students in one fourth grade at this elementary school. From this classroom, four students were selected for a treatment group based on multiple measures. A matrix was created that showed student scores on the following data:

- Pre-test using researcher-created test on multiplication and division concepts
- Timed multiplication fact assessment. Bottom 35th percentile were identified.
- First trimester math grades
- Teacher rating on scale of 1 to 4 indicating mathematics performance in class in comparison to grade level peers.
- Idaho Standards for Achievement (ISAT) mathematics score Spring 2011. Bottom 35th percentile of the class were identified.
- ISAT proficiency level: basic, below basic, proficient, or advanced

Noting those scores which were in the bottom 35th percentile of the population is consistent with the recommendations of leading researchers in this field (Mazzocco, 2005; Geary, 2005; Gersten et al., 2005). Based on this collection of data from multiple measures, the teacher and researcher selected four students who were performing below their peers in mathematics to participate in the tutoring group. Two of the selected students were female and two were male. Three of the four students were Hispanic and one was Caucasian. Three had been identified by the school as limited English proficient (LEP), and two were receiving special education services for speech articulation.

Parent Permission

Because this study involved children, all guidelines for research on vulnerable populations were closely followed. Parents of all students in the class needed to provide consent for their child's data to be used in this study. The parents of the students in the tutoring group needed to be fully informed of the study and the intervention program their child was participating in. A parent permission slip was sent home with all of the students in the class. All students were asked to return the permission slip showing either consent or no consent. Students were provided a small prize for returning the permission slip. Only students whose parents provided consent were included in the population sample. Students whose parents did not provide consent still took the same assessments as their classmates, but their data was not used in the study.

Once the students for the tutoring group were selected by the teacher and researcher, the researcher or an interpreter provided by the school called each parent and explained the study and the instruction their child would be receiving. Because some of the parents did not speak English, an employee of the school who spoke Spanish called instead of the researcher. After the phone conversation, a written permission slip was sent home to the parent. The parents of all four students provided consent. Once parents had signed the permission slip and returned it to the teacher, the researcher met with the student to discuss the study and obtain student assent. A copy of the parent permission slips, scripts for phone calls and conversations to obtain student assent are included in Appendix A. No student identifying information will be used in any presentation of this study and all identifying information will be kept confidential by the researcher and school staff.

Measurement Instrument

Research on valid and reliable measures of mathematical understanding and abilities beyond basic computation is in its early stages (Geary, 2004, 2005; Feifer & De

Fina, 2005; Gersten et al., 2005; Mazzocco, 2005; Chiappe, 2005). The lack of available measures of mathematical abilities limits the quality of research that can be conducted on the effectiveness of intervention programs. This study addresses this limitation by utilizing a multiple measures approach. Achievement was measured with multiple sources of data, which included a researcher-created test on the topics taught, a basic mathematics fact assessment, the standardized state mathematics assessment, teacher and researcher observations and ratings, grades, work sample, and informal teacher and researcher-created assessments. While none of these measures is considered statistically valid or reliable, the collection of these multiple sources of data provided information on the mathematics achievement of the sample population as well as changes in their understanding of multiplication and division. Copies of the assessments used in this study are included with the intervention curriculum in Appendix B.

Treatment

Students in the treatment group received 27 sessions of mathematics intervention based on the instructional framework created by the researcher. Students received supplemental instruction on two core topics for the fourth grade mathematics curriculum. The group met three or four times a week for 45 minute sessions for a total of 27 sessions. The intervention took place at a time arranged with the school that did not remove students from their grade level mathematics instruction. The students participated in two instructional units aligned to grade level targets from the Common Core Mathematics Standards, which Idaho has adopted. The first unit was called Number Patterns. It included 11 sessions focused on patterns with numbers, skip counting, and basic number combinations with addition, subtraction, and multiplication. Students were introduced to the concept of using an iconic model to represent numbers. Examples of such models included a picture, open number line, arrow math, tree diagram, an array, and a set model. These models were also used in the second unit on multiplication. The second unit was 16 sessions focused on the topic of multiplication. It provided students with contextual problem solving experiences related to multiplication. The curriculum for each unit and lesson plans for tutoring sessions are included in Appendix B. Informal formative assessments were utilized throughout the intervention period to monitor progress and mastery of objectives related to lessons.

Instructional activities for the intervention period were based on the instructional framework created from the comprehensive review of literature supporting this study. The key instructional practices used for the intervention were use of real-world problems, strategy instruction, using a CRA sequence, peer-assisted learning, and progress monitoring. Throughout the intervention period, the researcher recorded anecdotal observations and made notes on work samples and assessments, which demonstrated strength or weakness in the areas identified as a possible indicator of an MLD. These areas were fluency with basic facts, judgment of magnitude, determination of unreasonable results, use of multiple representations, procedural memory, and visual-spatial skills. Learning activities emphasizing these skills were strategically integrated into lessons. All student work was collected in a portfolio for each student in the tutoring group. Work samples and anecdotal observations were then coded based on the indicator areas. At the end of the treatment period, the patterns of strength and weaknesses for each child were summarized. Scores on the post-test were compared to pre-test scores to determine whether the students in the tutoring group had made changes in their understanding of multiplication and division. Based on the post-test, the qualitative data and the researcher's experiences with the child, a progress report for each student was prepared by the researcher and provided to the teacher, principal, and parent of each student. Copies of these progress reports are included in Appendix E.

Control Group

All of the other students in the fourth grade class acted as a control group. These students participated in the same mathematics lessons provided by the classroom teacher as the tutoring group. The fourth grade at this school has a mathematics intervention period for all students. The rest of the class participated in a 25 minute math intervention period four days a week during the time of the tutoring for this study. For these classes, the entire grade level was ability grouped and teachers worked with students on reteaching or enriching mathematical skills. During the treatment period, students in the tutoring group did not participate in this intervention program but instead worked with the researcher for an extended intervention period of 45 minutes. At the end of the treatment period, all students in the class took the same post assessments as the students in the tutoring group on the same schedule. The assessment data from the tutoring group was compared with the assessment data from the rest of the class.

Data Analysis

This study used a mixed-methods design to evaluate the effectiveness of the instructional approach used for the treatment group. Statistical analysis and a scatter plot was used to determine whether the students in the treatment moved closer to the group median as compared to the assessment given before the intervention. This analysis helped determine whether the treatment intervention was effective in closing the achievement gap between the low performing students and their peers.

The researcher created a pre-test and post-test to assess changes in understanding on multiplication and division. Copies of these assessments as well as their theoretical constructs are included in Appendix B. The students' ability to solve open ended contextual problems involving multiplication and division and the use of models to represent both equations and contexts was evaluated. Changes from pre-test to post-test for the treatment group were noted. Each subskill on the unit tests was evaluated using a four point rubric recommended by Van de Walle, Karp, and Bay-Williams (2010). Table 7 explains the rating for each level of the scoring.

Table 7. Four Point Rubric for Pre-Test and Post-Test Questions

Score 4 Got it	Excellent: Full Accomplishment Evidence shows that the student essentially has the target concept or idea. Strategy and execution meet the content, processes and qualitative demands of the task. Communication is judged by effectiveness, not length. May have minor errors.
Score 3 Got it	Proficient: Substantial Accomplishment Evidence shows that the student essentially has the target concept or idea. Could work to full accomplishment with minimal feedback. Errors are minor, so teacher is confident that understanding is adequate to accomplish the objective.

Score 2 Not there yet	Marginal: Partial Accomplishment Student shows evidence of major misunderstanding, incorrect concept or procedure, or failure to engage the task. Part of the task is accomplished, but there is lack of evidence of understanding or evidence of not understanding. Direct input or further teaching is required.
Score 1 Not there yet	Unsatisfactory: Little Accomplishment Student shows evidence of major misunderstanding, incorrect concept or procedure, or failure to engage the task. The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little or no success.

Note: From Van de Walle et al. (2010, p. 81)

Additionally, the work samples, informal assessments, and anecdotal notes collected for each student's portfolio provided qualitative data in a case study approach for each student. This data combined with the assessment data from the pre- and post-tests allowed the researcher to summarize each student's response to the intervention and make conclusions about patterns in strengths and weaknesses. The data for each student were coded and summarized based on the six indicators of an MLD. This qualitative data collection provided an opportunity to evaluate the effectiveness of organizing data about students according to the six indicators of an MLD listed on the framework.

CHAPTER 5: RESULTS

This research study focused on two research questions. The first question was how does low achieving fourth graders' participation in tutoring based on five key instructional practices for student with mathematics disabilities increase their understanding of multiplication and division? The second question was what are the patterns of deficits low achieving fourth graders have in relation to the six indicators of MLD? This review of results will first address the research question about changes in understanding based on results of the pre- and post-unit tests and the assessment of basic fact fluency. The test scores of the four students who participated in the study will then be shown in comparison to their peers to present how the students who participated in the tutoring made changes in their understanding in relation to their classmates. This information will help determine whether the four students made progress in closing the achievement gap between them and their peers. Next, the specific subtest scores for each student will be presented in more detail, and the qualitative data on the patterns of deficits for each student will be summarized along with the presentation of each student's subtest scores.

Multiplication and Division Unit Tests

The multiplication and division unit tests were designed to look at student proficiency on specific grade level mathematical knowledge and skills related to the Common Core Mathematics Standards. Therefore, results were analyzed looking at student proficiency on each subskill as well as an overall test score. The multiplication test evaluated student abilities in 11 subskill areas and the division test evaluated 6 subskill areas. Each subskill was rated on a scale of 1 to 4 based on an assessment method recommended by Van de Walle et al. (2010, p. 81). This scoring method was explained in the methods section. The overall test score was determined by averaging the subtest scores for each student. Tables showing all of the subtest scores for the pre- and post-tests, overall scores, and number of skills proficient for each student in the study sample are included in Appendix C. The class averages and standard deviations for each subtest and overall scores are also shown. The students who participated in the tutoring made up their own alias names. For the discussion of the results and findings, these four students will be referred to as Big Jay, Destiny, Happy Gilmore, and Katarina. The tables in Appendix C show the alias names for each of the four students who participated in the tutoring. All other students are referred to by student numbers.

Figures 4 and 5 show the number of skills proficient on the pre- and the post-tests for each of the four students who participated in the tutoring. The data reflects Big Jay, Destiny, and Happy Gilmore increased the number of multiplication skills proficient, and Katarina decreased in her number of multiplication skills proficient from pre-test to post-test.

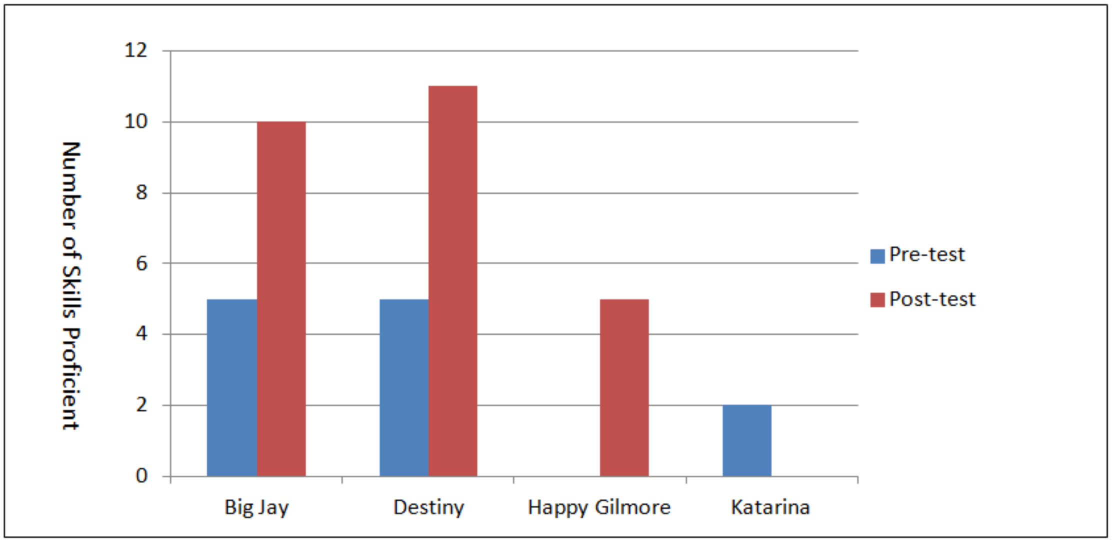


Figure 4. Multiplication Unit Test: Number of Skills Proficient
Note: Proficient was considered a score of 3 or 4 on the subtest.

On the division test, Big Jay and Destiny increased their number of skills proficient. Happy Gilmore showed no change, and Katarina decreased from one division skill proficient to zero proficient.

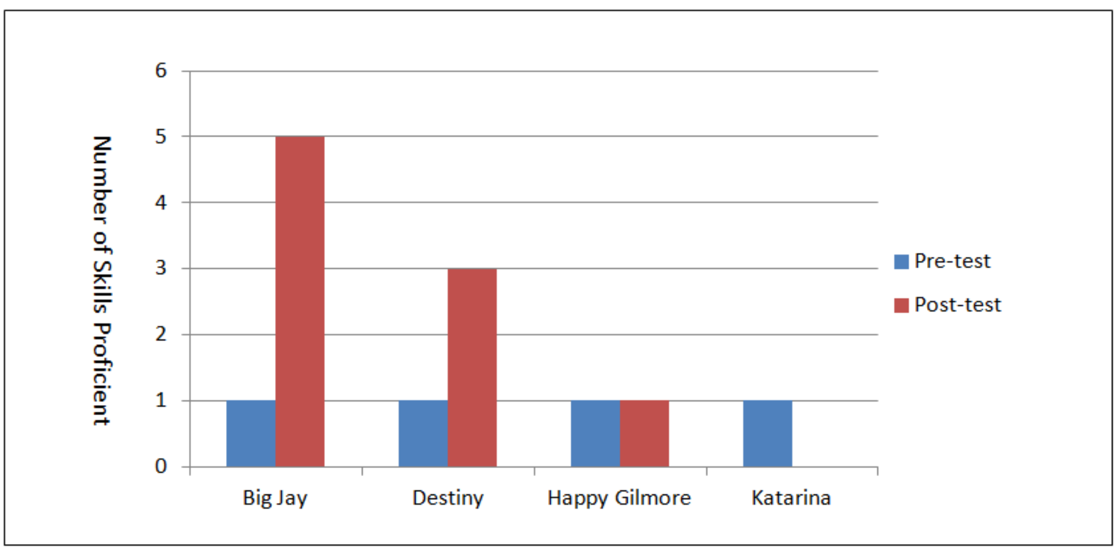


Figure 5. Division Unit Test: Number of Skills Proficient
Note: Proficient was considered a score of 3 or 4 on the subtest.

Two Minute Multiplication Facts Assessment

The scores for the whole class on the two minute multiplication fact assessment are shown on the same data sheet as the data for the multiplication unit subtests included in Appendix C. Figure 6 shows the comparison between pre- and post-test for the four students on this assessment. The data shows Big Jay increased from 53 to 55 correct facts in two minutes. Destiny increased from 32 to 50. Happy Gilmore increased from 23 to 29 correct facts, and Katarina completed 23 correct facts on both the pre- and the post-test, resulting in no change.

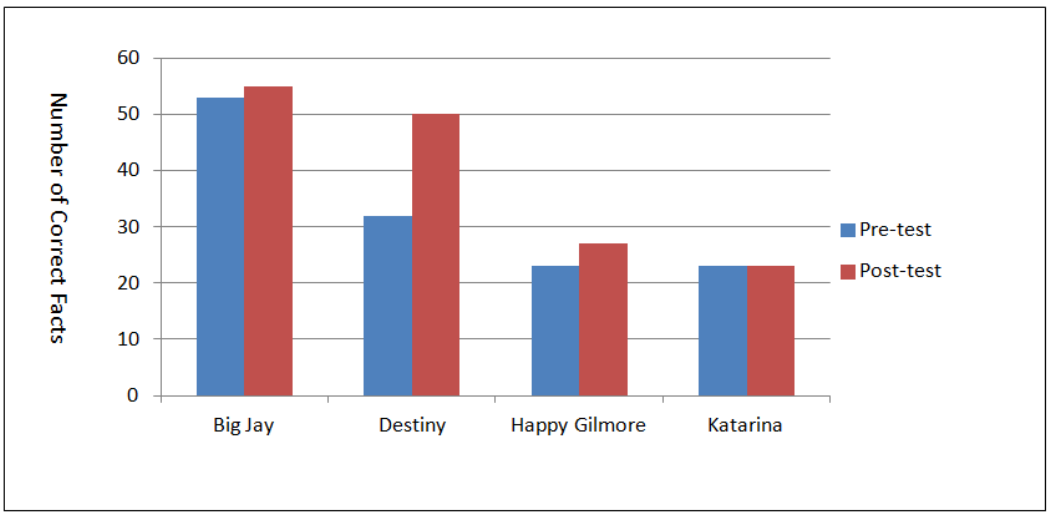


Figure 6. Two Minute Multiplication Facts Assessment

Comparison to Peers

When examining the results of academic intervention for low-achieving students, it is important to consider the changes in understanding that also occurred for the other students in the class during the same time period. While the results of a pre-test to post-test may show positive, even statistically significant, gains for students who participated in a treatment, these gains may not have been sufficient to close the achievement gap between them and their peers. In other words, while the students participating in the tutoring were learning mathematics, so were their typical peers. The scatter plots in Figures 7 through 12 show the changes from pre-test to post-test on the unit assessments and the multiplication fact assessment for the whole class. The scores of the students who participated in the tutoring are shown in red and the class average is shown in green on each scatter plot. Table 8 shows the numbers assigned to the four study participants and the number representing the class average.

Table 8. Assigned Student Numbers to Study Participants

Student Name	Number on Scatter Plots
Destiny	2
Happy Gilmore	10
Big Jay	14
Katarina	21
Class Average	22

One way to compare the performance of students in one class on an assessment is to look at quartiles. In this way, teachers can easily see which students are scoring lower than 75% of the class and which students are scoring higher than 75% of the class. Using quartiles allows teachers to determine which students may be in need of intervention and which students are advanced compared to peers in their class. Additionally, looking at the growth of a student from one quartile to another from pre-test to post-test can help a teacher determine whether the student was closing the achievement gap and scoring closer to the middle of the class in response to intervention. Quartiles were used to analyze the pre-test to post-test scores of the four students in the tutoring group in comparison to their peers. Because the scores for each assessment were not normally distributed, the median was used as the measure of central tendency instead of the mean. Table 9 shows the median, the 25th percentile, and the 75th percentile score for each pre-test and post-test. Additionally, the 25th percentile and the 75th percentile cut offs are shown on the scatter plots in Figures 7 through 12.

Table 9. Quartile Data on Pre-Test and Post-Tests

Test	Median	25th Percentile	75th Percentile
Multiplication unit pre-test Number of skills proficient	6	5	8
Multiplication unit post-test Number of skills proficient	8	6	10
Division unit pre-test Number of skills proficient	3	1	6
Division unit post-test Number of skills proficient	4	2	5
Multiplication fact pre-test Number of facts correct	44	32	51
Multiplication fact post-test Number of facts correct	47	39	55

On the multiplication unit test (Figures 7 and 8), the results show Destiny and Big Jay made notable progress in closing the achievement gap between them and their classmates. On the pre-test, both of these students performed below the class median of number of skills proficient and were in the third quartile of their class. Destiny and Big Jay were in the group of students with the seven lowest test scores in the class on the multiplication pre-test, which is why they were selected to participate in the tutoring. On the multiplication post-test, Destiny was the highest scoring student in the class, and Big Jay scored the same as six other high scoring students at the 75th percentile. Happy Gilmore was the lowest scoring student in the class on the multiplication pre-test with zero subskills proficient. On the multiplication post-test, he moved closer to the third quartile of his class and scored comparably with other low achieving students in the class with five subskills correct. The gap in achievement between Katarina and her peers increased from the pre-test to the post-test. She scored in the bottom quartile of the class on both tests and decreased from two skills proficient to zero skills proficient from the pre-test to post-test on the multiplication test. She moved to the lowest scoring student in the class on the multiplication unit post-test.

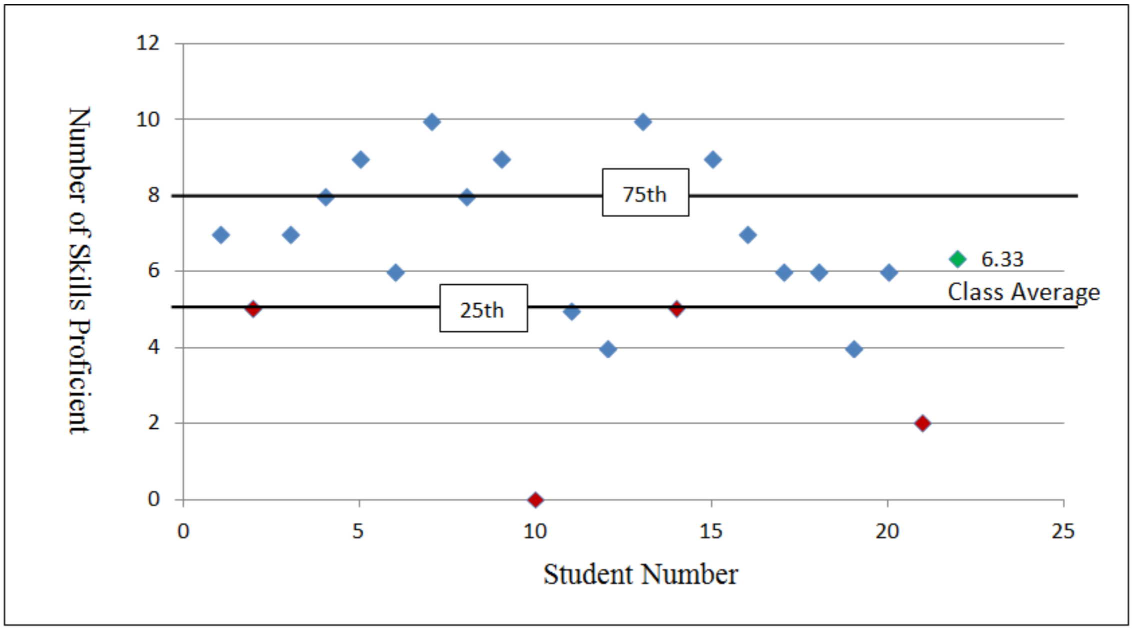


Figure 7. Class Graph - Multiplication Unit Number of Skills Proficient Pre-Test

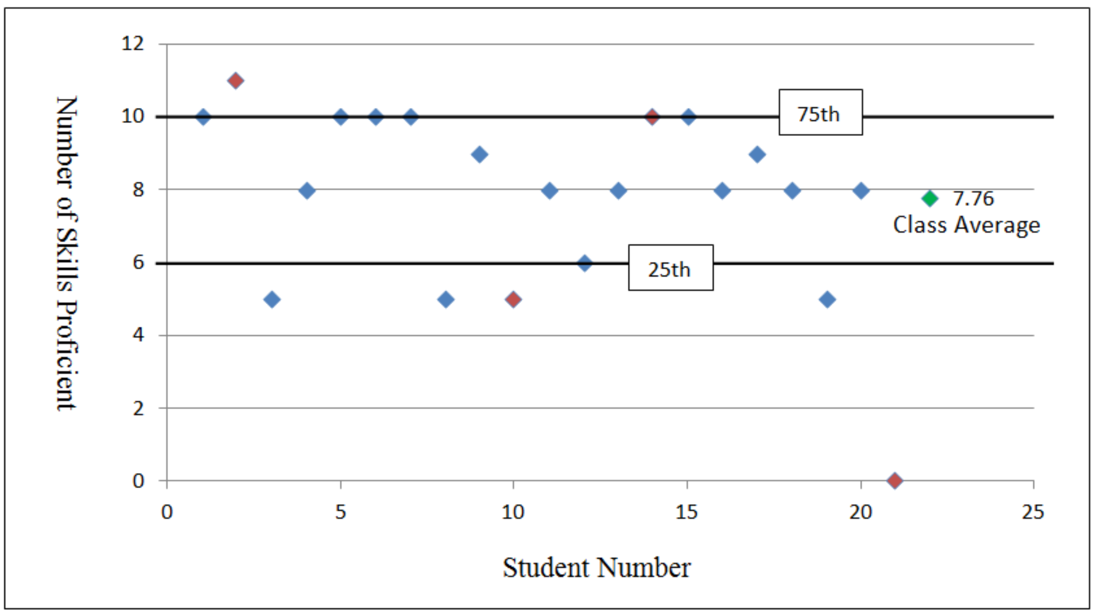


Figure 8. Class Graph - Multiplication Unit Number of Skills Proficient Post-Test

On the division unit pre-test (Figure 9), all four study participants had one skill proficient, skill proficient, scoring in the bottom quartile in the class. They were four of six students with a score of one. On the division post-test (Figure 10), Destiny scored at the 75th percentile. Her score was comparable to the seven highest scoring students on the post-test. Big Jay scored closer to the class median and improved from zero to three skills proficient from pre-test to post-test. Happy Gilmore and Katarina still scored in the bottom quartile of the class, with the two lowest scores in the class. Happy Gilmore had only one division skill proficient and Katarina had zero.

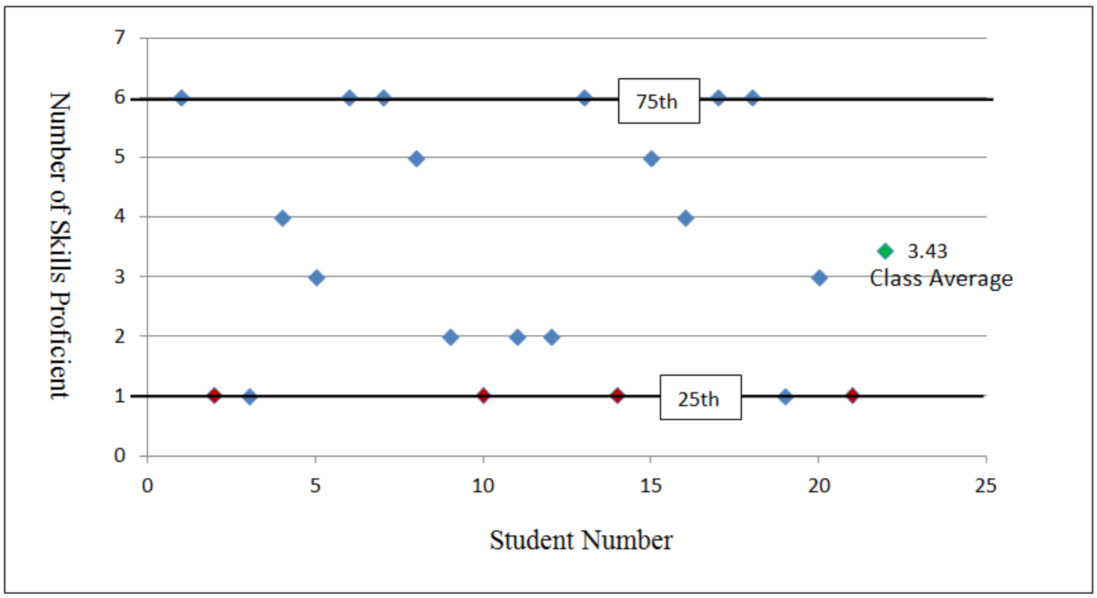


Figure 9. Class Graph - Division Unit Number of Skills Proficient Pre-Test

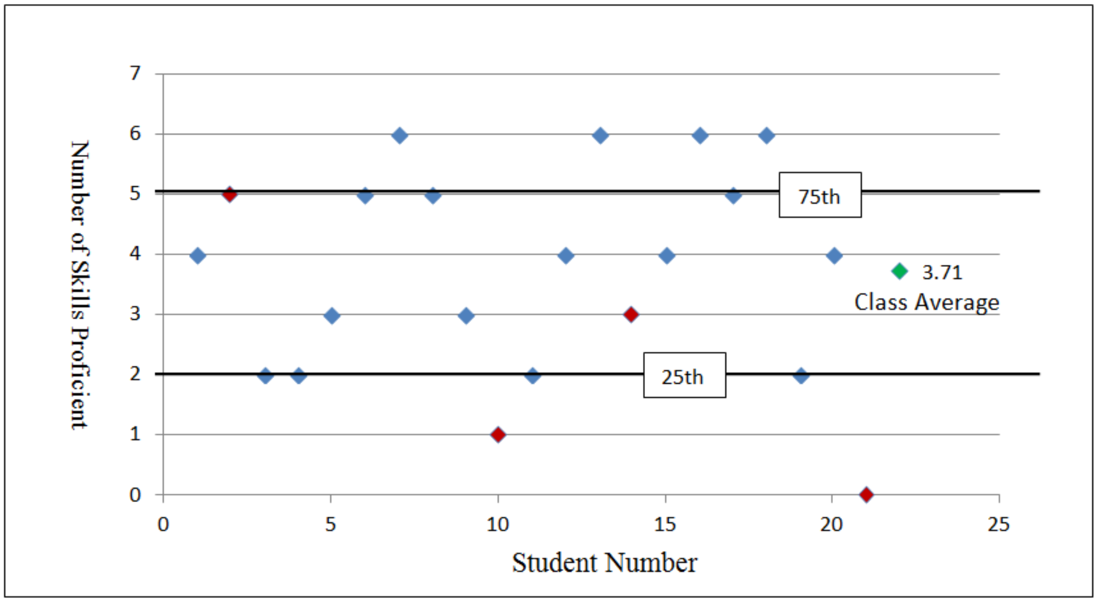


Figure 10. Class Graph - Division Unit Number of Skills Proficient Post-Test

On the two minute multiplication fact assessment (Figures 11 and 12), Happy Gilmore and Katarina, with their scores of 23 facts correct in two minutes, were the two lowest scoring students in the class on the pre-test. On the post-test, they maintained their ranking as the two lowest scoring students. Katarina had exactly the same score on the pre-test and post-test. Because the class median improved on the post-test, this means that the gap between her and her peers on this assessment increased. Happy Gilmore made a slight gain on his post-test, improving from 23 to 29 correct facts. In spite of this improvement, he made little movement toward the class median. On the pre-test he was 21 correct facts away from the class median score, and on the post-test he was 20 correct facts away. On this assessment pre-test, Destiny was exactly at the 25th percentile, scoring only slightly above the lowest scoring students. On the post-test, she improved her ranking to the second quartile and scored above the class median with a score of 50. Big Jay was slightly above the 75th percentile of the class on this pre-test and was one of the five highest scoring students in the class. On the post-test, he maintained his placement at the 75th percentile and again scored comparably to the highest scoring students in the class.

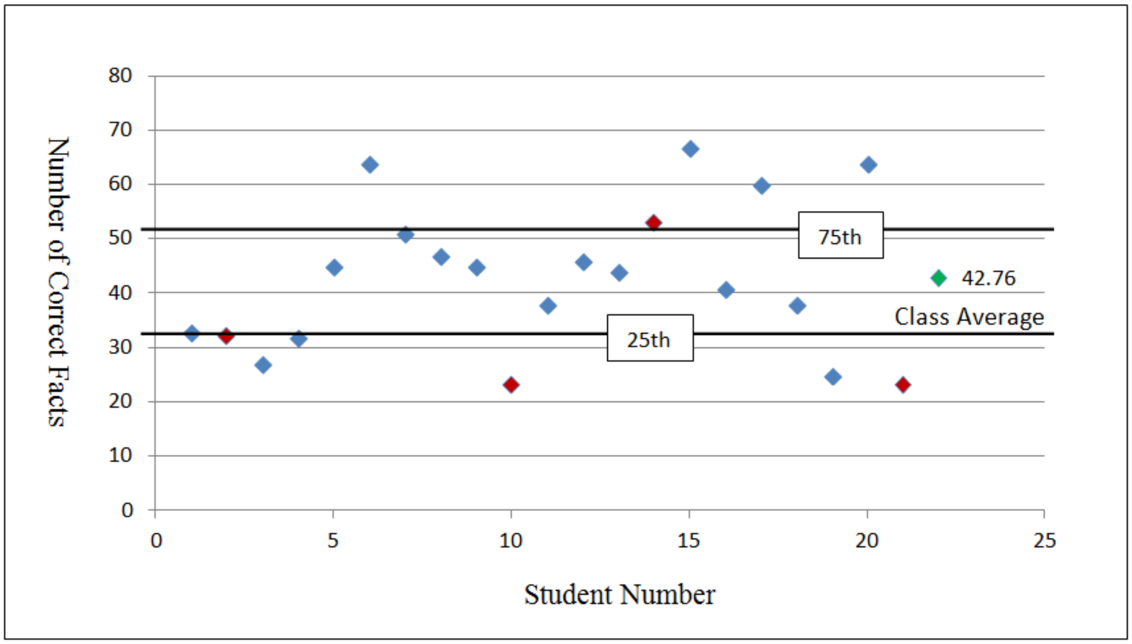


Figure 11. Class Graph - Two Minute Multiplication Fact Assessment Pre-Test

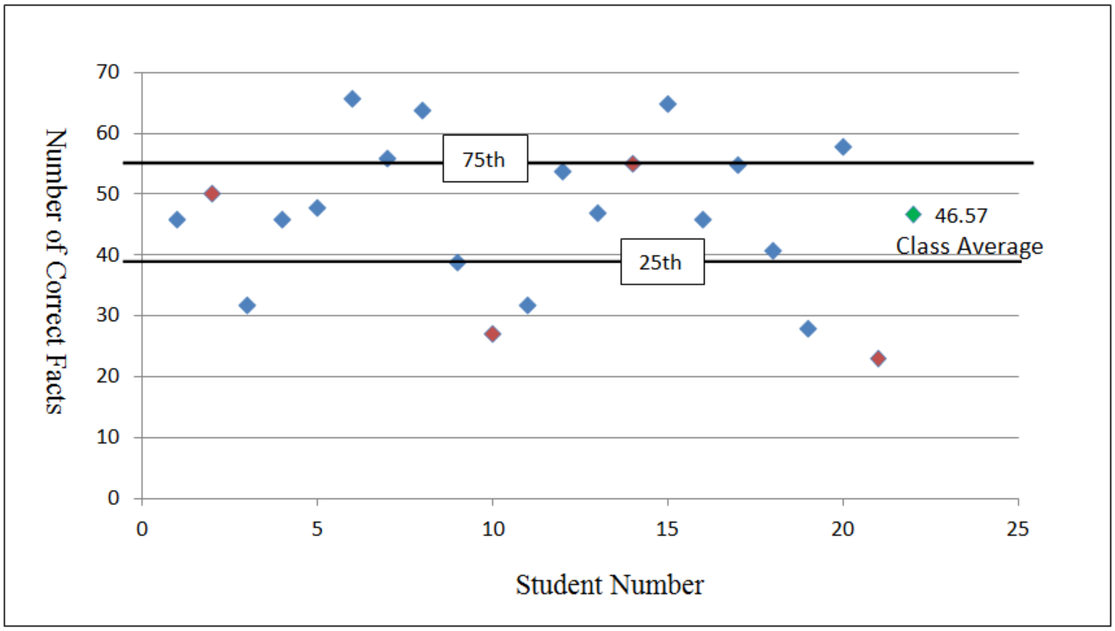


Figure 12. Class Graph - Two Minute Multiplication Fact Assessment Post-Test

Individual Student Results

The data comparing pre- to post-test scores for the four students as well as the data comparing their scores to their classmates suggests Big Jay, Destiny, and Happy Gilmore improved their understanding of multiplication and division, while Katarina did not. This section will look more closely at the subtest scores for each of the four students. These scores provide additional insight into the changes in understanding, which may or may not have occurred as a result of the tutoring. In addition to the subtest scores, the qualitative data for each student will be presented and the patterns of deficits in relation to the indicators of an MLD discussed. These data provide a greater understanding of each student's strengths and struggles in relation to the subtest scores. Therefore, it will be presented for each student along with the quantitative data on the unit subtest scores. Table 10 provides a key for the codes used to report the qualitative data.

Table 10. Codes Used for Qualitative Data Analysis

Code	Meaning
JM	Judgment of magnitude
MR	Multiple representations
FBC	Fluency with basic calculation
VSS	Visual spatial skills
PM	Procedural memory
PE	Procedural error
PAL	Peer assisted learning
EI	Explicit instruction
SI	Strategy instruction
DIS	Productive disposition
LB	Language barrier
ATT	Attendance

During the coding process, it was difficult to distinguish between judgment of magnitude and determination of unreasonable results. It was apparent during both the tutoring and the coding process that a student's ability to determine whether an answer was reasonable was related to their judgment of magnitude. Therefore, the indicators of MLD were condensed to five instead of six and determination of unreasonable results was not used in the data analysis. Each code was used to indicate a strength or a struggle for that student. For example, if the observation or work sample showed a strength in fluency with basic calculation, it was coded as +FBC. If an observation or work sample showed fluency with basic calculation as a struggle, it was coded as -FBC. This method allowed the researcher to organize the observations and work sample notes as strengths or struggles for each indicator of MLD. The strengths and struggles approach is adapted from Bryant et al.'s (2006) suggestions for examining student abilities needed to perform mathematical tasks in their Adaptations Framework. Other themes emerged during the coding process, which resulted in the addition of several other codes used along with the MLD indicator areas. The code ATT was used to show the attendance record of each student but these numbers were not used to report the total number of coded observations.

Big Jay

Figures 13 and 14 show Big Jay's scores on the multiplication and division unit subtests for both the pre-tests and the post-tests. This display of results from the unit assessments provides more detail on the specific mathematical concepts on which Big Jay showed changes in understanding. Appendix D provides both the pre-test answer and the post-test answer for several test questions to show more specifically the improvements in his understanding of multiplication and division.

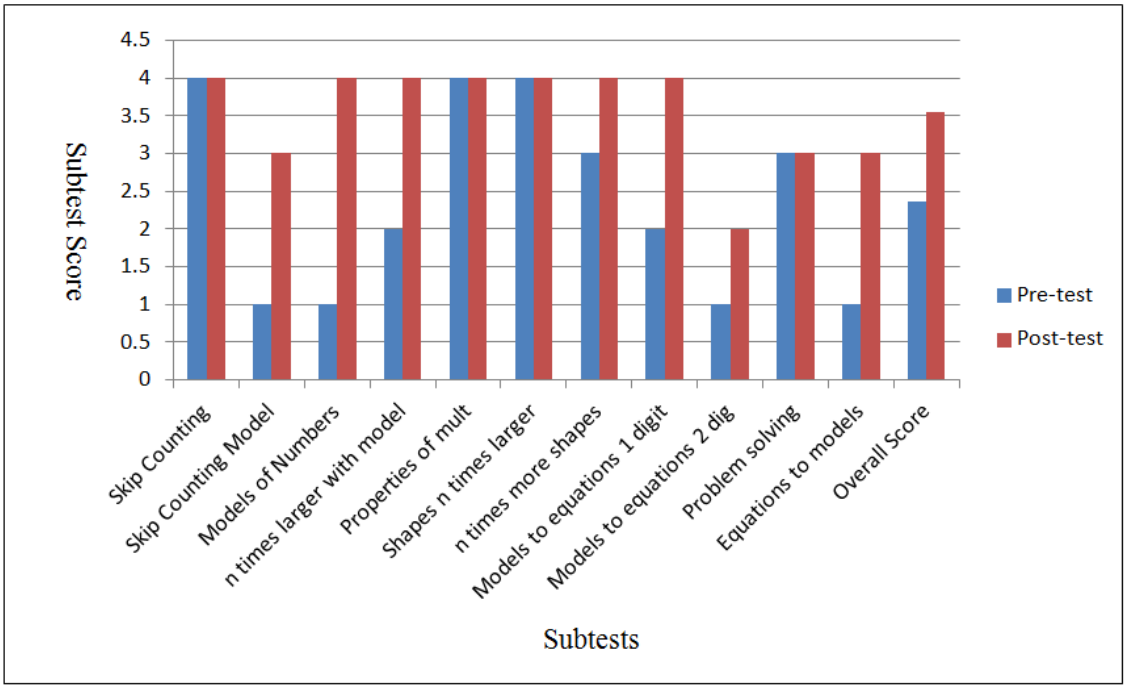


Figure 13. Big Jay Multiplication Subtest Scores

These results show Big Jay improved his understanding in seven subskills tested on the multiplication unit test. His score stayed a three on the problem solving section. On the subtests on which he had scored a four on the pre-test, he also scored a four on the post-test. Big Jay improved in four of the six subskills tested on the division unit test. In the other two subskills, making groups and equations to models, his score was lower on the post-test than the pre-test.

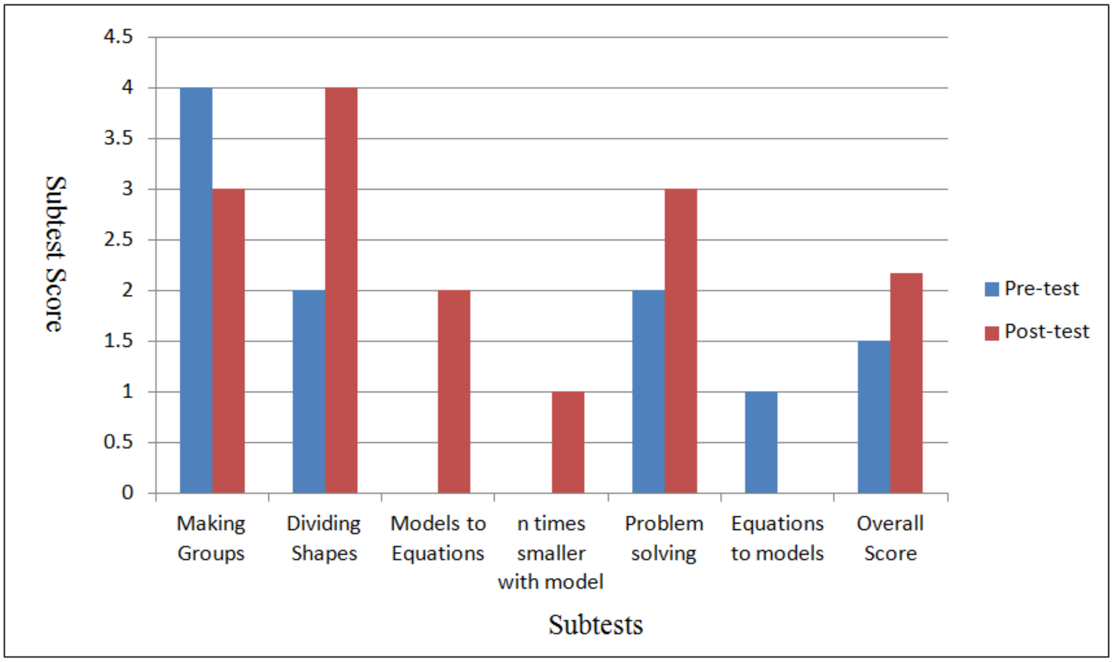
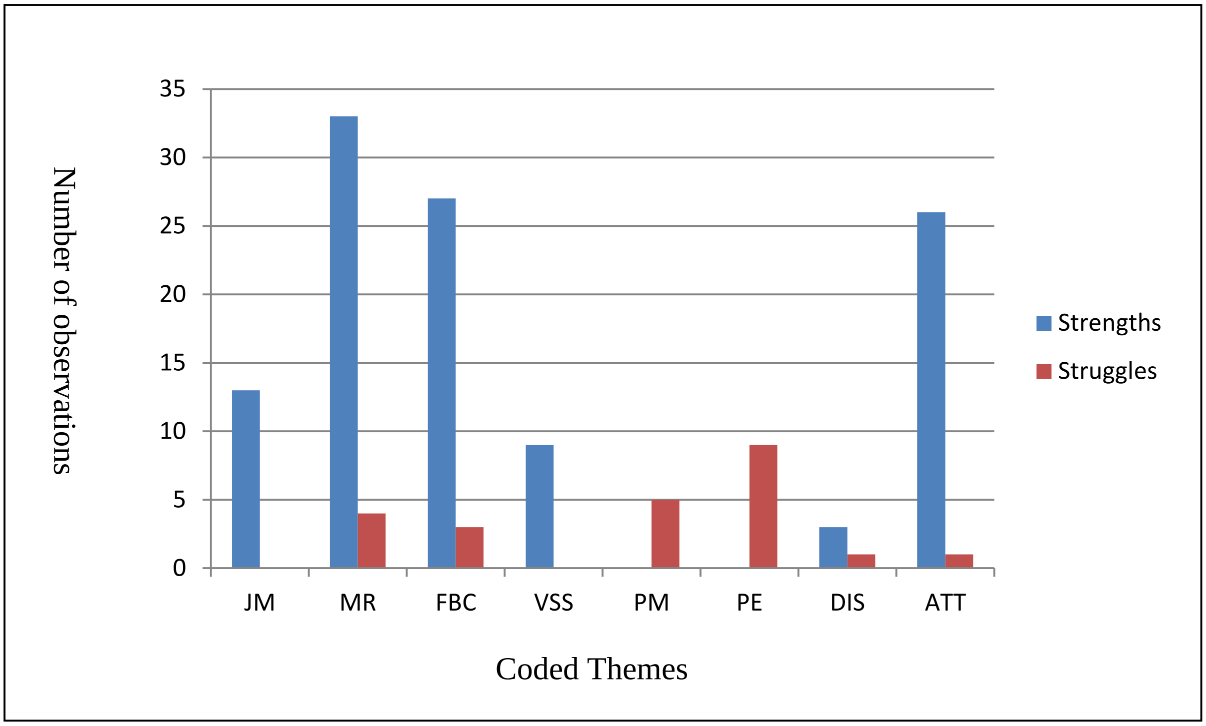


Figure 14. Big Jay Division Subtest Scores

The qualitative data gathered from observations and work samples throughout the tutoring provide greater insight into the reasons Big Jay's scores dropped in some subskills on the post-test. These data also provide information related to whether Big Jay displays a pattern of deficit related to the five indicators of MLD. Table 11 shows a summary of 107 coded observations and work samples from Big Jay. Figure 15 displays the same information as a graph.

Table 11. Big Jay: Coded Observations and Work Samples

Code	Strength	Struggle
JM	13	0
MR	33	4
FBC	27	3
VSS	9	0
PM	0	5
PE	0	9
DIS	3	1
ATT	26	1
Total observations: 107		



Note: Based on 107 coded observations and work samples and attendance.

Figure 15. Big Jay Strengths and Struggles

The qualitative data revealed Big Jay did not have a consistent pattern of weakness in judgment of magnitude, using multiple representations, fluency with basic calculation or visual spatial abilities. Many observations and work samples revealed strengths in these areas. He also had good attendance, and a productive disposition was noted three times. Big Jay did have some struggles in the indicator of procedural memory. A more detailed analysis of the observations related to procedural memory revealed that PE (Procedural Error) was a better coding for some of these observations and work samples instead of PM (Procedural Memory). For the observations coded PE, his errors were not related to memory or conceptual understanding. It was just a small procedural or counting error. Therefore, both the PM and PE codes were used to show Big Jay's struggles in his performance in mathematics.

Big Jay made the same kind of small procedural error on the one multiplication skill in which he did not show improvement on his subtest score. An example from the test item showing this procedural error is shown in Appendix D. In the section on making groups on the division unit post-test, Big Jay again made a procedural error on one test item and counted wrong. On another item in that section of the test, he did not complete the problem. The other division unit subtest in which Big Jay showed a drop in his score was going from division equations to a model. Big Jay did not attempt to answer any of the questions in that section, which is why he had a score of zero for that subtest.

Destiny

Figures 16 and 17 show Destiny's performance on the multiplication and division unit subtest for both the pre-tests and the post-tests. Destiny showed improvement in seven of the areas assessed on the multiplication unit test. On three skill areas that showed no improvement, she had already scored a four on the pre-test. On the other subtest, which showed no improvement, she made the same error on both the pre-test and post-test. This error was related to translating the set model of multiplication to a basic equation. Excerpts from the multiplication test showing Destiny's improvement from pre-test to post-test on selected items as well as some of the problems that showed misconceptions are included in Appendix D.

On the division subtests, Destiny also showed improvement on all but one of the skills assessed. Item analysis of this subtest revealed Destiny was able to correctly answer and make a model of two digit by one digit division problems with no remainders. She struggled to represent and solve problems with two digit dividends that had remainders and problems with three digit dividends. Work samples from some selected items from the division pre- and post-test that demonstrate Destiny's improvement in her understanding of division are also shown in Appendix D. Destiny's subtest scores show that she greatly improved her understanding of both multiplication and division as a result of the tutoring she received.

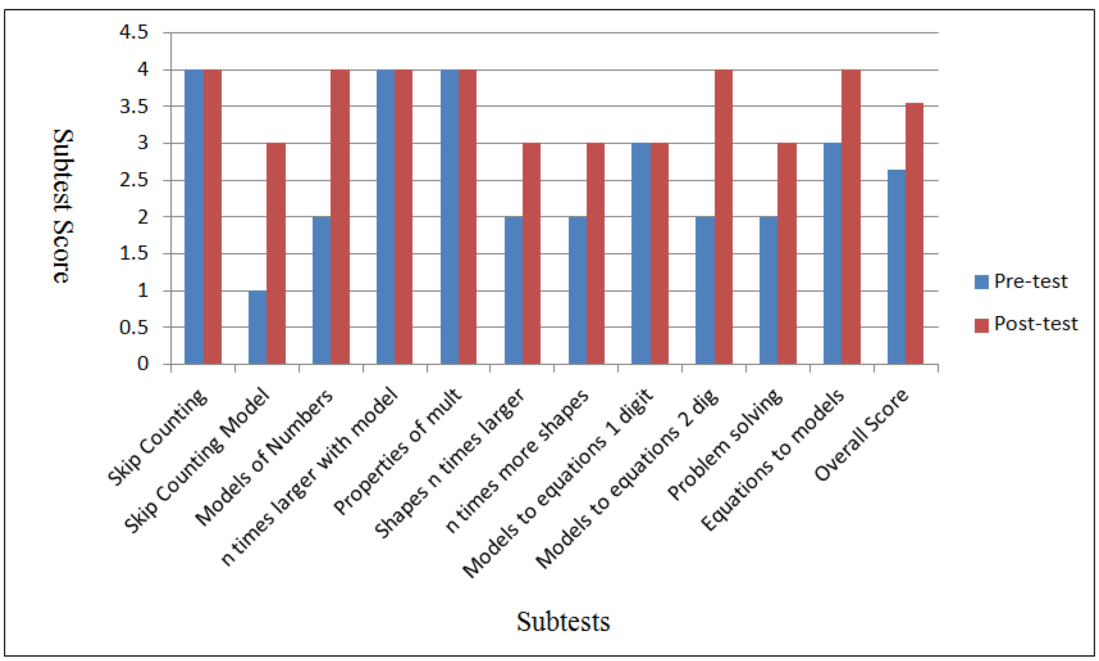


Figure 16. Destiny Multiplication Subtest Scores

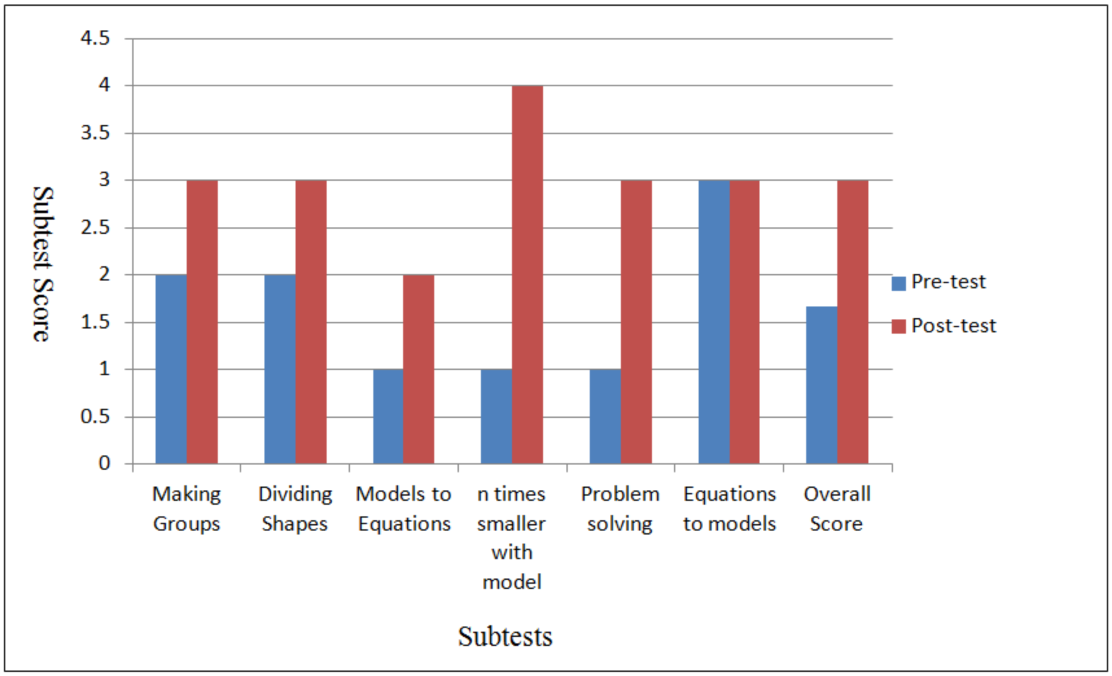


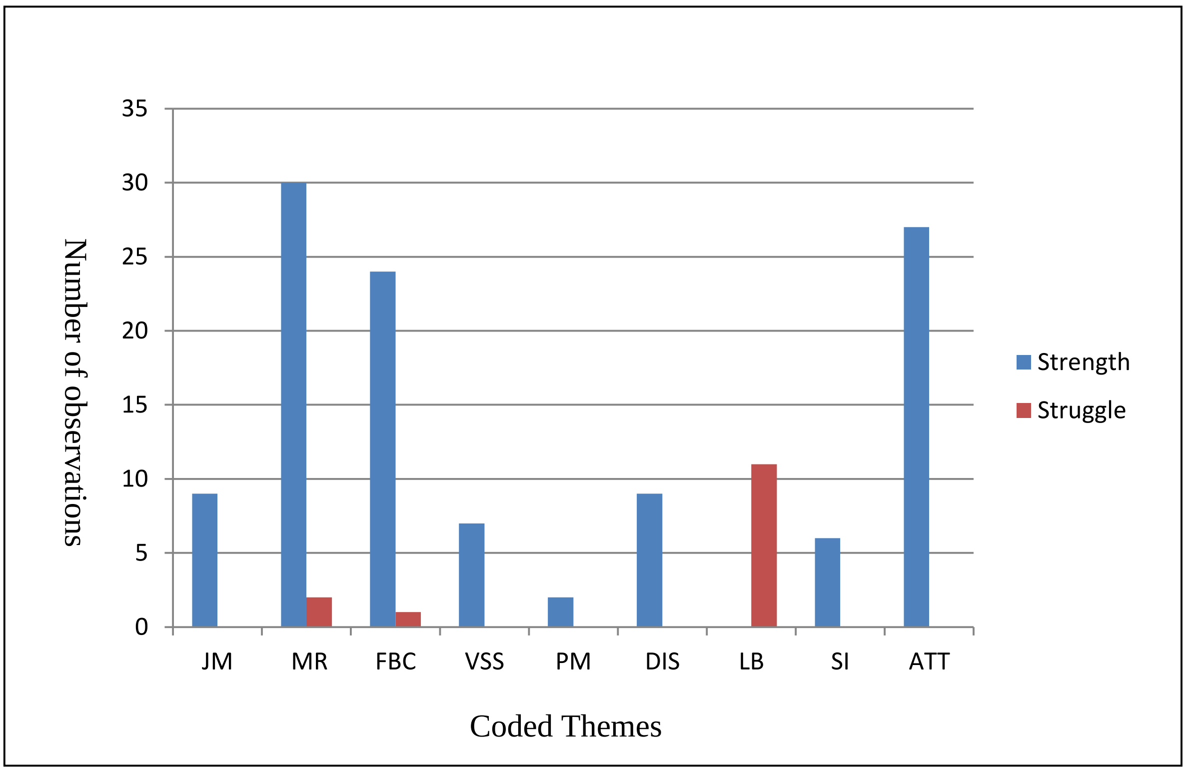
Figure 17. Destiny Division Subtest Scores

The qualitative data on Destiny yielded 101 coded observations and work samples. These data are summarized in Table 12 and shown as a graph in Figure 18.

Table 12. Destiny: Coded Observations and Work Samples

Code	Strength	Struggle
JM	9	0
MR	30	2
FBC	24	1
VSS	7	0

Code	Strength	Struggle
PM	2	0
DIS	9	0
LB	0	11
SI	6	0
ATT	27	0
Total observations: 101		



Note: Based on 101 coded observations and work samples and attendance.

Figure 18. Destiny Strengths and Struggles

Destiny's patterns of deficits show minimal struggles in the indicator areas related to MLD. She was especially strong in her ability to represent mathematical ideas with multiple representations and used words, models, and symbols effectively and interactively for the same problem. Several observations revealed she had good judgment of magnitude and the ability to be fluent and flexible when completing basic calculations. The qualitative data did not show any notable weaknesses in the areas of visual spatial skills or procedural memory. She had a productive disposition and good attendance. The code LB (Language Barrier) was used for observations that revealed Destiny's struggle to comprehend the English academic vocabulary used in mathematics. She had 11 instances where a language barrier was noted. Destiny's observational data also led to the introduction of the code SI to represent Strategy Instruction. This is not necessarily a strength or a weakness in a child. It is a teacher behavior, but this code was used to refer to times Destiny responded positively to explicit strategy instruction when working with basic calculation.

Happy Gilmore

Figures 19 and 20 summarize Happy Gilmore's subtest scores on the unit pre-tests and post-tests. He showed improvement in eight of the eleven subtest areas on the multiplication unit test after the tutoring. He improved in three of the six subtest skills on the division unit test. These scores provide evidence that Happy Gilmore improved his understanding of multiplication and division from the tutoring. Test items showing some examples of his changes in understanding are included in Appendix D.

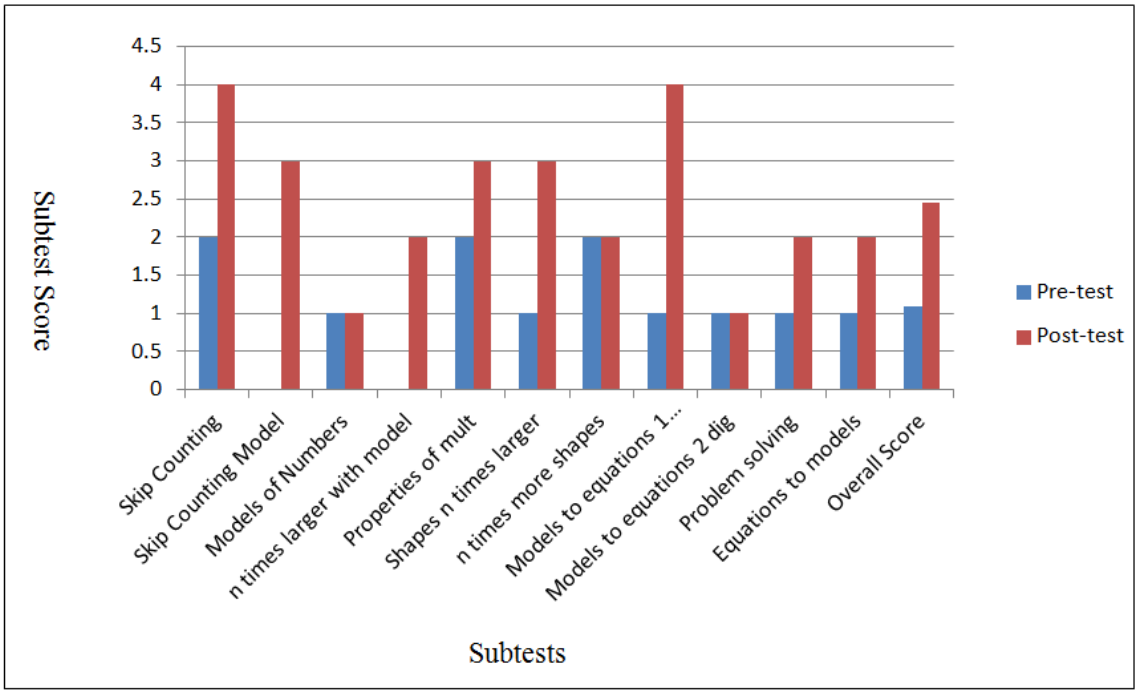


Figure 19. Happy Gilmore Multiplications Subtest Scores

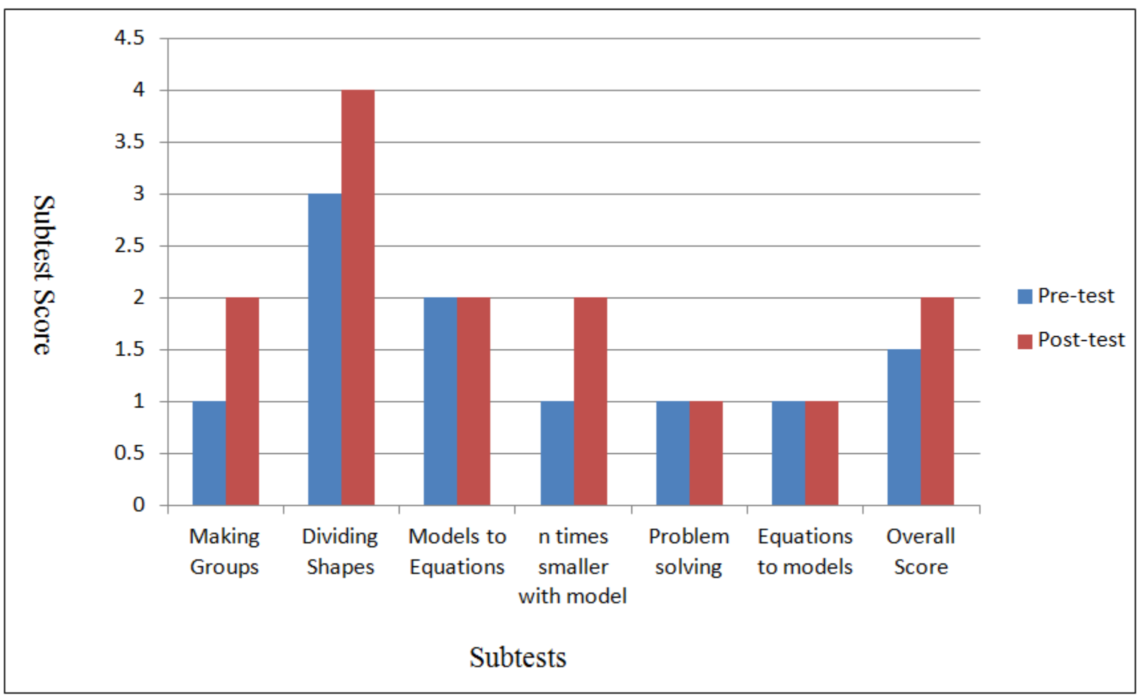


Figure 20. Happy Gilmore Division Subtest Scores

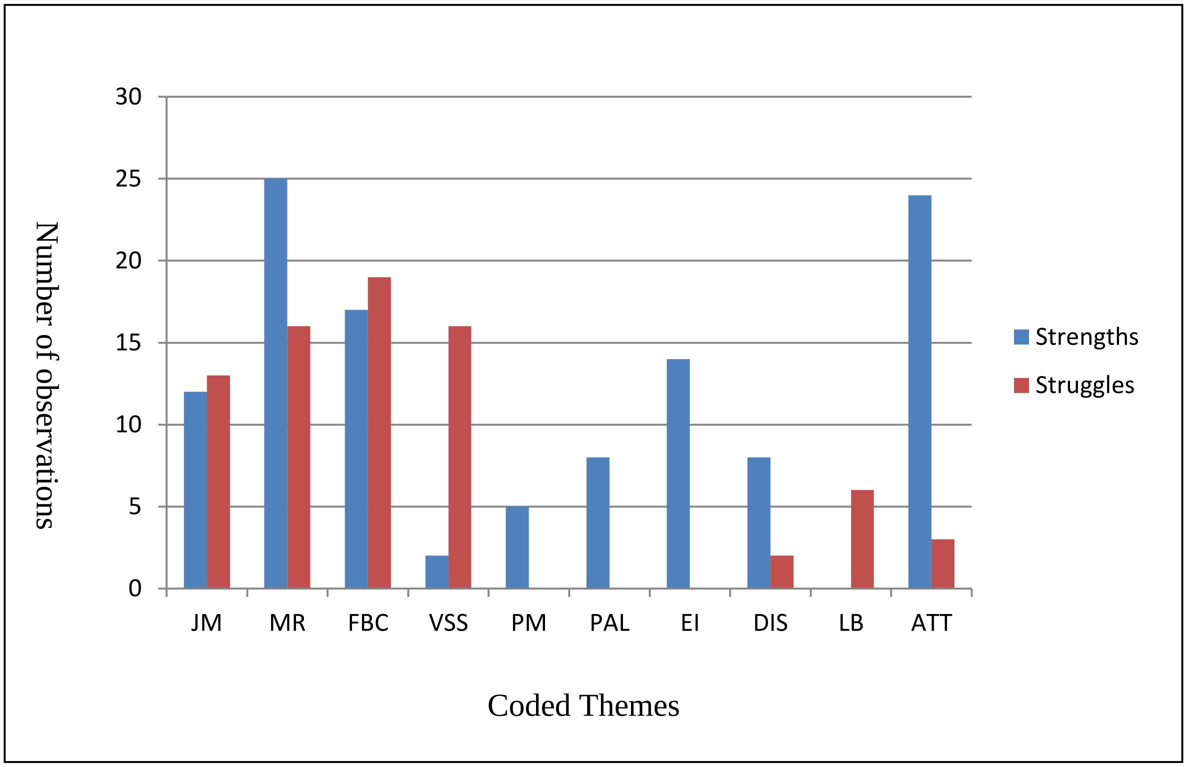
Two multiplication skills that Happy Gilmore showed no change in understanding were making models of numbers and interpreting the statement "draw n times more shapes" as a multiplicative relationship. He interpreted the statement as additive. He also showed no progress in his ability to interpret models of 2 digit multiplication problems represented with open arrays. On the division post-test, he showed no change from his pre-test score on converting models to equations. He showed the models as multiplication equations rather than division equations. Happy Gilmore also showed no changes in scores on the problem solving section of the division test, and converting division equations to models. On the division post-test, he did attempt to solve the division contextual problems using models more than he did on the pre test. An example of this change in his attempt to solve a division contextual problem with a model is shown in Appendix D. These struggles on the unit tests are consistent with what the patterns in the qualitative data suggest about Happy Gilmore's struggles in mathematics.

The qualitative data on Happy Gilmore yielded 163 coded observations and work samples. This data is summarized in Table 13 and shown as a graph in Figure 21. When coding observations and work samples for Happy Gilmore, additional codes were added. EI was used to code incidents when he responded positively to explicit instruction. PAL indicated a positive response to peer-assisted learning and student discourse.

Table 13. Happy Gilmore: Coded Observations and Work Samples

Code	Strength	Struggle
JM	12	13
MR	25	16
FBC	17	19
VSS	2	16
PM	5	0
PAL	8	0
EI	14	0
DIS	8	2
LB	0	6
ATT	24	3

The qualitative data on Happy Gilmore showed he had many struggles in several of the indicator areas related to MLD. There were several observations noted that showed patterns of deficits in judgment of magnitude, using multiple representations, fluency with basic calculations, and visual spatial skills. In addition, a language barrier when he did not know the meaning of an English word was noted six times. Happy Gilmore did not have any noted struggles with procedural memory and responded well to explicit instruction. He remembered procedures shown to him over time. His strengths in productive disposition and peer-assisted learning show his enthusiasm about mathematics and an ability to orally discuss the mathematical concepts in the tutoring lessons. He also had good attendance.



Note: Based on 163 coded observations and work samples and attendance.

Figure 21. Happy Gilmore Strengths and Struggles

Although the qualitative data on Happy Gilmore's struggles in mathematics are consistent with the indicators of MLD, he also had several incidents when he showed strengths in judgment of magnitude, using multiple representations, and fluency with basic calculations. The observations and work samples with these codes were looked at more closely and further categorized. First, the observations coded JM (Judgment of

Magnitude) were categorized by whether the task included the use of a visual aid. A visual aid was something like a hundreds chart or a model to refer to such as a number line drawn on the board. These data are shown in Figure 22 and show that without a visual aid, Happy Gilmore was often able to visualize magnitude. However, he also had many times when he was not able to visualize magnitude even with the use of a visual aid.

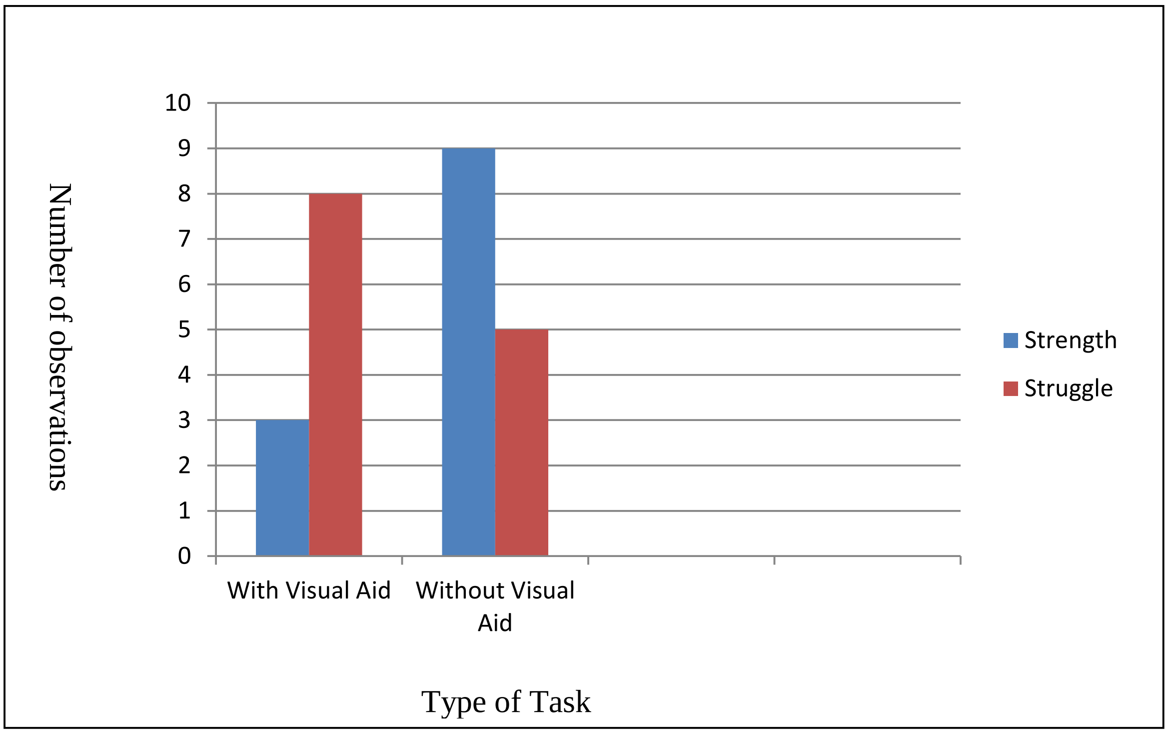


Figure 22. Happy Gilmore Task Analysis for Judgment of Magnitude

Next, the tasks that involved the use of multiple representations were analyzed based on the Triple-Code Model. Table 14 provides a key to the codes used for this analysis, and Figure 23 displays these results.

Table 14. Codes used for Multiple Representations Task Analysis

Code	Meaning
STL	Symbols to language
STM	Symbols to model
LTS	Language to symbols
LTM	Language to model
Code	Meaning
MTS	Model to symbols
MTL	Model to language

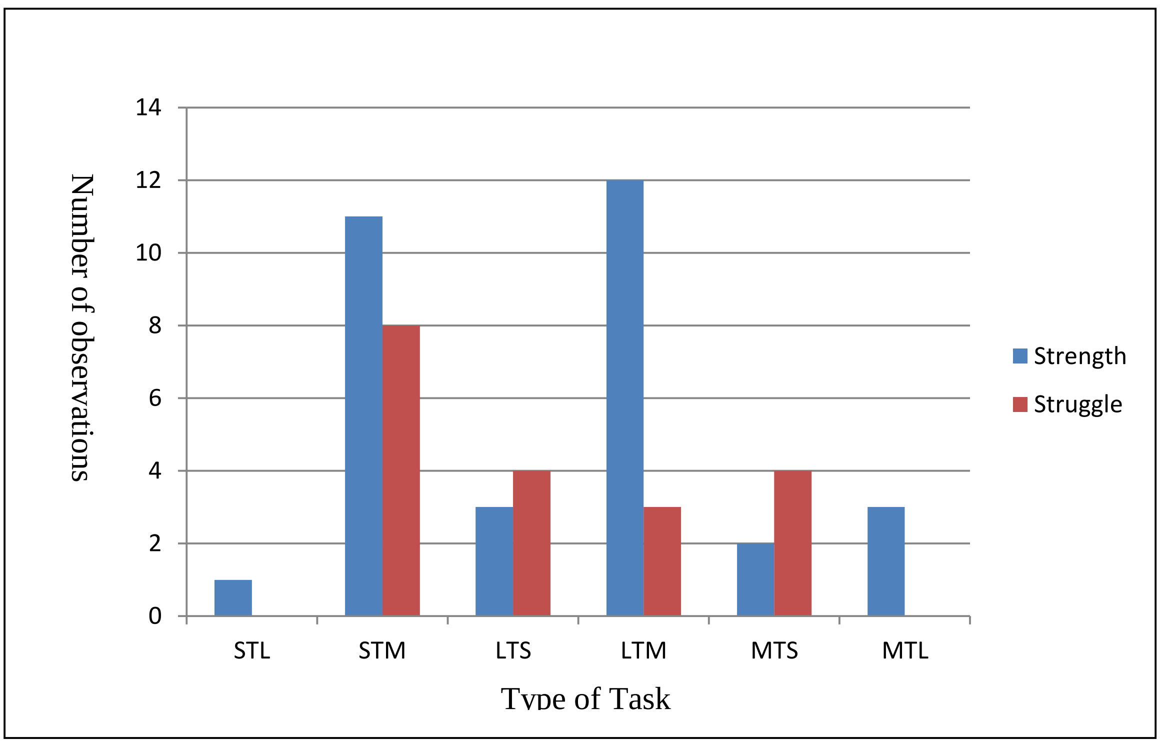


Figure 23. Happy Gilmore Task Analysis for Multiple Representations

These results show Happy Gilmore was more successful with tasks that did not involve the use of symbols. Two examples of such tasks are shown in Appendix D. When tasks involved the use of symbols, Happy Gilmore showed a consistent pattern of struggle. On the language to model tasks, he showed many more strengths than struggles and improved his abilities on these tasks from being taught how to draw models during the tutoring. The observations that showed a struggle with the language to model tasks were more frequent at the beginning of the tutoring period when he was initially learning to use models. The observations coded +MR (LTM) increased during the later tutoring sessions. This data suggests Happy Gilmore improved his ability to convert from language to a model as a result of the tutoring. This claim was further validated by item analysis on the problem solving sections of the pre- and post-tests. Work samples and test items supporting this claim are shown in Appendix D. Happy Gilmore's performance on the unit post-tests complements the qualitative data that shows his pattern of deficit related to the use of symbols. All but one of the subtest areas on which he showed no progress involved converting from or to symbolic notation.

On tasks that involved fluency with basic calculations, the observations were noted as a task in which he had the use of a visual aid or without a visual aid. These results are shown in Figure 24.

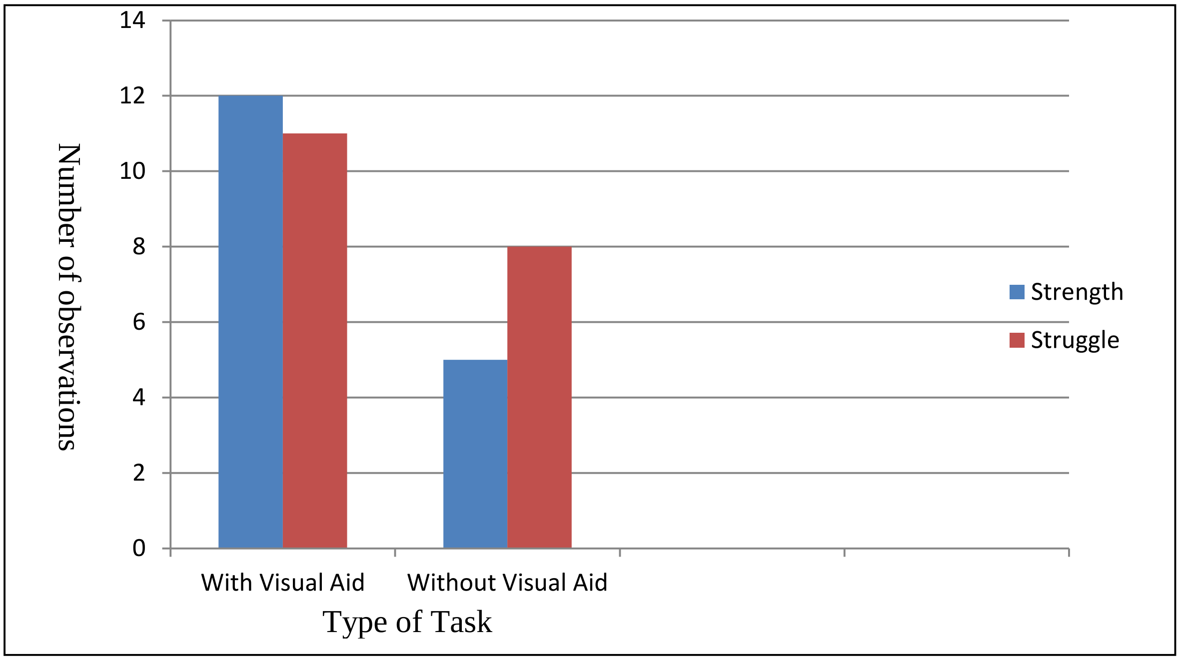


Figure 24. Happy Gilmore Task Analysis for Fluency with Basic Calculation

This task analysis revealed even when provided a visual aid, Happy Gilmore still struggled with fluency with basic calculation. The data suggest he did much better with basic calculation when he had a visual aid available compared to when he didn't. This error pattern is consistent with a deficit in fluency with basic calculation related to MLD. Happy Gilmore's minimal progress on the two minute multiplication fact assessment provides additional evidence supporting the claim that he has a consistent pattern of deficit in the indicator area of basic fact calculation.

Both the quantitative and qualitative data on Happy Gilmore show he made positive changes in his understanding of multiplication and division as a result of the tutoring he received. Although he had several noted observations when he was successful with judgment of magnitude, using multiple representations and fluency with basic fact calculation, the data revealed a consistent pattern of deficit in each of these areas. He also showed a deficit in his visual spatial skills.

Katarina

On the multiplication unit test (Figure 25), Katarina scored lower on the post-test than the pre-test in many skills. She regressed in her ability to skip count and interpreting a problem such as "2 times larger than 10." She also regressed in her ability to solve a multiplication equation and represent it with a model. In all other subtest areas, Katarina scored exactly the same on the pre-test and post-test. Test item analysis revealed there were a few problems that showed some change in her understanding from pre-test to posttest. These test items are shown in Appendix D. Although these few test items show some progress in her understanding of multiplication, her performance on other similar test items was inconsistent. Some of these test items are also shown in Appendix D for comparison. In other words, while Katarina was able to successfully solve a few problems on the post-test, she did not show the same understanding on other similar test items. Thus, the results of Katarina's multiplication post-test indicate no substantial change in her understanding of multiplication.

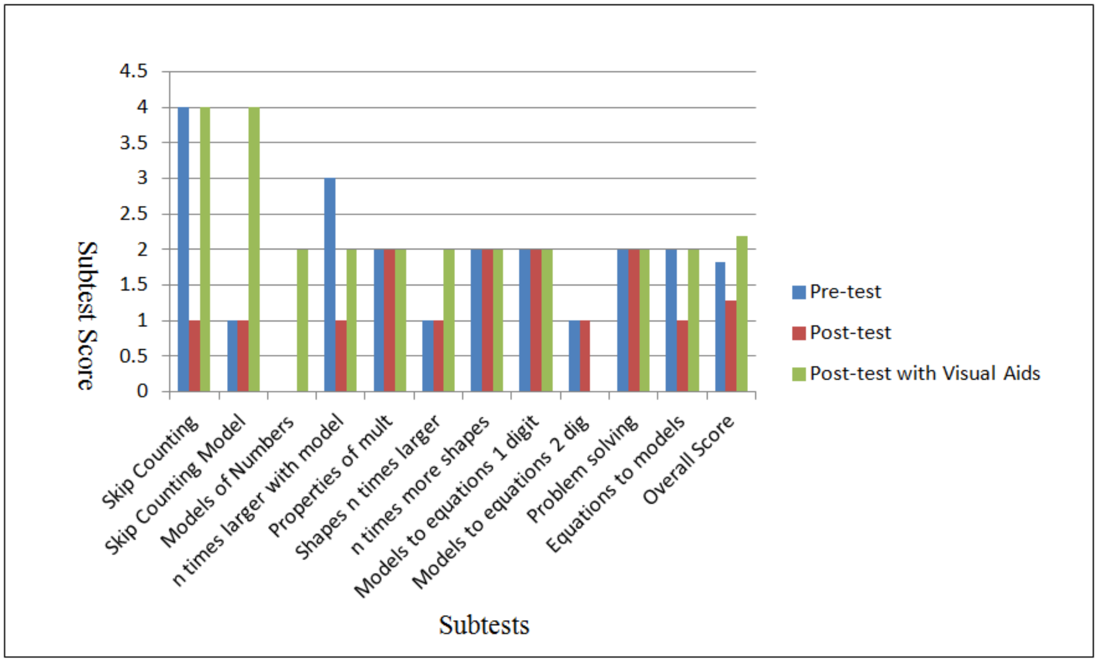


Figure 25. Katarina Multiplication Subtest Scores

The researcher gave Katarina the multiplication post-test a second time and utilized accommodations. Katarina was able to use a color coded 100's board and a flip book of multiplication models. The researcher also read the test to her and clarified vocabulary in the directions and problems. With these accommodations, Katarina was able to score a four on all of the skip counting questions. She was also able to make models of numbers under 100. She was able to correctly answer three of six multiplication contextual problems using her visual aids but did not correctly show models of the problems. She did show some attempts to make models of these problems, which are shown in Appendix D. Even with accommodations, Katarina only scored proficient on two skill areas on the multiplication post-test.

On the division unit tests (Figure 26), Katarina showed very little change in her scores from pre-test to post-test. Item analysis revealed she used the same incorrect procedures to solve problems on the post-test as she did on the pre-test (see Appendix D). She regressed in her ability to divide shapes into equal portions and to interpret statements such as "what number is 3 times smaller than 6?"

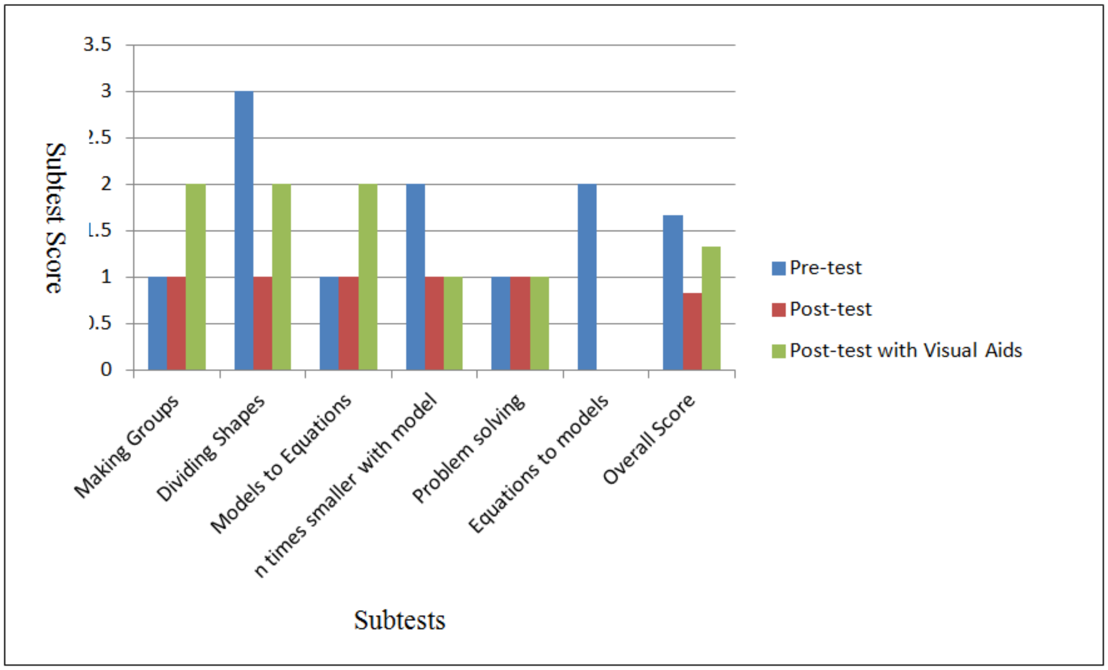


Figure 26. Katarina Division Subtest Scores

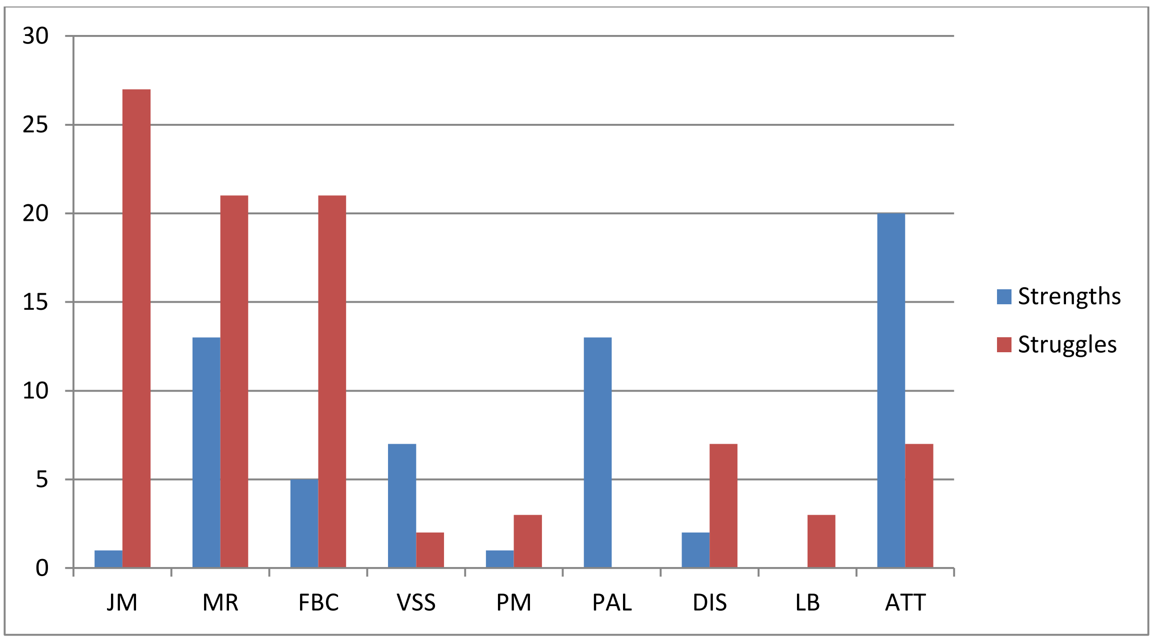
Katarina did not attempt to solve division equations and show the problem with a model on the post-test. On the pre-test, she did have some successful attempts to solve these types of problems. Work samples from the pre-test on this skill are shown in Appendix D to provide an example of the regression in her abilities. When given the post-test a second time with the same accommodations provided for the multiplication test, Katarina improved her score in making groups, dividing shapes, and converting models to division equations. On the other three subtests, she scored exactly the same and solved the problems in the same way as she did without the accommodations. Katarina's scores on the division pre-test and post-test show little change in her understanding of division. In some areas, she regressed in her demonstration of understanding from pre-test to post-test.

The qualitative data on Katarina revealed consistent patterns of deficits related to MLD. The data summarized in Table 15 and Figure 27 is the result of 126 coded observations and work samples. Katarina consistently struggled with tasks involving judgment of magnitude, using multiple representations, and fluency with basic calculation. She also had poor attendance during the tutoring period. Seven observations recorded a negative disposition. Katarina showed strengths at times in using multiple representations and had several occasions when she responded positively to peer-assisted learning and student discourse. She demonstrated good visual spatial skills. A few instances where a language barrier was obvious were also noted.

Table 15. Katarina: Coded Observations and Work Samples

Code	Strength	Struggle
JM	1	27
MR	13	21
FBC	5	21
VSS	7	2
PM	1	3
PAL	13	0

Code	Strength	Struggle
DIS	2	7
LB	0	3
ATT	20	7



Note: Based on 126 coded observations and work samples and attendance.

Figure 27. Katarina Strengths and Struggles

Because Katarina's strengths and struggles also revealed deficits related to MLD, the observations coded as judgment of magnitude, multiple representations, and fluency with basic calculation were further analyzed using the same procedure and codes used to analyze the data on Happy Gilmore.

The results of this additional task analysis for observations related to judgment of magnitude are summarized in Figure 28. Twenty-eight observations were coded as involving judgment of magnitude. Katarina only had one recorded incident when she was successful with this type of task. This task did not involve the use of a visual aid. Of the 27 observations coded as a struggle with judgment of magnitude, 16 of the tasks had a visual aid provided and 11 did not. These results show even when provided a visual aid, Katarina struggled with judgment of magnitude. This error pattern is consistent with the characteristics of MLD.

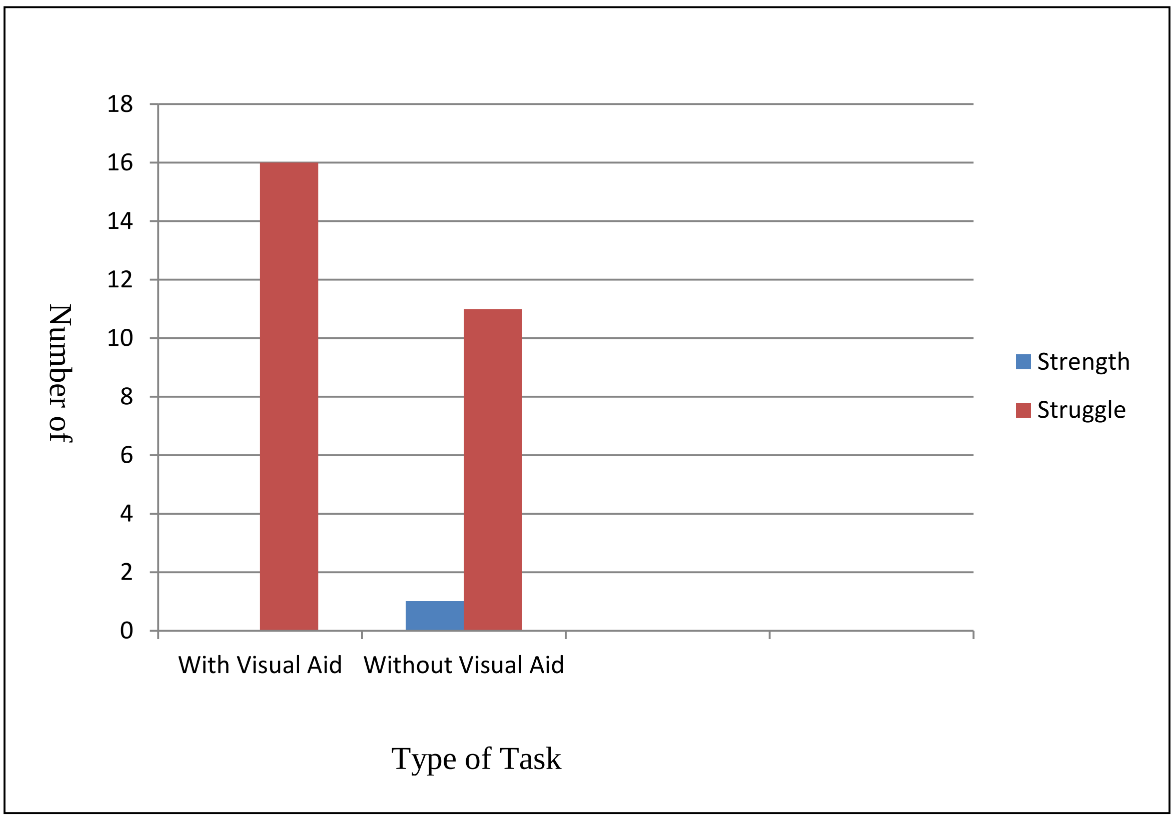


Figure 28. Katarina Task Analysis for Judgment of Magnitude

Item analysis of the tasks that involved the use of multiple representations (Figure 29) revealed Katarina struggled with all types of tasks that involved converting from symbols to language to models. She was successful with some tasks involving converting from symbols to models and language to models. Her struggles on all of these types of tasks are also consistent with the characteristics of MLD.

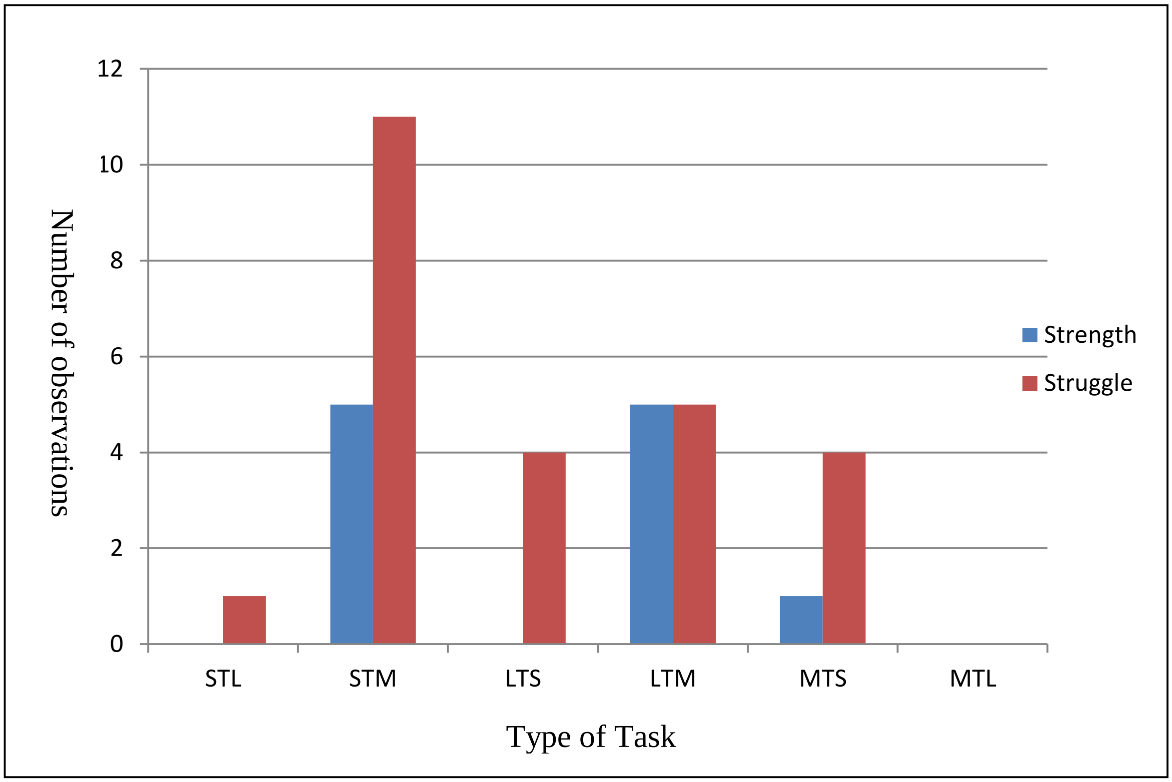


Figure 29. Katarina Task Analysis for Multiple Representations

On tasks that involved fluency with basic calculations (Figure 30), Katarina struggled with basic fact calculation even when provided visual aids. This error pattern is consistent with a deficit in fluency with basic calculation related to MLD. Katarina's lack of progress on the two minute multiplication fact assessments provide additional evidence supporting the claim that she has a consistent pattern of deficit in the indicator area of basic fact calculation.

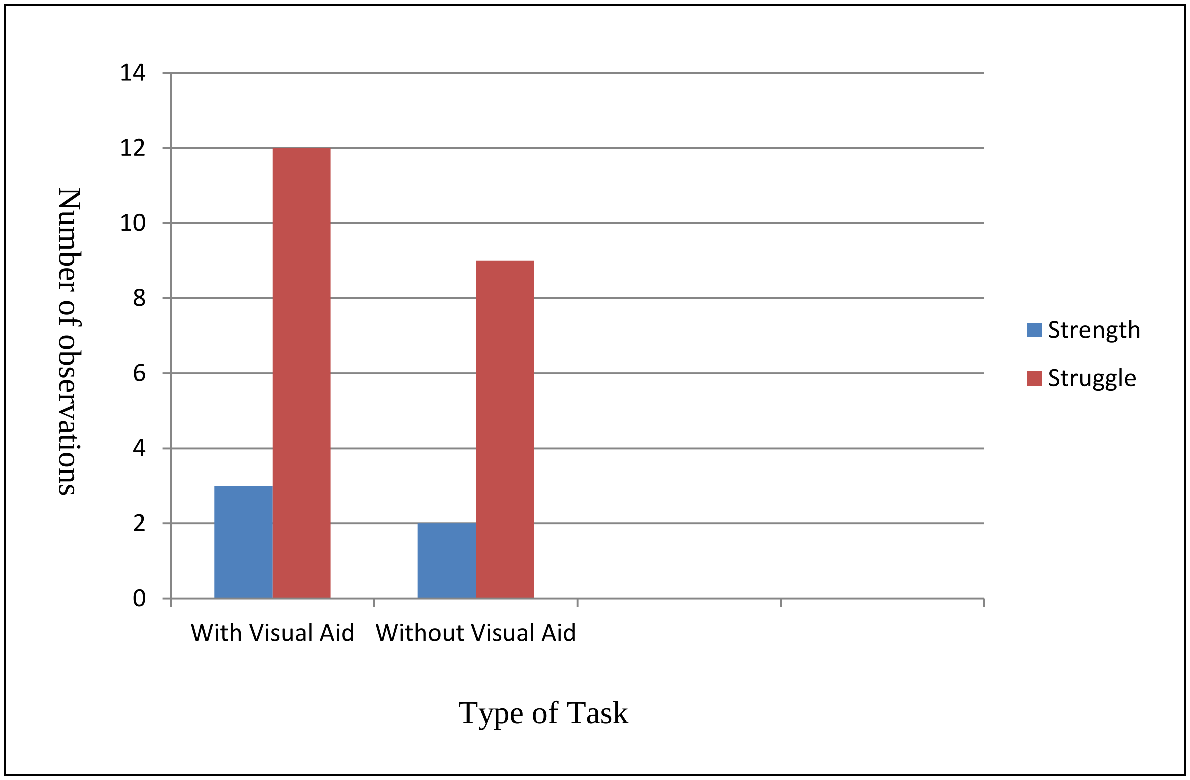


Figure 30. Katarina Task Analysis for Fluency with Basic Calculation

Both the quantitative and qualitative data on Katarina show that even after participating in tutoring, she continues to struggle in her understanding of both multiplication and division.

She showed very little improvement in her understanding from pre-test to post-test and in the qualitative observations. Katarina has patterns of deficits related to MLD.

Summary

This presentation of results has provided a summary of what was learned from both quantitative and qualitative data about each of the four study participants. Two research questions were addressed with these results. The first question was does low achieving fourth graders' participation in tutoring based on five key instructional practices for students with mathematics disabilities increase their understanding of multiplication and division? The results indicate that Destiny and Big Jay made substantial improvements in their understanding of multiplication and division from participating in the tutoring. Happy Gilmore made some improvement, and Katarina showed very few improvements in her understanding of these concepts. The second research question addressed with these data was what are the patterns of deficits low achieving fourth graders have in relation to the six indicators of MLD? As a result of this data analysis, the six indicators were reduced to five and the determination of unreasonable results was combined with judgment of magnitude. Katarina and Happy Gilmore showed patterns of deficits consistent with several of the indicators of an MLD. Destiny and Big Jay did not. The next chapter will discuss further what can be learned from the stories of each of these students and the implications this study could have on instructional practice for students who struggle to learn mathematics.

CHAPTER 6: STUDENT STORIES AND INDICATORS OF MATHEMATICAL LEARNING DISABILITY

Student Stories

Teaching and learning are interrelated. While learners learn from their teachers, teachers also learn from their students. Cochran-Smith and Lytle (2001) discuss the learning that occurs when teachers teach. This type of knowledge is generated when teachers make “their classrooms and schools sites for inquiry, connecting their work in schools to larger issues, and taking a critical perspective on the theory and research of others” (p. 49). In this view of knowledge of practice, the knowledge needed to teach well is generated when teachers investigate, interrogate, and interpret the knowledge and theory produced by others in their teaching. This research study involved this type of inquiry. The researcher interpreted knowledge and theory from many different genres of research related to MLD and created an instructional framework to guide Tier 2 mathematics intervention. This study investigated the effectiveness of this framework on four fourth grade students who were struggling in mathematics in their general education classroom. The knowledge gained from the experiences of the four students in this study can be connected to larger issues related to helping students who struggle to learn mathematics.

This chapter discusses what can be learned from this study that could guide instructional practice and professional development, as well as future research. Each case study will be reviewed and the systemic issues illustrated by the story of each student will be identified. Next, what was learned about the instructional framework and MLD will be examined. Lastly, implications for practice and suggestions for future research will be presented.

Collectively, the case studies of the students in this study illustrate the complexity of determining the cause of a student's failure in mathematics in the general education classroom setting. It is difficult to determine whether a child is struggling with the difficult content of mathematics or if they may have a cognitive deficit prohibiting their ability to understand the subject (Geary, 2005, p. 306). Individually, each of these students' cases highlight systemic issues in public education related to the implementation of RTI and effective mathematics intervention.

<u>Big Jay</u>

Big Jay is an example of a student with a domain general disability (Chiappe, 2005; Geary, 1993; Von Aster & Shalev, 2007), which may be impacting his performance in mathematics in the general education setting. When Big Jay was originally selected for the tutoring group, his teacher recommended him because of all the students in her class, she felt that he was the one she most suspected had an MLD. She knew that Big Jay was on an IEP, but she understood that it was for speech only, meaning articulation. His teacher reported that in class Big Jay often asked unusual questions during mathematics instruction and his performance was inconsistent. His inconsistent performance in the subject was apparent in the pre-testing for the study with his high basic fact score and his low unit test scores in comparison to his classmates.

A file review of Big Jay's school cumulative record and his special education paperwork revealed some important findings related to how Big Jay processes information. Big Jay's teacher was correct that he was only on an IEP for speech articulation. His report cards since kindergarten showed that he struggled with articulation from an early age. He has shown good progress in this area since he began speech therapy in kindergarten. Because Big Jay was making good progress and showing improvement in his speech articulation, no other cognitive assessment has been done by the school psychologist. All of Big Jay's testing for special education was conducted by the speech and language pathologist, which is appropriate for an articulation focus. However, there is an important note in Big Jay's special education eligibility report that could be a key factor in his performance in mathematics and other academic subjects. In his language testing, Big Jay showed above average abilities in his receptive language. He had a scaled score of 117. However, his articulation abilities scored in the 4^{th} percentile. This student has a huge gap between his receptive language abilities and his expressive abilities. It is possible that Big Jay has a more serious language impairment than just articulation that could be affecting his academic performance in mathematics as well as other subjects in school.

Big Jay's performance during the tutoring was consistent with this hypothesis. He did not show a pattern of deficit consistent with the indicators of a domain-specific MLD. At the beginning of the tutoring, Big Jay struggled to explain his thinking and use mathematical language, which is typical of a student with a language impairment. He was initially very resistant to drawing models. After about 12 tutoring session, he made a major shift in his attitude and went what he called "model crazy." He liked to see if he could show a problem with all of the different models he knew. Teaching Big Jay to draw models gave him a way to explain his mathematical thinking, first with the visual model, and then with language. The visual model provided scaffolding for his weakness in expressive language. He excelled in mathematics when he was able to express what he was thinking with a visual representation or symbols. He enjoyed the problem solving from real-world scenarios and was eager to orally communicate his thinking with the other students in the tutoring group. He easily understood the mathematics in contexts related to his background knowledge, such as buying cases of bottled water at Walmart.

Thirteen coded observations noted a strength in Big Jay’s judgment of magnitude. He seemed to have a very well developed internal number line that made real-world problem solving and mental arithmetic easy for him. He was very fluent with mental calculation, including two digit numbers. Often in the tutoring, Big Jay was given a harder problem than the other students to challenge him. When he was asked how he solved mental math problems such as 27 + 35, he said, "I just saw it." He seemed to think in pictures but couldn't always describe what he was visualizing with language. Two examples of his ability to visualize magnitude were his success with making a maze five times larger and solving mental math addition and subtraction on a hundreds chart. Big Jay’s success with these non- verbal tasks suggests he is very strong in his visual spatial abilities and with non-verbal tasks. As his language scores indicate, he struggles when he needs to use language to communicate what he knows.

Both in the tutoring and in his classroom, Big Jay's work often had small procedural errors. In the tutoring, he frequently made small counting errors such as drawing seven groups of four when the problem asked for eight groups of four. When his error was pointed out to him, he easily understood what he did wrong and fixed it. He often did not pay attention to details in his work, and his work was very sloppy. He does not seem to be the type of learner who will be successful in a mathematics classroom that focuses on step-by-step arithmetic procedures. During this tutoring, providing Big Jay visual tools by teaching him mathematical models helped him communicate about what he immediately sees non-verbally. Big Jay’s non-verbal strengths and visual spatial skills are cognitive abilities that could help him be very successful in higher level mathematics classes and career pathways involving math, science, and engineering. He seems to be an “out of the box” thinker evidenced by his creative approach to problem solving. He seemed to quickly interpret and visualize information related to numbers. This observation is supported by his above average receptive language score.

There were times during the tutoring when Big Jay would get frustrated when something was difficult and he would shut down. This occurred when he didn’t understand something. He wanted mathematics to make sense. If it didn't, he would give up and avoid a task. He showed this behavior on his division post-test. Big Jay knew that he didn’t know how to make models from division equations and did not attempt to answer any of the problems on that subskill. Throughout the tutoring, he was very cognizant of what he knew and didn't know. For example, one day he wrote a 2 x 2 digit multiplication problem on the board when he first came into the tutoring class. He said, "I need to learn how to do these better." He was trying to remember the steps to the traditional algorithm and could not remember them. This is an example of a concept that was not making sense to Big Jay and he was frustrated by it.

In order to be successful in mathematics classes in his future, Big Jay may continue to need help communicating what he knows with language. He may continue to need support connecting visual models, symbolic notation, and language. A concern about Big Jay as he heads toward middle school is what will happen if he does not get into a classroom that teaches mathematics in the visual way that he learns. It is possible that he will fail in a traditional, computation-based approach to mathematics.

Big Jay's learning needs could likely be met in the general education classroom if he has a general education mathematics teacher who understands his unique cognitive strengths and weaknesses. A general education teacher may need to spend a little extra time making sure Big Jay understands the math concepts being taught. He may need encouragement to use mathematical academic vocabulary. He excels when he is allowed to be creative in his approach to solving problems, but still will need to learn conventional mathematical notation. Big Jay will likely thrive in a general education classroom that teaches mathematics through the use of visual models and real-world contexts.

In order for general education teachers to fully understand Big Jay's learning needs in mathematics, it is probable they will need to be educated about his disability and provided with strategies for helping him be successful in a general education mathematics classroom. For example, a strategy that can easily be taught to a general education teacher is using visual models. Big Jay can be asked to draw a visual model of a problem and then have him explain his model with words to a peer or the teacher. This strategy will facilitate his use of academic vocabulary and develop his oral language skills as well as conceptual development. Another strategy is having him keep a personal notebook of academic vocabulary words where he can draw a picture or model to represent the definition of the word. Big Jay may also need someone to monitor his progress in mathematics and prevent failure. The support Big Jay may need to succeed in general education mathematics may require having a special education case manager. For this reason, it may be appropriate to do further cognitive testing on Big Jay before he goes to middle school and consider an eligibility category of language impairment rather than articulation only. This will allow Big Jay to continue to receive support from special education for all academic subjects through middle school and high school.

Several systemic issues are illustrated in Big Jay’s story. One is how data from a special education eligibility report can often be overlooked in the general education setting. The evaluation information in his file on the difference between Big Jay’s receptive and expressive language abilities is important knowledge that can help general education teachers understand Big Jay's strengths and weaknesses. The large discrepancy in his receptive and expressive language abilities could impact his success in all academic subjects. General education teachers may not understand the significance of those scores due to a lack of professional development on learning disabilities. This case study illustrates how a domain-general disability can impact performance in mathematics. To the general education teacher, this may look like an MLD.

Another key issue illustrated by Big Jay's case is the importance of good communication and collaboration between general education and special education. During the tutoring, Big Jay demonstrated strong skills in mental calculation, judgment of magnitude, using multiple representations, and visual spatial skills in numerous coded observations. Through collaboration and support from a special education teacher, Big Jay may be able to be very successful in a general education mathematics classroom. A special education case manager or IEP team would need to equip his teacher with knowledge about his disability and specific strategies for helping him. This raises the questions whether special education teachers have the knowledge they need to support general education mathematics instruction. Thus, the importance of high quality professional development on how to help both general education and special education professionals teach mathematics to SWD is emphasized by Big Jay's story.

Destiny

Destiny's case represents the complexity of diagnosing a learning disability for students who are LEP. When Destiny was selected for the tutoring group, her teacher commented that she was on an IEP but it was for speech only, meaning articulation. A file review revealed that this was not the case. She was eligible for special education under the category of language impairment. Her special education goals were more than articulation. They also included using multi-syllabic words, coming up with an antonym when provided a synonym, using conjunction words, and producing grammatically correct 5 to 7 word sentences. Her eligibility report stated that she had speech articulation problems not attributed to Spanish influence and she often had difficulty finding the right word, saying longer words, and finishing sentences even in her first language. Destiny tested at the Advanced Beginner level on the Idaho English Language Assessment (IELA) in third grade, also called Level 2. Compared to other LEP students with the same proficiency level, the IEP team determined that she had sound production differences even from other LEP students and was in need of speech services. Thus, she went to speech therapy through special education. Destiny was also on an English Language Proficiency (ELP) plan. Her English language goals were addressed through the school's daily reading intervention time. In fourth grade, Destiny was struggling in both reading and mathematics in her general education classroom.

In the tutoring, Destiny did not show any patterns of deficits related to an MLD. Observational data revealed strengths in judgment of magnitude, using multiple representations, visual spatial skills, and fluency with basic calculation. She quickly learned to draw models and used them flexibly from one concept to another, indicated by how she generalized the use of the models from multiplication to division on the unit post-tests. Several times, Destiny was able to easily represent a problem with more than one model with very little guidance. She also responded to strategy instruction for basic fact calculation and again demonstrated the ability to generalize. For example, she learned the doubling strategy with multiplication facts including twos and fours. We discussed in the group that if you know a number times two, if you just double the answer, you will have the four. A few days later, Destiny realized what she called "the trick" worked for threes and sixes as well. Discussing these types of strategies helped Destiny improve her fluency with basic calculation during the tutoring period.

Destiny also benefitted from explicit vocabulary instruction, connecting the content academic words being used in the mathematics lesson to her first language. The observational data recorded eleven instances when Destiny demonstrated a language barrier translating an English word. When explaining her models to other students, sometimes the researcher had her first explain it in Spanish to her classmates who understood Spanish. She then seemed more confident and articulate explaining it in English to the researcher and the other student who did not speak Spanish. This was a strategy the researcher used to promote student discourse in both the student's first and second language. During their explanations, when the students struggled with the use of an English word like "circle" or "opposite," we discussed the Spanish word that meant the same thing, then discussed the definition in English. These conversations seemed to greatly help her. A limitation with this strategy was when the students didn't know the Spanish word for a concept like "multiple." This demonstrates why students who have greater proficiency and academic vocabulary in their first language learn academic vocabulary more easily in their second language because they can connect new English vocabulary words to existing schema in their first language. For students like Destiny, she is not only learning a second language, she is also learning concepts and building schema on a concept like multiples at the same time. This is challenging. For this reason, explicit vocabulary instruction within mathematics lessons will greatly help students like her. Based on the noted observations during the tutoring, her difficulties in mathematics seem to be much more language based than due to an MLD. Whether her difficulty was related to second language acquisition or a language impairment is difficult to distinguish. Her academic records indicate she has a history of challenges with both issues.

Destiny's case again emphasizes the importance of good communication between special education and general education. During group discussions about problem solving, Destiny showed many instances of mastering her IEP goals. Many of her goals could have been practiced during the tutoring and progress documented if the researcher had been informed of her language goals from her IEP. The need for professional development that provides general education teachers with strategies to meet the needs of diverse learners is also highlighted in Destiny's case. For Destiny to be successful in general education mathematics, her teachers may need to use a combination of teaching strategies that meet the needs of LEP students as well as language impaired students. Fortunately, many of these strategies are the same. A general education teacher could benefit from professional development about how strategies such as using visual models and explicit vocabulary instruction meet the needs of both LEP and language impaired students. Core mathematics instruction that emphasizes visual models, student discourse, explicit vocabulary instruction, and strategy instruction greatly helps students like Destiny be successful. If these teaching techniques were used consistently in the general education setting, students like Destiny may not need supplemental instruction in mathematics.

Happy Gilmore

Happy Gilmore's case also represents the complexity of diagnosing a learning disability for a student who is also LEP. Unlike Destiny, Happy Gilmore had never been qualified as eligible for special education. His file revealed he has been referred for a special education evaluation several times throughout his school career and at more than one school. Through this process, he has received a variety of intervention services in both reading and math. He has participated in summer school programs and after school tutoring from private service providers. Progress reports from tutoring and meeting notes indicated Happy Gilmore made progress in response to tutoring, which shows his ability to learn. However, in spite of all of the intervention he has received, Happy Gilmore is still performing far below his grade level peers in both reading and mathematics.

Additional data to take note of are Happy Gilmore's scores on the IELA. In first grade, he scored as an Advanced Beginner, or a Level 2. In second grade, he scored a level 4, which is classified as Early Fluent. In third grade, he scored Intermediate, which is a level 3. These scores indicate he has made good progress in the development of his language skills in English in reading, writing, listening, and speaking. If a language barrier was the only factor affecting Happy Gilmore's performance in academics, he should be closing the achievement gap between him and his classmates. This is not happening. Another fact to take note of is Destiny had a lower score on the IELA in both second and third grade. If English proficiency was the only factor impacting Happy Gilmore's performance in mathematics, it would be reasonable that he would have made progress from the tutoring comparable to Destiny. Although Happy Gilmore showed some positive changes in his understanding of multiplication and division, his progress was not nearly as remarkable as Destiny's.

There is a note in Happy Gilmore's file from second grade commenting that phonics was a weakness for him. His first grade teacher noted struggles with numbers and number sense. His grades show him as improving or needs improvement in all academic subjects since first grade. He has very few subjects marked as satisfactory. In addition, there is a note from a vision test done in third grade that said that the school nurse was referring him to a physician for further evaluation. No evidence of follow up on this referral exists in his file. Another relevant report was from a private tutoring company who worked with Happy Gilmore in mathematics for ten tutoring sessions in third grade. These notes say he struggled with writing number words in correct form, rounding numbers in 10's place and hundreds place, telling time, counting money, and multiplication facts. During the tutoring, he made progress in place value, rounding, telling time, and adding money. There is no mention of progress in multiplication facts or converting numbers from symbolic to written form. The report also noted that Happy Gilmore struggled with the meaning of English words in mathematics and speaks both English and Spanish. The observations made in the tutoring in third grade were very consistent with the observational data on Happy Gilmore from this research study.

Happy Gilmore's overall academic records as well as the notes from the intervention he has received in both reading and mathematics indicate long term, pervasive difficulties with both reading and mathematics. The results of this study showed a consistent struggle with the use of symbols in mathematics. It is very probable that Happy Gilmore has a learning disability that may be impacting his academic achievement in both reading and mathematics. Happy Gilmore fits Feifer and De Fina's (2005, p. 39) description of students with semantic subtype of dyscalculia who may also struggle with sound-to-symbol pairing in reading. Students with this subtype of MLD frequently also have difficulty with language arts skills. Happy Gilmore's immature counting strategies and his inability to learn basic facts are also consistent with this subtype of MLD. The description of this subtype of MLD helps explain why Happy Gilmore was successful with some tasks involving multiple representations and judgment of magnitude, and he sometimes could fluently compute. A trend in the observational data on Happy Gilmore shows that he was successful with judgment of magnitude and converting between models, contexts, and symbols when the problem used smaller numbers. He was successful with basic calculation tasks when he could use his immature and often inefficient counting strategies, such as counting by 2's, 5's,10's, or 1's. Happy Gilmore consistently struggled when the numbers got larger and more abstract or he had to count by larger and more uncommon numbers like sixes, sevens, and eights. He understood conceptually the idea of skip counting but relied on his hundreds board to help him visualize numbers over twenty or count by anything other than 2's, 5's, 10's, and 1's. Happy Gilmore was confident, articulate, and remembered the concepts he understood, such as adding the zero when multiplying a number by 10. He struggled when procedures were more complex and abstract.

Happy Gilmore's success with smaller numbers suggests how he can be successful in mathematics. His general education teacher could be taught to adapt his mathematics assignments to involve smaller numbers that he can visualize and accurately make models to represent. For example, if the class is practicing division of two digit by two digit numbers with remainders, Happy Gilmore's assignment can be adapted so he is practicing problems that have a one digit divisor. He can have a shortened assignment, do fewer problems, and make a model to represent the problems. He can use a hundreds board as a visual aid to recall basic facts. He can utilize his strengths in language to write a story problem to match a practice problem. Based on his success with solving real-world problems, making models, using visual aids, and hands-on activities, Happy Gilmore seems capable of learning grade level concepts if the material is presented with smaller numbers and real-world contexts. His multiplication and division post-test demonstrated Happy Gilmore's ability to use models to solve problems and divide shapes correctly (see Appendix D). These work samples and his success with activities using manipulatives indicate Happy Gilmore can be successful with grade level measurement, data analysis, and geometry activities. He can also use visual models to understand basic fraction concepts.

There are many grade level skills and content Happy Gilmore can learn using accommodations and adaptations to curriculum. In the general education setting, these accommodations and adaptions would be appropriate for a student on an IEP. Happy Gilmore will most likely need support to learn mathematics throughout his school career. In order for this to occur, he will need to qualify for special education services. In addition, any classroom teacher working with Happy Gilmore will need to be educated by a special education case manager on how to provide accommodations and adaptions in mathematics that are helpful for Happy Gilmore. This is assuming that the special education teacher is well versed in strategies for accommodations and adaptations in mathematics for SWD. This assumption is related to the special education teacher's preservice education and professional development experiences.

In addition to adapted grade level instruction, Happy Gilmore may also need targeted, specially designed instruction to help him develop better number sense and judgment of magnitude. He may benefit from relearning addition and subtraction concepts and models now that he is more proficient in English. He may have missed those concepts in first and second grade due to his limited English abilities at that age and changing schools a lot. Special education could contribute greatly to Happy Gilmore's success throughout his schooling in mathematics.

Happy Gilmore's story illustrates some shortfalls with the RTI process. How did a child who is so academically needy make it to fourth grade without being evaluated for a learning disability? Two factors may have contributed to this scenario. One, the fact that Happy Gilmore is LEP and was making some progress justified waiting to evaluate him for special education services. He did respond to intervention and made some progress in all of the tutoring he received. The question that perhaps should have been asked was whether the progress was enough to close the achievement gap between him and his peers. It is possible that the gap continued to grow as the material to be learned became more complex in third and fourth grade. Another factor that may have contributed to Happy Gilmore being overlooked for a special education evaluation when he was younger was his mobility and home situation. Happy Gilmore had changed schools several times. He was living with his aunt for this school year and his mother was in prison. He did not have a strong adult advocate. His aunt did not speak English, and the classroom teacher had never met her. Happy Gilmore knew he would be moving to a different school next school year when he went to live with his mom again. His file review showed that whenever the RTI process was started on him, he would change schools and the whole process would start over at his new school. An example of this is the referral from the school nurse for a more extensive vision evaluation by a physician. This was never followed up on by a problem solving team. Happy Gilmore's family will probably need help from social services in order to set up that evaluation. The school can assist with this. In order for Happy Gilmore to receive a special education evaluation, an RTI team will need to utilize the data from all of the schools he has attended and the progress reports in his file. It is reasonable that any problem solving team would have determined that limited English abilities, family upheaval, and mobility may have contributed to Happy Gilmore's low academic achievement. It is hoped that providing the school data from the results of this study will complement file data and classroom data to support a full special education evaluation for Happy Gilmore.

Happy Gilmore's case illustrates that sometimes the RTI process can miss students who perhaps should have been receiving special education services much earlier in their schooling. It also demonstrates that for some students, several interrelated factors can contribute to low achievement, which may mask a learning disability. It emphasizes the importance of submitting documentation of Tier 2 intervention to the cumulative record. A report summarizing the student's progress, the content of the intervention, and recommendations for meeting a child's future needs should be prepared at the end of the Tier 2 intervention. This documentation should be placed in the student's cumulative file so that any educators concerned about the child in the future can utilize the data. This documentation is critical for students who move a lot.

Katarina

Katarina is a student who has been a victim of high mobility. She attended six different schools in her five years of schooling. Several times she was at a school for only a few months before moving again. Throughout her file are comments about poor attendance. There are also gaps in her enrollment when she moved from one school to another. It is obvious from reviewing her file that Katarina has missed a great deal of instruction in grades K-4. For this reason, an RTI problem solving team could not confidently determine that the achievement gap between her and her grade level peers was not due to lack of instruction. This would make her ineligible to qualify for special education. In addition, Katarina is also LEP. That is a second factor that would make an RTI problem solving team hesitant to consider a disability as a primary factor contributing to her low achievement. Katarina's school records do not show any intervention in reading or math had been provided to her. This is likely because she would move before teachers realized how low she was and could go through the appropriate paperwork process in order to get a child into Tier 2 intervention programs. The 2004 guidelines for qualifying a child for special education through an RTI process can take up to two school years before a team determines a child has a learning disability. For most students, this process insures that a learning disability is not being misdiagnosed. However, highly mobile students like Katarina may not stay in a school long enough for the process to be completed. According to her file, this school year was the first year Katarina was participating in intervention groups in reading and mathematics. Katarina's mobility has prevented her from fully participating in the RTI process of evaluating a student as learning disabled.

Even in this study, Katarina was absent frequently. However, some data were collected to provide evidence to support the hypothesis that she may have a learning disability. Her lack of progress on multiplication tasks even after targeted instruction on the subject indicate serious learning difficulties. The observational data had a clear pattern of deficit in her ability to judge magnitude, show a mathematical context with both symbols and models and fluency with basic calculation. She struggled with converting symbols to meaning. This could also be a factor in her struggles with reading. One incident clearly illustrates Katarina's struggles with interpreting numbers. She was working one-on-one with the researcher to make up an assessment she had missed from an absence called the number trail test (see Appendix B) from Feifer and De Fina (2005). This assessment is essentially a connect-the-dot activity where she had to draw a line connecting the numbers scattered on the page in order of magnitude. The numbers were both two and three digit numbers. On the first try, she seemed to guess and completed the trail in 24 seconds, but it was all incorrect. The researcher clarified the directions and asked her to do it again. The second try she took 43.9 seconds but was still incorrect in ordering the numbers. The third time the researcher asked her to say the numbers out loud as she went. When she did this, she took 1 minute 19.6 seconds to complete this task but she got the numbers in order correctly. In comparison, Destiny and Big Jay completed the same task correctly with one attempt in about 30 seconds whereas Happy Gilmore completed it in 1 minute 3 seconds but he was correct the first time he completed the task. When given cards with two digit numbers on them to put in order of magnitude, Katarina again was much more successful when she said the numbers out loud than when she tried to arrange them in her head mentally. This brings to mind Von Aster and Shalev's (2007) description of some children who do not develop an "internal number line." In another observation, Katarina was making a number line on the board and I asked her what zero plus six was orally. She could not come up with an answer or draw that problem. These are three examples that demonstrate Katarina's serious deficits in her understanding of numbers. She uses immature counting strategies and does not seem to understand basic addition and subtraction or place value. She did not know what a rectangle was. She seems to survive school math by copying and sometimes making up procedures to get an answer that looks like she knows what she is doing. She did not like to ask for help or admit when she did not know something. Several times during the tutoring she displayed a negative attitude and shut down. Her file has comments from teachers on her poor attitude and behavior at times.

Although this tutoring did not seem to make a difference for Katarina in terms of her academic achievement, it did provide documented evidence of her struggles in mathematics. In an RTI process of qualifying a student as learning disabled, documenting lack of progress is just as important as showing progress in response to intervention. In this case, the report provided to the school on her struggles could help a problem solving team justify a special education evaluation. When factoring in her LEP status, she can be compared to both Destiny and Happy Gilmore. According to the IELA, she scored as Advanced Beginner in first grade, Level 2. In second grade, she scored as Intermediate, Level 3, and in third grade, she also scored an Intermediate, Level 3. According to this assessment, her English language proficiency is higher than Destiny and comparable to Happy Gilmore. If lack of English proficiency were the only factor impacting Katarina's performance in mathematics, it would be reasonable to expect she would have made improvement in her understanding comparable to Happy Gilmore and Destiny. This was not the case.

Katarina needs a full cognitive assessment to determine whether she may have a processing deficit interfering with her ability to make progress academically. Her behavior issues would also warrant a social emotional evaluation. She seems to be a student who is lost in the world of school, and therefore may be avoiding it whenever she can with poor attendance. During the last week of tutoring, she said she was moving in the summer again. It is hoped that her next school can use the data collected and reported from this study to get her the help she needs. The special education eligibility process works best when students stay in a school with a consistent problem solving team who can complete the process according to the law. Based on her pattern of mobility, this may not happen for Katarina. Tragically, she may be a student who goes through her schooling with an undiagnosed learning disability.

Katarina exemplifies the type of student who many educators describe as one who "falls through the cracks." This implies the cracks in the system of public education. If she does have a learning disability, it is masked by so many other factors that could also cause her to be performing academically below her peers. She does not stay in a school long enough for even the most efficient school problem solving team to follow a special education eligibility process. She has mastered the ability to cover up what she does not know by looking like she is working and not bringing attention to herself. The future prospects for this student are heartbreaking, but she represents many other students just like her.

What can be learned from Katarina for schools is to again reinforce the practice of documenting all progress from all Tier 2 interventions and placing records in the student's cumulative records. Another thing the present school can do is to pay attention to when her records are requested from the new school. A teacher, principal, or special educator could call the new school and inform them of their concerns about a possible learning disability and encourage the new school to accelerate an eligibility process. This type of communication between schools is very helpful, but rarely occurs as students move from one school to the next. Most students with special needs have parents or other adults who advocate for them. Sadly, this student does not have that adult support.

Indicators of Mathematics Learning Disability

The stories of these students bring to light many issues related to providing Tier 2 intervention in mathematics in an RTI model of systemic support. Looking at these topics through the lenses of student stories makes the issues real for educators. We can then analyze the changes we have made to the system and come to a greater understanding of how the changes impact children, both positively and negatively. While there are many topics that could be discussed in more detail in relation to the stories of these four students, the next section will focus on the purpose of this study, which was to contribute to the pedagogical content knowledge needed to teach students who struggle in mathematics (Hill & Ball, 2009; Danielson, 2007). Specifically, what was learned about each of the identified indicators of MLD from the researcher's experiences tutoring these four students will be presented. Dialogue about the key instructional practices will be integrated with the discussion about the indicators of MLD.

Fluency with Basic Calculation

One of the purposes of this study was to provide an alternative to common practice in special education mathematics instruction that have traditionally focused intervention classes heavily on drill and practice of basic arithmetic facts (Maccini & Hughes, 1997; Woodward & Montague, 2002; Woodward, 2006). However, because fluency with basic calculation is a significant indicator of an MLD (Geary, 2005; Gersten et al., 2005), it was important that both instruction and practice of basic facts was a part of the intervention curriculum.

Fluency with basic calculation was addressed in many ways during the tutoring lessons. Peer-assisted learning was used when the students played a variety of games and shared strategies for solving facts. A CRA sequence was used by practicing basic multiplication, addition, and subtraction facts using manipulatives and coloring number patterns on 100's charts and multiplication charts. The students also created visual models of arithmetic problems and wrote equations to match models. Literature and contextual problems engaged students in solving real-world problems related to multiplication facts.

Problem strings were used to lead to strategy instruction and explicit vocabulary instruction. Counting, number patterns, and reasoning strategies were discussed, taught, and encouraged. For example, strategically giving the students a problem string of 2 facts and 4 facts (See Appendix B, day 16) led to an opportunity to explicitly teach, model, and practice a mental strategy called doubling. If you can't remember a fact such as 4 times 8, you can cut the four in half, compute 2 times 8 and then double the product. Discussions about problem strings naturally led to instruction on mathematical vocabulary words such as factor, product, multiple, greater than, less than, even, odd, and opposite. As students gained confidence with basic multiplication facts, they practiced taking a timed basic fact test, which led to progress monitoring and student involvement in tracking their progress in a two minute time period. We analyzed their timed tests and openly discussed as a group strategies for figuring out each student's "tricky facts" - the facts they personally struggled to recall identified by their assessments.

Although all of the students participated in the wide variety of engaging activities designed to develop fluency with basic calculation, telling differences emerged between the students. Destiny and Big Jay consistently generalized the thinking strategies we discussed to new situations. Big Jay used the doubling strategy with 4 x 65 (See Appendix D) on his multiplication post-test, and Destiny was using it on a timed test one day for the "sixes." She realized if she knew her threes, she just had to double a three fact to her six fact. In contrast, Happy Gilmore and Katarina did not develop the same fluency and flexibility with basic facts that Destiny and Big Jay did in response to the same experiences. However, Happy Gilmore's willingness to use his 100's board and make models when he couldn't recall answers allowed him to participate fully in all basic fact activities. He was slower in his work when he had to use his visual aids compared to Destiny and Big Jay but he was able to do the same work. Katarina never developed the same confidence with the visual aids and willingness to participate as Happy Gilmore did although she may have if given more time. Her inconsistent attendance or emotional issues may have contributed to her lack of confidence and willingness to participate.

The approach to basic facts mastery used in this study is consistent with a cognitive approach to helping children develop rich conceptual schema and good number sense while they practice basic facts (Baroody et al., 2009; Baroody, 2006; Gersten & Chard, 1999). The variety of activities provided many opportunities to observe each student's proficiency with basic calculations in response to explicit instruction. The researcher was confident that Happy Gilmore and Katarina's pattern of struggle to recall basic multiplication facts was not related to lack of instruction. Several observations in the coded data suggested recalling basic facts from memory was a consistent struggle for both Happy Gilmore and Katarina.

An interesting reflection on the tutoring related to basic fact instruction is an observation regarding the interaction between the two students who had strengths in basic fact fluency and the two students who didn't. It may seem reasonable to create an intervention group based on several students' lack of fluency with basic facts. In this study, Big Jay showed a strength in this area on the multiplication fact pre-test. It was apparent very early in the tutoring that Destiny had good fluency and number sense but had not practiced her facts enough to have them memorized. When the group did basic fact activities such as a problem string or playing a game, Big Jay and Destiny played a valuable role in helping Happy Gilmore and Katarina learn how to think about problems. Because Destiny and Big Jay both had weaknesses in language, discourse that involved them explaining their thinking about how to solve a basic arithmetic problem helped them as much as it helped Katarina and Happy Gilmore. If Destiny and Big Jay had not been in the group, the researcher would not have been able to utilize peer assisted learning to provide capable peers who influenced Katarina and Happy Gilmore's mathematical thinking and vocabulary. Putting a group of students who all have poor number sense in an intervention group is comparable to putting a group of students who are all poor oral readers together. If there is not a good reader in the reading group, the poor readers never get to hear a peer model good oral reading. The same thinking could be applied to grouping students for mathematics intervention. If a teacher were to group all of the students in a class who struggle with fluency with basic calculation, he could inadvertently take away the rich learning that occurs from the student-to-student discourse on thinking strategies with more capable peers. This reflection is consistent with the work of others on peer assisted learning (Fuchs et al.,1997; Kroger & Kouch, 2006) and student discourse (Kazemi, 1998; Sims, 2008)

Judgment of Magnitude

Several lessons in the tutoring were designed to assess and develop the students' ability to visualize magnitude. These activities were aligned to grade level common core standards such as understanding of place value, analyzing patterns, measurement, and understanding the difference between a multiplicative comparison and an additive comparison (See Appendix B). A variety of activities such as games, ordering numbers, making models, measuring, and using manipulatives required students to utilize judgment of magnitude. These activities allowed the researcher to make observations regarding the students' strengths or struggles in this indicator area.

One such lesson was the making mazes activity. This lesson focused on interpreting a multiplication equation as a comparison. A statement such as 5 times as many as 7 is different for a child than thinking about multiplication as 5 groups of 7. This activity provided a real-world context for this grade level common core standard with manipulatives. The students made a maze with connecting cubes. They then had to make the same maze five times larger. On the second day, the students had to first recreate a maze with cubes from a model drawn on the board and then make it three times larger. This activity integrated all five of the key instructional practices from the framework. It allowed the students to progress from concrete to representational to abstract models of the context. They discussed and developed strategies for both informal and formal notations of the multiplicative relationship between their original and enlarged mazes. This activity provided a context for peer-assisted learning and mathematical discourse. The students worked together to share strategies and build their models. The researcher was able to effectively assess their abilities and understanding in a lot of areas through this one activity, which led to baseline data for progress monitoring. She was able to assess each student’s ability to create a plan to solve a problem. She was also able to observe their ability to judge magnitude and their ability to mentally compute a number times five. Furthermore, the activity allowed for observations of visual spatial skills while the students were creating mazes from models drawn on the board and from enlarging their original designs. Because this activity involved no symbols, it allowed the students to demonstrate their abilities in judging magnitude and mental computation without using formal mathematical notation. They could rely on their informal number sense to complete the activity. This activity was powerful as an assessment method as well as building strong conceptual understanding about the difference between an additive relationship and a multiplicative relationship.

There is a great deal of research suggesting judgment of magnitude could be a strong predictive indicator of mathematical abilities (Chiappe, 2005; Gersten et al., 2005; Sousa, 2008; Feifer & De Fina, 2005). Involving the students in activities that required them to compare and order numbers, compose and decompose numbers, count, make models, enlarge and reduce quantities, estimate measurements, and then measure provided an opportunity to observe their skills in judging magnitude. Through these activities, it was very apparent that Happy Gilmore and Katarina struggled to visualize numbers on an "internal number line" (Von Aster & Shalev, 2007). However, they did seem to have more success with ordering numbers once they were taught the visual models. This fits with Von Aster and Shalev's suggestions that visual imagery has a role in the development of the mental number line. It was also obvious from the activities that Big Jay and Destiny did not consistently struggle with judgment of magnitude based on their confidence and success in activities requiring the use of judgment of magnitude. They both showed strengths in this indicator area.

The historical tradition in special education remedial math classes includes instruction focused on below grade level computational skills and rote practice of computation (Warner et al., 1980; Wagner, 1995; Cawley & Miller, 1989; Cawley et al., 1992; Woodward & Montague, 2002; Woodward, 2006; Maccini et al., 2008; Maccini & Hughes, 1997). One focus of the instruction in this study was on involving students in engaging problem solving activities that allowed the researcher to assess and helped the students improve their judgment of magnitude. This difference in focus makes the intervention experience the students in this study participated in distinctively different than most remedial math classes. At the same time, carefully designing lessons that integrated judgment of magnitude required highly specialized pedagogical content knowledge. It may be difficult for most intervention teachers to design learning experiences based on grade level standards that involve judgment of magnitude without a great deal of professional development.

Use of Multiple Representations

The use of multiple representations was a key component of all of the learning activities included in the intervention curriculum used in this study. The students daily participated in activities that involved moving across representational systems and using language, visual models, and symbols to communicate about mathematical concepts. They solved contextual problems related to their real world. They used manipulatives and learned to draw visual models of the problems they solved. They discussed strategies for solving problems and evaluated which strategy was the most efficient or which model best represented the problem. Peer-assisted learning was embedded in these discussions.

These conversations provided a context for explicit vocabulary instruction as the mathematical vocabulary emerged as a result of the discourse. They created an opportunity for explicit instruction on different types of models the students could use to represent numbers, contexts, and equations. A part of the explicit vocabulary instruction was the use of terms such as tree diagram, arrow math, open number line, and array to describe the visual models the students learned. This instruction provided a common vocabulary to discuss visual representations of problems. In all of the problem solving experiences, the students showed their work in their math notebooks. This allowed the researcher to analyze their work samples daily, monitor progress as well as struggles, and plan for what needed to be taught the following day.

An example of a problem solving activity utilizing multiple representations was what the four students and the researcher started calling Walmart math. These problems evolved from a mathematical discussion started by one of the students one day during tutoring. Happy Gilmore was saying that he was doing multiplication at Walmart the night before. The researcher asked him how, and he gave some examples of shopping with his mom and multiplying to see how much they were spending. Thus, the kids began making up Walmart math problems. The researcher began integrating more Walmart math problems into the tutoring sessions. We often began or ended class with a Walmart problem (see Appendix B). These were contextual problems with contexts often generated by the students. They connected to their world and integrated their language and culture. They were able to make models of the mathematics taking place in the problems and had rich discourse about their strategies, solutions, and models. They used mathematical vocabulary. The Walmart math problems allowed the researcher to effectively monitor their progress in using models to represent multiplication contexts and their development of more formal strategies. The Walmart math problems met all of the components of the five key instructional practices and allowed the researcher to observe the students' abilities in moving across the representational systems of language, symbols, or visual models.

As the tutoring progressed, the researcher became more aware of deliberately constructing lessons that required students to covert from language to models to symbols in a variety of ways. Based on the Triple-Code Model, there are six different conversions between systems that learners make when processing numerical information. The six different conversions and the coding used in the data analysis are as follows:

- Language to symbol (LTS)
- Language to model (LTM)
- Symbol to language (STL)
- Symbol to model (STM)
- Model to symbol (MTS)
- Model to language (MTL)

These different types of tasks became apparent to the researcher when she was attempting to understand the struggles of Katarina and Happy Gilmore in the indicator area of use of multiple representations. Both of these students had some coded observations of success converting between different systems and many instances of struggle. In contrast, Destiny and Big Jay consistently had very little difficulty converting between all three representational systems. Once they learned the different models, they used them flexibly and confidently.

Analyzing the coded observations related to use of multiple representations for Katarina and Happy Gilmore was educational for the researcher. This process revealed that for a student who struggles with the use of symbols, making a visual model from a contextual problem is much easier than translating a context to symbolic form. This observation fits with Geary's (2004) and Feifer and De Fina's (2005) description of a procedural subtype of MLD. Students with this subtype struggle to link numeric systems into a meaningful language system. A deficit revealed during the data analysis process was that the tutoring in this study did not provide very many opportunities for students to convert from symbols to language. A simple task of asking the students to write a story problem, even a Walmart math problem, that matches a given equation would have involved this symbol to language conversion.

When comparing Happy Gilmore's and Katarina's strengths and struggles within the indicator of use of multiple representations, some interesting trends emerged. The data suggested Happy Gilmore seemed more successful with symbol to model and language to model conversions. He struggled to convert both language and models to symbols. Considering the theory that the recall of basic facts is a verbal language skill (DeHaene & Cohen, 1995), recalling a basic fact can be a symbol to language conversion if the problem is presented visually but it can also be a three way language to symbol to language conversion if the fact is presented orally. Based on this theory, it is understandable that Happy Gilmore struggles to recall basic facts due to his consistent struggle in converting between language and symbols. However, Happy Gilmore's data indicates that he can learn mathematics by relying on the use of visual models and language more than symbols. In contrast, Katarina showed a pattern of difficulty in representing numbers with models. She did not show a clear pattern of success in any of the six types of conversions. It is possible that more time with Katarina would have revealed a more consistent pattern in how she can learn mathematics. Another way to determine how she can be successful would be to work with her on below grade level foundational knowledge such as addition and subtraction to determine what number knowledge she does have. This would move Katarina into the Tier 1 intensive level of support in an RTI model.

This method of analyzing mathematical tasks, student work samples, and observations can be helpful for a teacher to understand a student's strengths and struggles in mathematics and consider accommodations that might help a student be successful in her classroom. However, teachers should be cautioned to not attempt to use this information to diagnose an MLD in a child. This area of research is still very theoretical and unvalidated. While this study provides an opportunity for rich discourse related to diagnosing MLD and could serve as a foundation for future research in this area, it should not be interpreted as a method of diagnosis that would replace a cognitive assessment by a psychologist and a comprehensive evaluation by a special education IEP team.

Procedural Memory

In this study, only 16 observations from all of the students were coded as procedural memory. With such a small number of observations, it is difficult to confidently make conclusions from patterns in the data. A few themes emerged that should be interpreted guardedly. The observations coded procedural memory for all of the students related to remembering how to do a specific task. For example, several observations of Destiny and Happy Gilmore noted they remembered something they were taught from one day to the next after being given explicit instructions. For example, Destiny remembered how to use the partial products method of multiplying a two digit number after being shown this strategy only once. Happy Gilmore had five instances of remembering how to use a model to solve an equation or a problem after explicitly being shown how to make the model. In contrast, the observations coded as struggles for both Big Jay and Katarina noted times when they demonstrated the ability to do something, such as draw a set model, but then could not do it again the next day. In this study, strong procedural memory was not necessarily needed for the content of these units. In addition, open-ended problem solving activities allowed students to rely on multiple ways to solve a problem. They didn't need to remember only one procedure. As the students progress to multi-digit operations and more formal mathematics, it is possible that procedural memory may become a bigger barrier for Big Jay and Katarina and a strength to capitalize on for Destiny and Happy Gilmore. Although procedural memory was not as evident in this study as some of the other indicator areas, it is still an important component of success in mathematics and should be kept as a part of the instructional framework. Future studies based on multi-digit operations or solving multi-step equations may contribute to a better understanding of this indicator area for elementary and middle school students.

Visual Spatial Skills

Feifer and De Fina (2005) discuss visual spatial skills as "the ability to represent and transform spatial information" (p. 45). During this study, the researcher found it very difficult to distinguish between difficulty with visual spatial skills and judgment of magnitude. This may be because there could be a relationship between these two indicator areas as some researchers suggest (Sousa, 2008; DeHaene & Cohen, 1995; Feifer & De Fina, 2005). During the tutoring, it was apparent that while these two skills may be related, they are also very different.

Big Jay is an example of a student who appears to have poor visual spatial skills. His work was consistently sloppy and lacked attention to details. His work samples in Appendix D demonstrate his typical pattern of having his numbers different sizes, not writing in the boxes provided, or not lining up his place value columns. He often had small errors in his models. In one problem solving activity, he drew a 2 x 8 array but labeled it 2 x 4. He frequently made these types of errors. When they were pointed out to him, he quickly fixed them. He could do neat work when he was asked to redo something. Big Jay consistently demonstrated the ability to use visual spatial skills to solve problems, noted in nine coded observations. In one activity, he led the whole group in a multi-step addition and subtraction problem using a hundreds chart. He was mentally computing and making up the problems as we went, marking our answers on the hundreds board with a magic math finger. Big Jay took the whole group and the researcher all over the hundreds board adding and subtracting two digit numbers and purposefully ended up back on the number he started with. Another indication of Big

Jay's visual spatial abilities was his creative use of the models he learned in the tutoring. His multiplication post-test in Appendix D shows his combination of arrow math and an open number line to combine his hundreds, tens, and ones when multiplying three digit numbers.

An activity that clearly illustrates the distinction between Big Jay's unorganized work habits and a perceived weakness in visual spatial skills appeared when he was doing the making mazes activity. When presented with the task of enlarging his maze five times larger, he immediately had a strategy to complete the task. He copied his maze five times. Then, he took each leg apart. The first time he did it he got all of the individual legs mixed together and couldn't get his original design back together. He told the researcher "I know what it should look like but I just don't know what piece goes where!" On his second attempt, Big Jay was much more careful in his process. He again made five replicas of his maze but kept his original in front of him. He broke off each leg from the smaller mazes sequentially one at a time and put them together to make an enlarged leg of the maze. In this way he created his maze five times larger. The making mazes activity illustrates Big Jay's approach to much of his work. He would often jump into a task, solve the problem, and not pay attention to details. When his errors were pointed out to him, he could easily fix them. He had the visual spatial abilities to easily complete the making mazes problem and other similar visual spatial tasks.

Contrast Big Jay's making mazes experience with Katarina. Katarina seemed to have no idea how to go about enlarging her maze with the manipulatives. She was very frustrated and completely shut down during the activity the first day and just watched the other students. On the second day of the activity, she was able to copy the maze drawn on the board onto graph paper correctly and then built it with cubes. Based on this observation, the researcher originally thought Katarina had a struggle with visual spatial tasks, but over time she demonstrated strong abilities in handwriting, drawing, making arrays, and copying off the board. Her work notebook was very neat and organized. She often drew very neat, precise models that were conceptually immature or inaccurate. She consistently struggled with judgment of magnitude, which may have been the cause of her frustration in the making mazes task. Thus, Katarina appears to be a student with poor judgment of magnitude but good visual spatial skills.

Happy Gilmore had the most observations of a struggle with visual spatial skills of all of the students, with 16 noted observations. He frequently reversed the direction of a number line and individual numbers. He struggled even at the end of the tutoring with drawing the lines to connect the numbers in the tens place and the ones place on a tree diagram even though he conceptually understood he was adding the tens together and the ones together and verbally explained it correctly. He once drew a square with all four sides labeled seven but one side was obviously shorter than the others. When this error was pointed out to him, he didn't seem to understand what was wrong with his drawing, and another student ended up fixing it for him. While at first glance, Happy Gilmore's notebook and Big Jay's notebook look very similar because they are both sloppy, Happy Gilmore's work shows much greater difficulty with visual spatial skills than Big Jay's work samples show. Because Happy Gilmore also showed struggles in judgment of magnitude, it was often difficult to determine if tasks were hard for him due to an inability to visualize an internal number line or challenges with a visual spatial task. An example of a task in which judgment of magnitude and visual spatial skills were integrated was when he was frustrated playing the game greater than/less than. He struggled to use the symbol cards correctly to show which number was greater than or less than the other. The correct direction of the symbols and arranging the playing cards to make the largest number possible confused him. It is possible that Happy Gilmore is a student whose struggles with judgment of magnitude are related to his struggles with visual spatial tasks. The fact that he did show success in judging magnitude in 12 noted observations provides some support for this hypothesis.

The relationship between visual spatial skills and judgment of magnitude is a topic worthy of additional research. Both Fiefer and De Fina (2005) and Sousa (2008) discuss visual spatial limitations as a domain-general disability. In this study, the researcher was able to observe a difference between visual spatial skills that might affect all subjects and struggles with judgment of magnitude specifically related to visualizing numbers. Happy Gilmore's ability to draw straight lines to connect his tens in his tree diagrams was different from his ability to visualize and understand combining the tens from two numbers in an addition problem. When he was playing the card game, he knew which number was bigger but struggled with getting the direction of the less than or greater than symbol correct. These examples illustrate how a visual spatial disability impact performance in mathematics. Happy Gilmore's progress in the tutoring demonstrated that drawing models seemed to develop his ability to visualize magnitude. However, his limitations in his visual spatial skills may have impacted the pace of this development in comparison to his peers who made more progress in the same intervention experience.

Another confounding variable in determining whether a student had a clear pattern of weakness in visual spatial skills was the fact that the students seemed to have had very few prior experiences with tasks involving manipulatives, geometry, and measurement. The Van Heile model of developing abilities in geometry suggests visual spatial skills can be developed from instruction (Crowley, 1987). Von Aster and Shalev (2007) support this theory. In this study, all four of the students seemed to improve in their ability to draw accurate models representing magnitude and solve problems with manipulatives. Activities such as making mazes, measuring with cubes, pattern block designs, using manipulatives, and drawing visual models to represent numbers and equations may have helped the students develop visual spatial skills while developing the ability to visually represent magnitude. These are the types of tasks Woodward (2006) and Bryant et al. (2006) are referring to when they claim SWD need a broader range of mathematical content in order to develop their mathematical knowledge and abilities.

CHAPTER 7: LIMITATIONS AND IMPLICATIONS

The instructional framework used in this study was designed to be a tool for teachers of mathematics intervention classes. It was created to help intervention teachers plan for effective instruction as well as understand the students they are teaching, who may or may not have MLD. The results of this study suggest intervention based on the key instructional practices identified in the framework led to positive changes in student understanding of mathematics for three of the four students. Recording observations and evaluating work samples based on the indicators of an MLD identified in the framework led to a greater understanding of the strengths and struggles of each student. In this section, the usefulness of the instructional framework as a tool for planning instruction as well as analyzing student strengths and struggles will be considered. Limitations of this study and suggested revisions to the framework will also be addressed. This section concludes with a discussion of implications for practice and suggestions for additional research.

Limitations

When considering the usefulness of this instructional framework, it is important to keep in mind some significant limitations of this study. The small student sample size makes generalization of this study to other populations difficult and should be done cautiously. Larger scale studies with a greater student sample size would be needed to provide reliable quantitative support for the effectiveness of this instructional framework in increasing achievement in mathematics. Related to that is the need for a statistically valid and reliable measurement of achievement that also aligns to the cognitive and social learning theories on which this framework is based. The scoring method for this study was subjective. The reliability of the results would have been improved if more than one person had scored the tests and interrater reliability evaluated.

Another limitation of this study is the provider of the tutoring was also the researcher who had developed the instructional framework used to plan lessons and record and organize observations about the students. A depth of knowledge related to effective instructional practices and MLD was formed by the substantial review of literature, which led to the creation of the framework. Therefore, for the researcher to effectively evaluate the usefulness of the framework is not objective. There is a strong researcher bias influencing any evaluation of the framework.

Another caution that should be noted when considering the use of this framework is to emphasize this instructional framework is not a diagnostic tool. The diagnosis of an MLD is made by physicians, psychologists, and IEP teams, not general education teachers. This framework is designed to help teachers organize and record observations about a student related to MLD, which can then be used by the appropriate professionals to assist with further evaluation and possible diagnosis. This framework provides teachers with professional vocabulary to contribute useful information regarding patterns they may observe in student work or behavior. Although these limitations exist, this study can serve as a foundation for future research and future professional development for teachers. Therefore, reflecting on the usefulness of the instructional framework and its connection to other topics in this genre of research is worthwhile.

Suggested Changes

While many components of the framework were helpful for planning the tutoring sessions and evaluating student strengths and weaknesses, the researcher identified a few areas that should be revised. A suggested revised framework integrating the recommended changes is shown in Figure 31. As noted in the results section, judgment of magnitude and determination of unreasonable results were difficult to distinguish between. Therefore, determination of unreasonable results was removed as an indicator area and demonstrations of this were considered as a weakness or strength in judgment of magnitude.

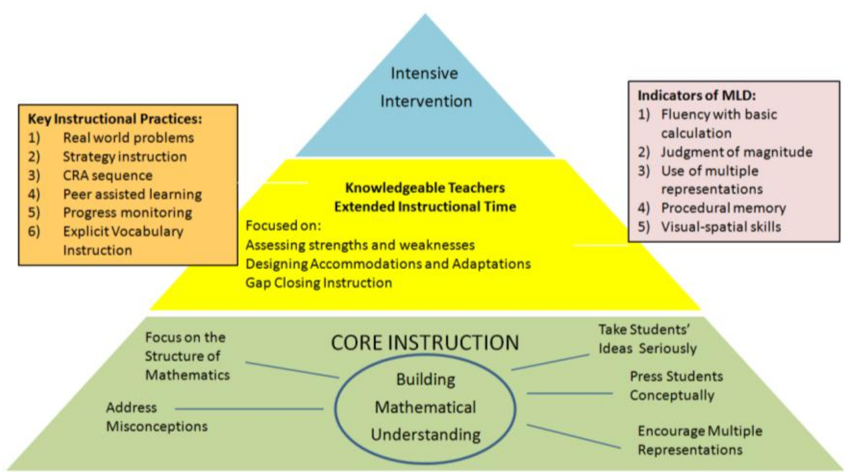


Figure 31. Revised Instructional Framework for Tier 2 Mathematics Intervention

Another suggested change to the framework would be to add a sixth key instructional practice, explicit vocabulary instruction. While this practice was not specifically addressed in the review of literature related to teaching students with MLD, it was very important when teaching the students in this study who were all either LEP or language impaired. In order to fully understand the mathematical concepts being discussed, the students first needed a good grasp of the mathematical content vocabulary being used each day in the tutoring sessions. The role of explicit vocabulary instruction and its relationship to student mathematics achievement is a topic worthy of additional research. It is also a critical component of effective instruction for students teachers suspect may have MLD who are also LEP. Since students who are LEP are often referred to Tier 2 intervention in mathematics as was the case in this study, explicit vocabulary instruction should be a key instructional practice encouraged by an instructional framework for Tier 2 instruction. Furthermore, since many students with language-based disabilities also struggle in mathematics, explicit vocabulary instruction in a Tier 2 setting will help determine whether a student’s struggles in mathematics are related only to language or if there are more serious problem with numerical processing.

A third revision to the framework is to reword fluency with basic facts to fluency with basic calculation. This change is wording is more consistent with the research of current leaders in this genre of research (Geary, 2005, 2004, 1993; Gersten & Chard, 1999; Baroody et al., 2009; Baroody, 2006). Furthermore, the tutoring experiences revealed fluency with basic calculation is much broader than just the ability to memorize and recall basic facts. For example, when Katarina could not quickly add "zero and six" when said to her orally by the teacher, her struggle revealed deficits in number sense and visualization of magnitude as well as lack of fluency with a basic fact. Zero plus six is not a fact that needs to be memorized because students can rely on their number sense to find the answer. In contrast, Happy Gilmore showed a consistent struggle with the ability to memorize more difficult math facts, but when he could rely on his number sense and number patterns, he was very fluent with basic calculations with small numbers or numbers that fit the patterns he knew. For this reason, the framework was revised to say fluency with basic calculation rather than basic facts. Additional discussion about the instructional framework in the rest of this chapter is based on the revised framework.

Strengths and Struggles

Going into the tutoring experience with the indicators of an MLD in mind was very helpful. These indicators provided a guide for the researcher to know what to look for as she was working with the students and what to record. It helped her evaluate both strengths and struggles for each student, as recommended by Bryant et al. (2006). Throughout the tutoring, the researcher noticed behaviors and examples in work that may not have otherwise been noteworthy. For example, recording Katarina’s inability to answer and draw a model of zero plus six was a significant observation showing her lack of ability to visualize magnitude. On the other hand, noting Big Jay’s amazing ability to judge magnitude with his 10 step hundreds chart problem was also important to note. Noting the students’ strengths and struggles in each of the indicator areas made it easy for the researcher to daily reflect on her experiences with the students and record significant observations. During any 45 minute tutoring experience with students, there are many interactions, conversations, work samples, and examples of student thinking that occur. The sheer number is overwhelming. Using these indicator areas as a way of organizing all of the different types of data available daily on these four students was easy to implement. The researcher was able to record observations on a daily basis for each area.

The framework made it easy to organize data and observe patterns at the end of the tutoring. A suggestion for future studies would be to create a daily recording sheet for each student organized by the indicator areas. That would have made organizing the data on each child easier to categorize at the end of the tutoring. The results of this study show that the coded data supported the researcher's subjective impressions of the students and the two students who seemed to have an MLD did show patterns of weakness in the indicator areas. The two students who did not seem to have an MLD and who showed great gains in achievement from the tutoring did not show patterns of deficit in the indicator areas. The framework's guidance on recording students' strengths and struggles in the indicator areas was helpful and effective in this study.

Instructional Design

A recurring theme in the discussion of the indicator areas is the importance of good instructional design for mathematics intervention (Fuchs et al., 2008; Carnine, 1997; Woodward & Montague, 2002; Bryant et al., 2006). Designing rich learning experiences for students who struggle to learn mathematics requires specialized pedagogical content knowledge for the teachers of intervention classes. When designing Tier 2 mathematics instruction, choosing tasks that maximize the instructional time and reveal student strengths and struggles is essential. Worthwhile tasks create not only rich conceptual understanding, they also provide authentic opportunities for assessment and progress monitoring. These types of tasks are what Woodward (2002) was referring to when he claimed that SWD need engaging tasks, a broader range of content and better use of instructional time. This framework provides teachers a useful tool for designing high quality mathematics intervention experiences.

Implications for Practice

The following recommendations are made to teachers of Tier 2 intervention classes based on what was learned from this study:

- Maximize instructional time in intervention classes by choosing worthwhile tasks based on grade level standards. These tasks should integrate the six key instructional practices from the framework and allow for both assessment of student abilities as well as conceptual development simultaneously.
- Organize observations and work samples around the five indicator areas of an MLD to support the need for further evaluation or demonstrate that the student does not show symptoms of an MLD.
- Document progress or lack of progress for all Tier 2 intervention and place progress reports in students' cumulative files.
- Thoroughly read each student's cumulative file before tutoring to gain a good understanding of all special education evaluation data, unique learning needs, and previous intervention the child has participated in. Be aware of LEP status and how a domain-general disability may impact a child's success in mathematics.

- Use observations and work sample notes on strengths and struggles to design accommodations to help the student in grade level instruction.
- Learn about the Triple-Code Model and Bruner's theory of enactive to iconic to symbolic to better understand how children process numerical information. Keep in mind the role of language in developing mathematical understanding and provide instruction that balances language development in mathematics with visual models and symbolic notation.
- Become knowledgeable of the three different subtypes of MLD.
- Plan learning experiences that integrate real-world contexts based on the students' culture.
- Plan intervention lessons based on grade level standards and scaffold instruction during learning activities to reteach needed skills and concepts.
- Carefully consider grouping students for intervention groups to provide students with poor number sense peers who can model effective mathematical thinking strategies.

Future Studies

A great deal of additional research would be needed to validate the effectiveness of mathematics intervention based on this framework. In this study, the framework's guidance for designing instruction and recording students' strengths and struggles in the indicator areas was helpful and efficient. The question remains whether it would be helpful to a teacher who was not the author of the framework who may have a more limited depth of background knowledge on the topic of MLD.

All research leads to more questions than answers. This study provides a rich background for continued research. Some studies that could be conducted based on this study are as follows:

- How does explicit vocabulary instruction in mathematics instruction impact student understanding of multiplication?
- How does the use of visual models impact student understanding of multiplication?
- What are practicing teachers' perceptions of the usefulness of the instructional framework for Tier 2 mathematics instruction?
- Does the implementation of the instructional framework for Tier 2 mathematics instruction lead to an increase in achievement for SWD and/or LEP students?
- How does the instructional framework for Tier 2 mathematics instruction developed for this study compare to best practices for ESL students?
- Do experiences with geometry and measurement lead to greater student achievement in multiplication?
- How does preteaching vocabulary and advance organizers impact the success of low achieving students in whole class problem solving activities?

Conclusion

The purpose of this study was to investigate the effectiveness of mathematics instruction based on an instructional framework for Tier 2 mathematics intervention. The study addressed two research questions. The first question was does low achieving fourth graders' participation in tutoring based on five key instructional practices for students with mathematics disabilities increase their understanding of multiplication and division? The second question was what are the patterns of deficits low achieving fourth graders have in relation to the six indicators of MLD? The results showed two of the students made remarkable progress in their understanding of multiplication and division from pre-test to post-test. One of the students showed some improvement, and one showed very little improvement. Two of the students showed a pattern of struggle related to indicators of an MLD, and two students did not. This study provided an opportunity for the researcher to put theory into practice by designing and conducting tutoring based on the instructional framework and then reflecting on what was learned. The findings provide guidance for professional practice and lay a foundation for future research.

The focus of this study was to contribute to the challenge of improving instruction for students in need of mathematics intervention. The suggested instructional framework provides an alternative to traditional drill and practice remedial math classes. This study has demonstrated that the reasons students are failing to learn grade level mathematics are complex. These students need high quality intervention classes that help them close the achievement gap between them and their peers. Intervention should also assess a student's strengths and weaknesses and find ways to help them be successful in their mathematics education. This framework can be used to educate teachers about MLD and the type of instruction students who struggle to learn mathematics need to experience.

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APPENDIX A

Parental Consent Documents

Letter to Parents in One Fourth Grade Classroom

sent via school email system and sent home with students in January, 2012

Dear Parents/Guardian of Fourth Grade Students:

I am an education professor at Northwest Nazarene University and a doctoral student at Boise State University. I am currently completing a research project at your child's school for my doctoral degree. For this study, I will research teaching methods which help students who struggle to learn mathematics. This research study has been reviewed and approved by the Research Review Committee at both Boise State University and Northwest Nazarene University. It has also been approved by the XXXX School District Deputy Superintendent and XXXXX principal. These committees are responsible for making sure that all research being conducted with children is completely safe for the participants and that parents are fully informed about the research in which their child may be participating.

All students in your child’s classroom at Sherman take math assessments such as the ISAT (Idaho Standards Achievement Test) as part of their class curriculum. I am asking for your permission to use your child’s data and scores from these assessments in my research. If you agree, I will be using teacher comments about your child, your child’s demographic and achievement data from the school database as well as your child’s scores from other math assessments given to the whole class to determine the effectiveness of instructional techniques. These math assessments include fourth grade multiplication and division concepts as well as basic math facts.

If you do not want your child to participate in my research, your child will still take all of these assessments, I will just not use their data in my research. Also, if you do decide to allow me to use your child’s data but later change your mind, you or your child may choose not to participate at any time without adverse effects. In this case, your child will still take all assessments that are part of their normal classroom activities, but I will no longer use their scores in my research. In the event that this study is published, no identifying information will be disclosed. No discomfort, stress or risks are anticipated.

Some students in the class may benefit from additional math tutoring and will be selected by myself and your child's teacher to participate in math tutoring and further testing. The parents of those students will be contacted by phone, provided with more information, and given the option to have their child participate in the study further. The initial testing for this study will begin in February and the post tests will be given to all of the fourth grade students in May. Signing this consent form provides permission for your child's teacher to give me your contact information if your child is selected for the tutoring group.

I will need written parent consent in order to use your child's data for this research. You can provide this consent via email or by returning this form. If you choose not to allow your child to participate, please complete the form at the bottom of this letter and return it to school or email it back to your child's teacher. No information about your child will be provided to the researcher by the school. There will be no negative consequences for not participating in the study. All children who bring back a signed consent form or remind their parents to send an email will receive a candy bar even if their parents choose to not have them participate in the study.

If you have questions about this research project, please, do not hesitate to contact me at Northwest Nazarene University at 467-8250, or via email at cbeals@nnu.edu. The results of my research will be available after October 1, 2012. If you would like to have a copy of the results of this study, please contact me.

Sincerely,

Catherine Beals, EdS
Northwest Nazarene University
Boise State University
208-467-8250
cbeals@nnu.edu

Please sign one of the sections below and return this form to your child's teacher or copy the appropriate section into an email and send it to your child's teacher or to cbeals@nnu.edu

Consent Given

I give consent for my child ________________________________ to participate in this study on teaching mathematics.

Child’s name: ____

Parent/Guardian printed name: _______________________________________

Parent/Guardian signature: (Not needed if sent via email) __________________________

Date: ___________________________

Consent Not Given

I do not want my child ________________________________ to participate in this study on teaching mathematics.

Child’s name: ____

Parent/Guardian printed name: _______________________________________

Parent/Guardian signature: (Not needed if sent via email) __________________________

Date: ___________________________

Script for Parent Phone Call to Tutoring Group

The investigator will not read the script word for word but convey the important information in a conversational style with the parent.

- Thank you for allowing your child to participate in my research study.
- I have been studying ways to teach students who struggle with mathematics for many years as an elementary and high school teacher and administrator. I am doing a formal research study at your child's school on effective ways to teach children who struggle to learn mathematics as part of my doctoral program at Boise State University. .
- Based on several different types of information, you child's teacher and I agree that your child may benefit from some extra tutoring in mathematics.
- Your child will be in a small group who will work with me for 6 to 8 weeks in 24 math tutoring sessions. I will keep careful records of your child's progress and give them some assessments every 8 sessions. At the end of the tutoring sessions, I will compare your child's scores with the rest of the class who did not participate in the tutoring with me to see if the teaching methods I used help the students I work with. I think this will be an enjoyable, meaningful experience for your child in mathematics.
- In order to best help your child, it would be helpful if you can provide consent for me to be able to talk to your child's teacher and other school employees who work with your child about their performance in math and other subjects and review your child's cumulative records and (if applicable) IEP information. Access to these records is completely optional and not necessary to participate in the study. It simply provides me with additional insight into your child's strengths and weaknesses in school and will contribute to helping others understand students who may have or have learning disabilities in mathematics.
- All information about your child will be kept in strict confidence. When the results of the study are shared outside of the school, your child's name will not be used. At the end of the study, I will meet with you so you know exactly what information will be shared about your child and you will have access to the results of the study.
- What I would like to do is send you an informational letter about the study with a permission slip. If you are comfortable having your child participate in the study, you can complete the permission slip and mail it back to me. There is a place on the permission slip to sign indicating that you have talked to your child about this opportunity. After I receive your permission slip, I will meet with your child individually and answer any questions.
- I am happy to meet with you at any time during the study or you may call me on my cell phone - that number will be in the letter sent home. You are also welcome to visit during the tutoring sessions.
- Are you comfortable with me mailing this information home to you?

- If you choose not to have your child participate, there will be no negative consequences for your child. I also need to inform you that your child will receive a small surprise gift at the end of the study as a way of saying thank you for participating in the study with me.

Script for Initial Contact with Students in Tutoring Group to Gain Student Assent

This individual conference will take place before the first tutoring session at a time arranged with the classroom teacher. I will meet with the child after receiving the parent consent form.

- Introduce myself and explain that I teach teachers at Northwest Nazarene University. I am doing a study to make sure that I am helping teachers learn good ways to teach math to kids.
- Did your parents talk with you about a research study we are doing at your school?
- What do you know about the study?
- Do you have any questions?
- Your job in this study will be to work with me for several weeks to see if we can help you be better at math. You will also take some tests with me every now and then. After awhile, I will compare your scores with other students in your class to see if there are differences in your scores. By doing this, I will find out if the way I think people should help kids learn math actually works better then the way we help kids now. In science we call you the experimental group - have you heard of that before? Explain the idea of a control and experimental group as much as the child can understand based on their background knowledge.
- Are you willing to help me with my study?

Letter to Parents of Tutoring Group

To the parent of ___________________________

Thank you for considering allowing your child to participate in my dissertation research study. The purpose of this letter is to provide more detailed information about the study, the role your child will have in the study and to obtain your legal consent for me to work with your child and have access to confidential information about your child.

I am a certified teacher and administrator in the State of Idaho. Throughout my twenty-three year career in education, I have been intrigued by the challenge of helping students who have difficulty learning mathematics. My doctoral program at Boise State University has allowed me to research and study the most effective teaching techniques for students who are struggling in this subject. This study is the culminating project of my program and an opportunity to apply all I have learned. What I learn from this study will be extremely helpful in my role as an education professor at Northwest Nazarene University. The classes that I teach at NNU are about teaching mathematics to elementary children. I hope to use the results of this study in professional development classes for practicing teachers, classes for preservice teachers and in further research and presentations. When this study is shared in these forums, all student identifying information will be removed and an alias used to refer to your child. If you are willing, I would like to use pictures of your child to share the results of the study and give it more meaning for the audience. This is completely optional.

Your child was selected from their fourth grade class as a student who would benefit from some additional tutoring in mathematics. Part of this study is to provide tutoring to a small group of 3 or 4 students. This group will participate in a small group math intervention class for six to eight weeks, beginning in March. I will personally be teaching this group and you are welcome to come observe any of the sessions. The teaching techniques which I will be using with these students will involve talking about strategies for solving problems, using hands on materials and drawings or models, working in groups or partners, solving real world problems and frequent informal progress monitoring assessments. The lessons will focus on three important fourth grade topics in mathematics: number patterns, multiplication and division. The group will meet 24 times and will complete an assessment after every eight sessions. At the end of the study, the students will take a post test and their scores will be compared to the other students in their class. From the post test, we will be able to see if the students in the tutoring group made more or less progress than other students in their class. My hope and hypothesis is that from participating in this intervention class, your child may be able to catch up to their peers in their mathematics skills.

Throughout the learning activities, which will often be problem solving, games and hands on challenges, I will be assessing your child informally by giving them problems such as the following: *Create a number line that shows me the placement of these four numbers: 12, 43, 27, 17*

Your child will complete most of their work and assessments in a math work spiral. I will save all of their work samples and at the end of the tutoring experiences, review what patterns I notice in how your child approaches mathematics. I have learned through my research that students who struggle in mathematics often have a pattern of struggle in one of the following six areas: fluency with basic facts, judgment of magnitude, determination of unreasonable results, use of multiple representations of a problem, procedural memory, and visual spatial skills. As I work with and assess your child, I will be carefully looking for patterns in these key areas. I will record observations and possibly direct quotes from your child. At the end of the intervention period, I will summarize what I learn about your child and provide some recommendations on how you and their teachers might help them be more successful in mathematics. I will prepare a written report for you and conference with you about it.

The risks to your child for participating in the study are minimal. They will be missing some class time but it will be arranged with their teacher to be at a time where it won't adversely affect their performance in key academic areas. Sherman's principal and your child's teacher are very supportive of this study. Your child may feel shy or anxious about testing and working with an adult they don't know at first, but in a small group setting this is usually overcome quickly.

The benefits for participating in this study include some extra help in mathematics from a highly qualified teacher and you and your child will hopefully gaining a greater understanding of his/her strengths and weaknesses in this subject. In addition, I will be giving your child a thank you present at the end of the study and may give them some small prizes throughout the intervention period to encourage progress and good behavior. I am required to fully disclose all incentives given to the participants.

The results of this study will be presented to my doctoral committee at Boise State University in the Fall of 2012. You may also have a copy of any part of my dissertation you would like to read and are welcome to attend my dissertation defense at BSU if you are interested.

I am greatly looking forward to working with your child. I love to teach mathematics, and I love working with children. I hope you will allow your child to participate in this study, but if you are not comfortable providing consent, there are no negative consequences for you or your child. Please complete and sign the attached form to let me know of your decision.

If you do provide consent, there is a place on the consent form to indicate that you have talked with your child about the study. After receiving your permission slip, I will meet with your child and get their signature on the form.

Thank you for your consideration and for reading this information. Please contact me if you have any questions or concerns.

Sincerely,

Catherine Beals, EdS
Northwest Nazarene University/Boise State University

Consent Form: Tutoring Group

I give consent for my child ______________________________ to participate in a research study on mathematics intervention at Sherman Elementary school. I have read the informational letter describing my child's role in the study and the description of the study.

I give my consent for the following: (Please initial all that apply.)

____ My child may participate in the small group tutoring sessions and assessments given to this group.

____ The researcher may review my child's school records in order to understand my child's educational history and needs. This includes any special education records if applicable.

_____ The researcher may freely discuss my child's progress with other school and district personnel who also have access to confidential information about students. This may include but is not limited to my child's teacher, educational assistants, counselors, the school principal, or support teachers such as special education teachers, ESL teachers or instructional coaches.

____ I understand that an alias name will be given my child when this study is shared.

____ I have discussed this study with my child and they are willing to participate in this study.

Parent signature: ____________________________________ Date: ________________

Best phone number to reach you if needed: ____________________________

Email: ________________________________

No Consent:

__________ I do not wish to have my child participate in this research study.

Parent signature: ____________________________________ Date: ________________

APPENDIX B

Assessments and Curriculum Units

Multiplication Test Constructs and Subscales

Construct	**Subscale**	**Number of Items**
Fluency with Basic Calculation	Recognition of number properties	6 Test Items: 12 - 17
	Skip counting	5 Test Items 1 - 5
	Total	11
Judgment of Magnitude	Draw objects a given number of times larger	6 Test Items: 18- 23
	Identify a number a given number of times larger	5 Test Items: 7 - 11
	Total	11
Use of Multiple Representations	Create a model of a number	10 Test Items: 4 - 11
	Create a model from a multiplication context	5 Test Items: 30 - 34
	Create a multiplication equation from a model	6 Test Items: 24 - 29
	Create a model of a multiplication equation	7 Test Items: 35 - 41
	Total	28

Multiplication Pre-test/Post test

Skip Counting:

1) Count by 6's and list 5 multiples of 6:

______ ______ ______ ______ ______

2) Count by 3's and list 5 multiples of 3:

______ ______ ______ ______ ______

3) Count by 4's and list 5 multiples of 4:

______ ______ ______ ______ ______

4) Show skip counting by 5's to 30 using arrow language or an open number line:

5) Show skip counting by 10's to 50 using arrow language or an open number line:

Models of Numbers:

6) Draw 2 different ways to show the numbers given:

A) 12 B) 42 C) 120

7) What number is 2 times larger than 10? ___________
Show your answer with a model.

8) What number is 3 times larger than 5? ___________
Show your answer with a model.

9) What number is 6 times larger than 2? ___________
Show your answer with a model.

10) What number is 2 times larger than 7? ___________
Show your answer with a model.

11) What number is 5 times larger than 4? ___________
Show your answer with a model.

Number Properties

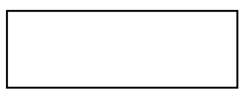
Fill in the blanks:

12) 3 x 7 = ____ x 3   13) 6 x ____ = 0   14) 8 x ____ = 8

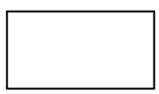
15) 341 x 8 = 8 x ____   16) 15 x ____ = 0   17) 16 x ____ = 16

Drawing Shapes:

18) Draw this bar three times longer:



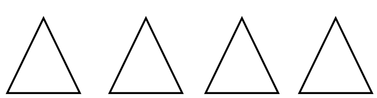
19) Draw this bar five times longer:



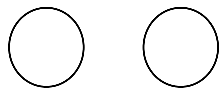
20) Draw this line 2 times longer:

21) Draw this line 10 times longer:

22) Draw 3 times more triangles:



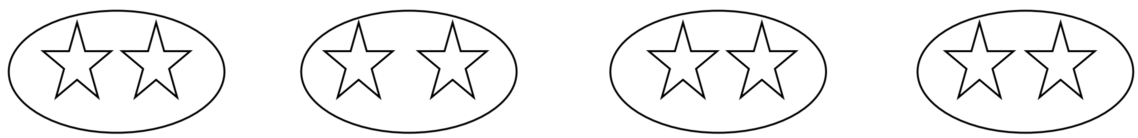
23) Draw 4 times more circles:



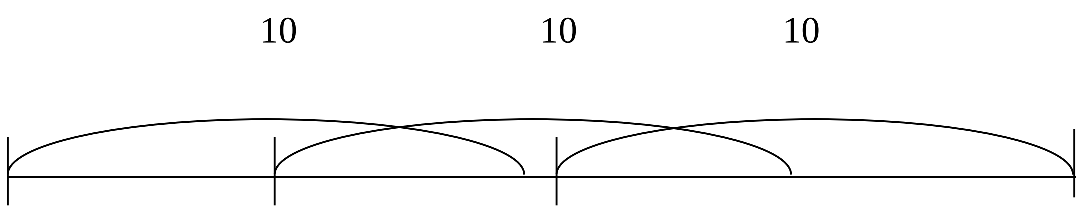
Models to Equations:

Write a multiplication equation that matches each model:

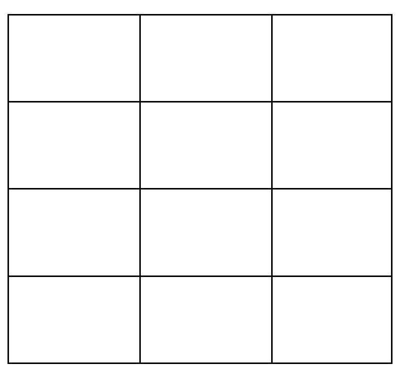
24) ______________________________



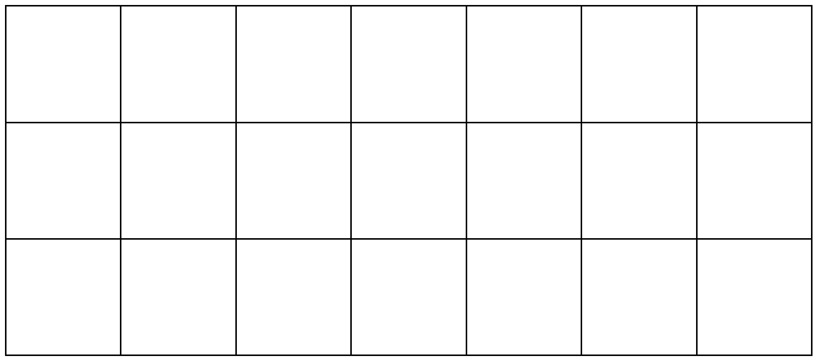
25) ______________________________



26) ______________________________



27) ______________________________________



28) ______________________________________

10	3	
60	18	6

29) ______________________________________

200	20	3	
800	80	18	4

Problem Solving:

Solve the problems and show your answer with a model.

30) Mrs. Compton had 4 cups. She put 8 mini Oreos in each cup. How many Oreos did she have all together?

31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

33) Packages of mini doughnuts are on sale for 65 cents. Dustin wants to buy 4 packages. He has $2.00. Does Dustin have enough money to buy 4 packages of mini doughnuts? How much money will he need?

34) Carmella is planting a garden. She has 6 packages of sunflower seeds. Each package has 15 seeds. How many sunflower seeds does Carmella have all together?

Equations to Models:

Solve the problem and make a visual model of these equations:

35) 4 x 7 =

Answer:	Model:

36) 5 x 6 =

Answer:	Model:

37) 14 x 4 =

Answer:	Model:

38) 23 x 5 =

Answer:	Model:

39) 162 x 4 =

Answer:	Model:

40) 328 x 2 =

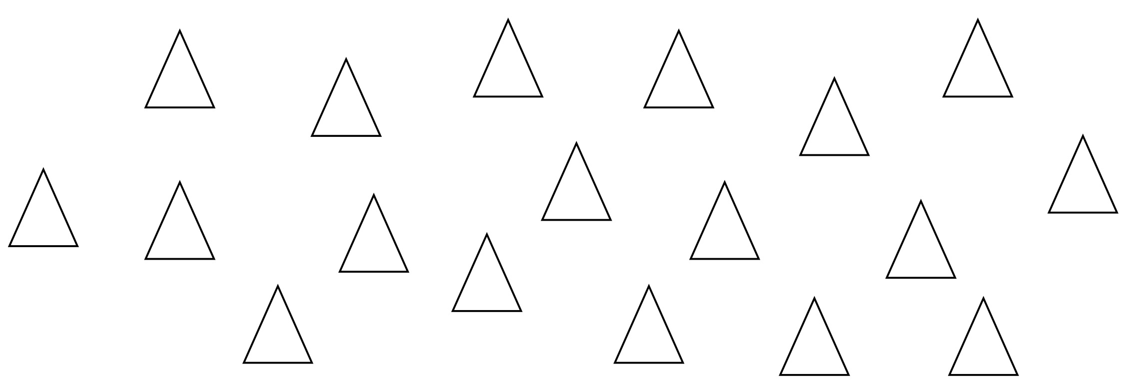
Answer:	Model:

Division Test Constructs and Subscales

Construct	Subscale	Number of Items
Judgment of Magnitude	Divide a given number of objects into equal groups	5 Test Items: 1 - 5
	Partition a shape into equal pieces	5 Test Items 6 - 11
	Identify a number a given number of times smaller	5 Test Items: 17 - 21
	Total	16
Use of Multiple Representations	Create a model from a division context	5 Test Items: 22 - 26
	Create a division equation from a model	5 Test Items: 12 - 16
	Create a model of a division equation	6 Test Items: 27-31
	Total	16

Division Pre-test/Post test

Making Groups

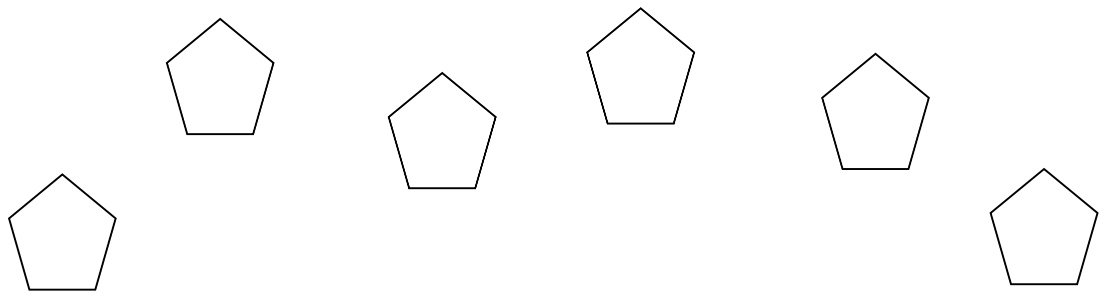


1) How many groups of 3 can be made from the shapes? ________

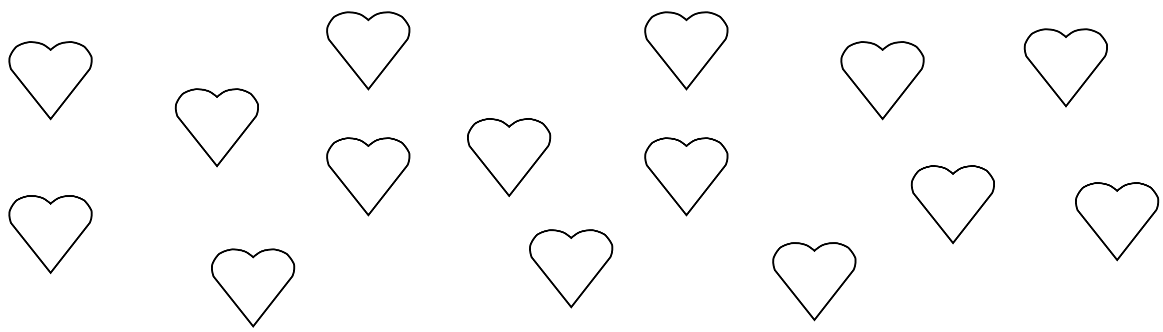
2) How many groups of 5 can be made from the shapes? _________

3) How many groups of 8 can be made from the shapes? _________

4) Divide these shapes into 3 equal groups.



5) Divide these shapes into 5 equal groups.



Dividing Shapes:

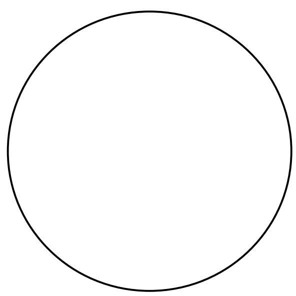
6) Divide this bar into 4 equal sections:

7) Divide this bar into 6 equal sections:

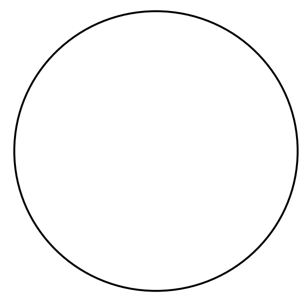
8) Divide this line into 3 equal sections:

9) Divide this line into 5 equal sections:

10) Divide this circle into 2 equal equal sections:



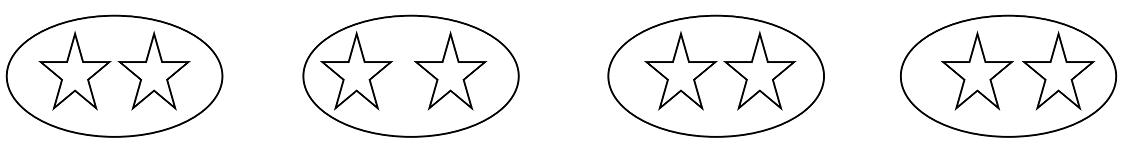
11) Divide this circle into 4 sections:



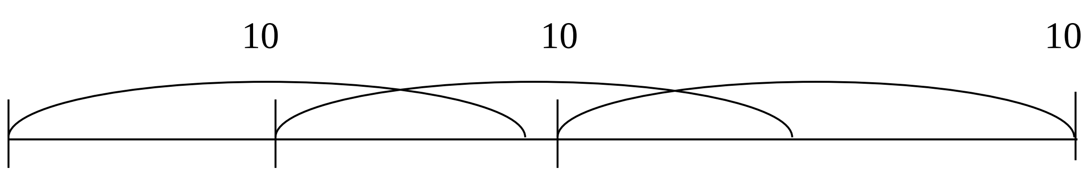
Models to Equations:

Write a division equation that matches each model:

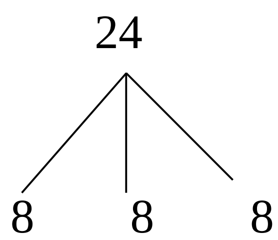
12) ________________________



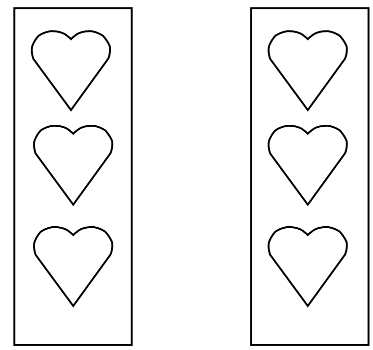
13) ________________________



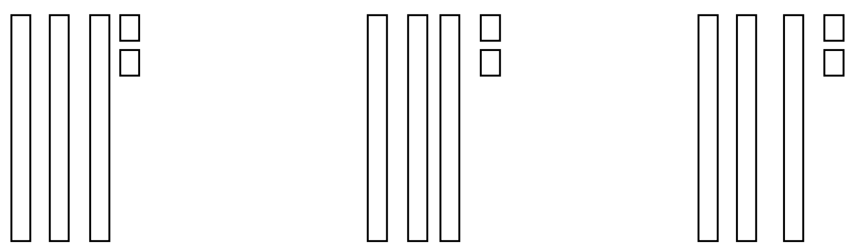
14) ______________________



15) ______________________



16) ______________________



Models of Numbers

17) What number is 2 times smaller than 10? __________
Show your answer with a model.

18) What number is 3 times smaller than 6? __________
Show your answer with a model.

19) What number is 6 times smaller than 12? ____________
Show your answer with a model.

20) What number is 2 times smaller than 14? ____________
Show your answer with a model.

21) What number is 5 times smaller than 20? ____________
Show your answer with a model.

Solve the problems and show your answer with a model:

22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?

23) Molly had a line of connecting cubes 28 cubes long. She wanted to break the strip into 4 equal pieces. How many cubes should each piece be?

24) Molly then made a line of 72 cubes. She broke the line into smaller lines of 10. How many smaller lines did she make with her 72 cubes? How many cubes were left over?

25) Mrs. Compton had 60 Mamba candies to give to 5 students. If she divided the Mambas equally between the children, how many Mambas would each student get?

26) Mrs. Compton's class was going on a field trip. She needed some parent helpers. She had 25 students and she wanted each parent to have 6 students in a groups. How many parent helpers will she need for the field trip?

27) Lucas had a penny jar. He wanted to trade his pennies in for dollar bills. He had 323 pennies. How many dollar bills will Lucas get for his pennies? How many pennies are left over? (Remember: 100 pennies = 1 dollar bill)

Equations to Models:

Solve the problem and make a visual model of these equations:

27) $18 \div 9 =$

Answer:	Model:

28) 30 ÷ 5 =

Answer:	Model:

29) 26 ÷ 4 =

Answer:	Model:

30) 32 ÷ 3 =

Answer:	Model:

31) $426 \div 3 =$

Answer:	Model:

32) $325 \div 5 =$

Answer:	Model:

2 Minute Multiplication Fact Assessment

Form 1

Multiplication Facts

All The Facts

Name: ______________________________

Time: ______________ No. Correct: ________/100

8×9	5×5	2×2	3×4	5×4	7×6	9×1	2×0	4×3	6×7
6×5	6×1	3×4	3×1	2×3	1×0	5×8	3×0	2×1	6×8
5×2	4×8	2×5	1×1	9×0	3×8	2×2	4×5	2×6	1×9
3×7	9×7	8×1	7×3	4×3	1×5	2×4	9×5	8×4	7×1
5×9	9×3	8×2	2×9	1×2	8×0	7×6	6×6	4×2	6×3
8×8	3×6	1×7	8×3	6×9	8×7	5×6	1×6	8×9	7×5
3×3	1×3	9×4	7×8	3×5	9×8	7×7	7×2	6×0	5×1
5×7	7×4	5×0	4×9	2×8	9×9	8×6	6×4	5×3	9×2
9×1	7×0	6×2	5×5	1×4	4×6	7×9	6×7	4×4	2×0
8×7	4×7	8×8	7×8	2×7	4×8	9×9	3×9	2×7	4×1

Source: Unknown

Number Patterns Unit Overview

Unit Title: Number Patterns Grade Level: 4 Length of Unit: 11 days

Common Core Standards Addressed in Unit

<u>Number and Operations in Base Ten</u>

Generalize place value understanding for multi-digit whole numbers.
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by apply concepts of place value and division.

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number; and multiply two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

<u>Operations and Algebraic Thinking</u>

Use the four operations with whole numbers to solve problems.
4.OA.1 Interpret a multiplication equation as a comparison, e.g. interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Gain familiarity with factors and multiples.
4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Generate and analyze patterns.
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features for the pattern that were not explicit in the rule itself. For example, given the rule "add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Measurement & Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Big Ideas:
1) Our number system is based on patterns.
2) Number patterns help us solve problems
3) Multiplication is a method of counting by equal groups of a number.
4) Models can be used to represent numbers and magnitude.
5) Multiplication shows a comparative relationship.

Essential Questions:
1) What patterns emerge from a base 10 number system?
2) How can we use patterns to solve problems?
3) What is multiplication?
4) How can we represent numbers and magnitude?

Learning Objectives	Assessments	Learning Activities
SWBAT order numbers based on magnitude. Tutoring Days: 1, 2, 3, 4, 5, 7	• Order numbers given orally • Sequence number cards • Number trail task • Show five numbers given as symbols and words on an open number line	• Hundreds chart patterns • Hundreds chart mental math • Open number lines • Place value war
SWBAT represent an addition or subtraction problem with a model. Tutoring Days: 2, 3, 4, 5	• Show addition and subtraction problem from hundreds chart mental math as arrow math and tree diagram. • Show problems with open number line.	• Hundreds chart addition and subtraction. • Bare and contextual problems - tree diagram, arrow math, open number line
SWBAT skip count forward and backward by a given number. Tutoring Days: 6, 7, 8, 9	• Orally skip count by 4's to 36. • Orally skip count backwards by 3's from 36. • Pre-test/Post test questions 1-3	• Multiplication quilt • Warm ups with quilts and math fingers • Oral practice forwards and backwards • Identify missing number in sequence • Unifix cube counting
SWBAT make a model to represent skip counting by a given number. Tutoring Days: 6, 7, 8	• Show skip counting by 3's to 21 using arrow math or open number line. • Show skip counting by 8's to 24 using arrow math or open number line. • Pre-test/Post test questions 4-5	• Arrow math, tree diagram, open number line, area model. • Unifix cube counting • Skip counting to models
SWBAT identify two factors and two multiples of a given number. Tutoring Days: 3, 6	• What are 2 multiples of 3? • What are 2 factors of 30? • List six multiples of 3, 10, 7, 9 • Pre-test/Post test questions 1-3	• Multiplication Quilts • Mystery numbers • Warm ups • Hundreds chart mental math with math fingers.

SWBAT represent a given number with a model. Tutoring Day 2, 3, 4, 6, 7, 8, 9	• Make two different models of the number 14. • Pre-test/Post test questions 6 a, b, c	• Arrow math, tree diagram, open number line, area model. • Choose numbers to show as models • Connect to multiplication quilts and factors and multiples. • Unifix cube activities
SWBAT represent a multiplicative comparison with an equation and a model. Tutoring Days: 10, 11	• Make an area model of "3 times more than 2" • 5 problems like: What is 10 times more than 7? Write as equation and the answer. • Make a model of 10 times more than 6 • Pre-test/Post test questions 7-11, 18-23.	• Unifix cube activities • Making Mazes • Problem strings • Arrow math, tree diagram, open number line, area model.

Number Patterns Unit Block Plan

Tutoring Day 1	Tutoring Day 2	Tutoring Day 3
Content Objective: -Order numbers based on magnitude. **Language Objective:** **-**Orally describe patterns found on a hundreds chart to the group. **Key Vocabulary:** Base 10, even, odd, greater than, less than **Warm Up:** Me & Math pictures - show in a picture what you think of when you think about math. **Lesson:** • Identify and describe patterns on a hundreds chart. • Hundreds chart addition and subtraction with math fingers **Assessment:** 1) Orally say 4 numbers - students write them in order (42, 23, 69, 36) **Materials needed:** Spiral notebooks, pencils, hundreds charts, chips, math fingers, dry erase pens & eraser	**Content Objective:** -Order numbers based on magnitude. -Represent an addition or subtraction problem with a model. **Language Objective:** Orally say two and three digit numbers on cards. **Key Vocabulary:** Base 10, even, odd, greater than, less than, arrow math, tree diagram **Warm Up:** 100's chart mental math with math fingers – add and subtract **Lesson:** • Introduce Arrow math & Tree diagrams with addition and subtraction • Start Hundreds Chart Quilts **Assessment:** 1) Sequence decks of number cards. (vary level of difficulty) 2) Number Trail Task 3) Students show an addition or subtraction problem using arrow math and a tree diagram - same problem two ways. **Materials needed:** Spiral notebooks, pencils, hundreds charts, chips, math fingers, dry erase pens & eraser, number card decks, Number Trail Task, construction paper, mini hundreds charts, colored pencils, glue stick	**Content Objective:** -Order numbers based on magnitude. -Represent an addition or subtraction problem with a model. **Language Objective:** Explain the difference between a factor and a multiple orally or in writing. **Key Vocabulary:** arrow math, tree diagram, open number line, factor, multiple **Warm Up:** 100's chart mental math with math fingers - addition and subtraction **Lesson:** • Introduce open number line model with warm up problems • Discuss using 10's a friendly numbers and using 10's and 1's patterns to add and subtract mentally. • Work on Hundreds Chart Quilts • Discuss factors and multiples in relation to quilts. **Assessment:** Make an open number line that shows these five numbers in order: 41, 2, thirty-four, 72, twenty-nine, 56 **Materials needed:** Spiral notebooks, pencils, hundreds charts, chips, math fingers, dry erase pens & eraser, number card decks, Number Trail Task

Tutoring Day 4	Tutoring Day 5	Tutoring Day 6
Content Objective: -Order numbers based on magnitude. -Represent a given number with a model. -Represent an addition or subtraction problem with a model. **Language Objective:** -Use words greater than and less than correctly when comparing two numbers. -Describe strategy and model for solving a contextual problem and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** arrow math, tree diagram, open number line, greater than, less than, equal **Warm Up:** Place Value War **Lesson:** • Have students show 34 + 52 and 56 - 41 as arrow math, open number line and tree diagram. • Contextual problem: *Kelle Moore won 50 games when he was quarterback for the Broncos.* *He started his last season with 38 wins and ended with 50. How many football games did he win in his last season as quarterback?* **Assessment:** Observation: ability to make models of problem **Materials needed**: Spiral notebooks, pencils, hundreds charts, dry erase pens & eraser, problem typed	**Content Objective:** -Order numbers based on magnitude. -Represent a given number with a model. -Represent an addition or subtraction problem with a model. **Language Objective:** -Describe strategy and model for solving a problem and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** model, arrow math, tree diagram, open number line **Warm Up:** Place Value War **Lesson:** • Show 69 - 37 and 42 + 26 with open number line. • Discuss different ways students made their open number lines. **Assessment:** Make an open number line that shows these five numbers in order: sixty-two, 26, sixteen, 67, 9 **Materials needed**: Spiral notebooks, pencils, hundreds charts, dry erase pens & eraser, assessment typed	**Content Objective:** -Skip count forward and backward by a given number. -Make a model to represent skip counting by a given number. -Identify two factors and two multiples of a given number. **Language Objective:** -Work with a partner to read clues about mystery numbers and figure out which number fits all the clues. -Listen to a story about skip counting. **Key Vocabulary:** skip counting, factors, multiples, model, tree diagram, arrow math, open number line **Warm Up:** Hundreds chart mystery numbers - Level 1 and 2 **Lesson:** • Read *Amanda Bean's Amazing Dream* • Skip count by different numbers using math quilts - discuss factors and multiples of different numbers (ie If we count by sixes we get to 24 so 24 is a multiple of six. Six is a factor of 24) • We can use what we know about skip counting to make models of numbers (show making a model of 15 and 24 as tree diagram, arrow math, open number line) • Have students choose numbers and show two different ways as models. **Assessment:** -Make two different models of the number 14 -What are 2 multiples of 3? What are 2 factors of 30? **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, hundreds charts, math fingers, chips, mystery number cards

Tutoring Day 7	**Tutoring Day 8**	**Tutoring Day 9**
Content Objective: -Order numbers based on magnitude. -Skip count forward and backward by a given number. -Make a model to represent skip counting by a given number. -Represent a given number with a model. **Language Objective:** -Orally describe another student's pattern made with cubes. -Orally explain how they used skip counting to count their cubes. **Key Vocabulary:** greater than, less than, skip counting, factors, multiples, model, tree diagram, arrow math, open number line **Warm Up:** Greater than/Less than Comparing Numbers Game **Lesson:** • Unifix patterns - make a pattern, copy each other's patterns • Estimate and measure objects with unifix cubes - chunk strips of cubes into bigger numbers to count them **Assessment:** -Show skip counting by 3's to 21 using arrow math or open number line -Show skip counting by 8's to 24 using arrow math or open number line **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser hundreds charts, unifix cubes, greater than/less than cards, playing cards	**Content Objective:** -Skip count forward and backward by a given number. -Make a model to represent skip counting by a given number. -Represent a given number with a model. **Language Objective:** -Discuss different ways to break up a large number. **Key Vocabulary:** skip counting, groups and members, factors, multiples, models - tree diagram, arrow math, open number line **Warm Up:** Make strip of 36 cubes. Skip count by fours to 36. Break strip into other numbers we can skip count by. **Lesson:** • Estimate and measure table together. Discuss how strip can be broken into a number to count by. What numbers would be efficient? • Write different ways of breaking up (decomposing) number as equations. Example: (9 x 10) + 6 = 96 **Assessment:** -Individual - orally skip count by 4's to 36, orally skip count backwards by 3's from 36. (Go to 2's, 5's 10's if too hard) **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser hundreds charts, unifix cubes	**Content Objective:** -Skip count forward and backward by a given number. -Represent a given number with a model. **Language Objective:** -Describe strategy and model for solving a contextual problem and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** comparison, sequence, n times larger **Warm Up:** (Use hundreds charts if needed) Fill in missing numbers in the sequence: 6, ___, 18, 24, ___, 36 3, 6, _____, 12, 15, ___, _____ 30, 25, ___, 15, ___, 5, _____ 40, 32, ___, 16, _____ What am I counting by? How did you know? **Lesson:** • Open ended – make a model of 24. Discuss • Introduce area model using cubes - show how to draw 8 x 3. Emphasize language 3 times larger than 8 is 24. **Assessment:** Observation: ability to make a model of 24. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, unifix cubes

Tutoring Day 10	Tutoring Day 11
Content Objective: -Represent a multiplicative comparison with an equation and a model. **Language Objective:** **-**Explain and compare strategies for making maze five times larger than original. **Key Vocabulary:** comparison, sequence, n times larger **Warm Up:** 1) Make an area model of 3 times more than 4 2) List six multiples of 3, 10, 7, 9 **Lesson:** • Make a maze with five turns with connecting cubes • Then make the maze 5 times larger • Discuss what happened to measurements • Relate to multiplication • Draw some of the path measurements as models and equations **Assessment:** Given orally, write equations and answers: What is 10 times more than 7? What is 10 times more than 12? What is 10 times more than 6? What is 10 times more than 14? What is 10 times more than 325? **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, unifix cubes	**Content Objective:** -Represent a multiplicative comparison with an equation and a model. **Language Objective:** Describe one way the area model is the same as or different from arrow math, open number line or tree diagram. **Key Vocabulary:** comparison, n times larger, number property, property of zero, identity property **Warm Up:** Problem Strings: 0 x 8 = 0 x 9 = 0 x 3 = 1 x 8 = 1 x 9 = 1 x 3 = 10 x 8 = 10 x 9 = 10 x 3 = Discuss properties **Lesson:** • Copy from a model of a maze on the board to cubes 3 times larger – then draw on graph paper. • With cubes - make a line of any number less than 10 – make your number six times larger, four times larger • Show as models on paper - arrow math, open number line, tree diagram • Introduce area model using cubes - show how to draw **Assessment:** What is six times more than 10? Make a model **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, connecting cubes

References for Activities in Number Patterns Unit

Math and Literature Story:

Neuschwander, C. (1998). *Amanda Bean's Amazing Dream*. New York: Scholastic Press.

Number Trail Test adapted from: Feifer, S. and De Fina, P. (2005). *The Neuropsychology of Mathematics: Diagnosis and Intervention*. Middletown, MD: School Neuropsych Press, LLC.

Hundreds Chart Quilt adapted from:

Burns, M. (1991). *Math By All Means Multiplication Grade 3.* Sausalito, CA: Math Solutions.

Mystery Numbers 100's chart problems from:

Goodman, J.M. (1992). *Group Solutions*. Berkley, CA: Lawrence Hall of Science.

Multiplication Models from:

Idaho State Department of Education (2011). *Mathematical Thinking for Instruction Workbook: 4 – 8*. Boise, ID: Idaho State Department of Education.

Materials for Activities Number Patterns Unit

Number Trail

Make a path from the smallest to the largest number.

512

62

12 764

9

322

83

72

7

111

65

Time to complete: ______

Ordering Numbers - Set 1 (Numbers on Hundreds Chart)

17	3	41	47
69	19	90	88

17	3	41	47
69	19	90	88

Ordering Numbers - Set 2 (Numbers on Hundreds Chart)

11	78	55	6
56	20	74	32

11	78	55	6
56	20	74	32

Ordering Numbers - Set 3 (with 3 digit numbers)

52	10	137	147
68	143	520	212

52	10	137	147
68	143	520	212

Ordering Numbers - Set 4 (with 3 digit numbers)

256	14	180	318
73	312	604	82

256	14	180	318
73	312	604	82

Place Value War

Objectives:

- Identify the value of digits in the ones, tens and hundreds place in three digit numbers.
- Say three digit numbers correctly.

Materials:
Playing cards, place value dice -

To make place value dice use blank cubes. Label two sides as 1, two sides as 10, and two sides as 100.

Rules:

- Remove the face cards and tens from a deck of cards and shuffle the deck. Place the deck face down in front of the two players.
- Each player draws three cards and places them on the table in any order to make a three digit number.
- Players take turns saying their three digit number correctly and help each other.
- Players take turns rolling the place value dice.
- The player with the larger number in the place value column rolled collects all the cards.
- If the numbers are equal, players draw three cards each again and roll the dice again. The player who wins on the second draw gets all the cards from both turns.
- The player with the most cards when the deck is gone is the winner of the game

Comparing Numbers Game

Objective: Compare numbers to 99 using the symbols <, > and =
Materials: Playing cards, symbol cards

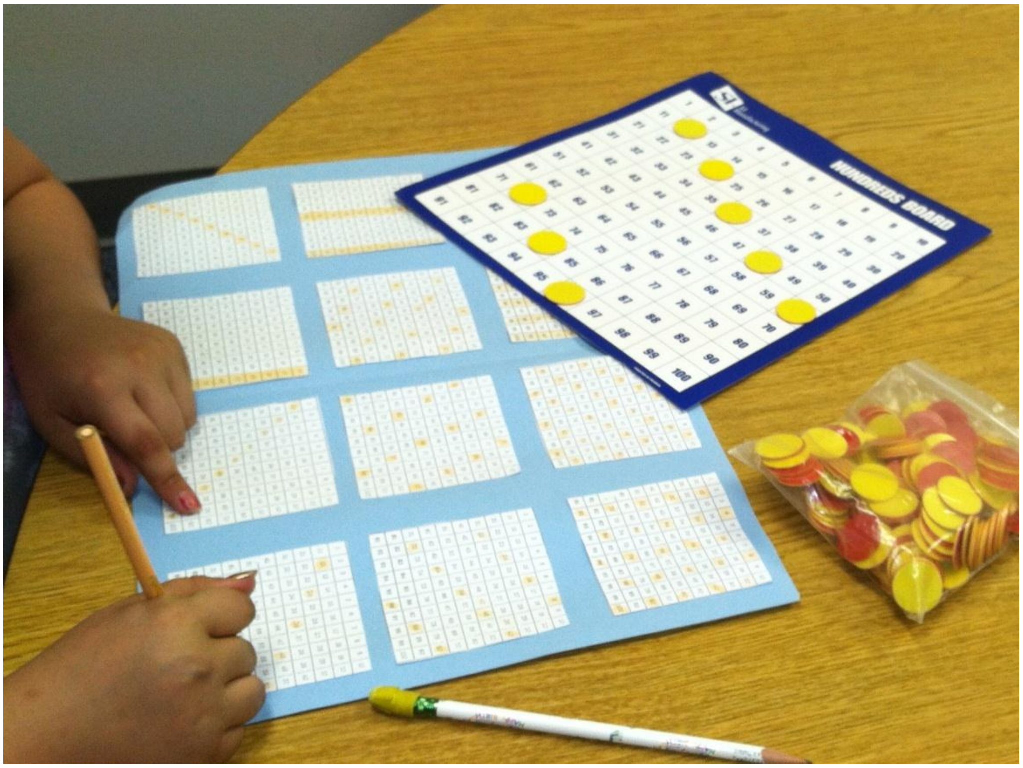
Rules:

- Remove the face cards from a deck of cards and shuffle the deck. Place the deck face down in front of the two players.
- Player one draws two cards and places them on the table in any order to make a two digit number. A ten creates a three digit number.
- Player two draws two cards and places them on the table to make his/her number.
- Players decide together which symbol card should go between the two numbers.
- Players take turns and help each other reading the number statement with the correct symbol.
- The player with the larger number collects all the cards.
- If the numbers are equal, players draw two cards each again and use the correct symbol card between them. The player with the larger number on the second draw gets all the cards from both turns.
- The player with the most cards when the deck is gone is the winner of the game.

Symbol Cards:

<	>	=
less than	greater than	equal to

Sample hundreds chart quilt:



Blackline master:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiplication Unit Overview

Unit Title: Number Patterns Grade Level: 4 Length of Unit: 16 days

Common Core Standards Addressed in Unit

Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number; and multiply two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Operations and Algebraic Thinking
Use the four operations with whole numbers to solve problems.
4.OA.1 Interpret a multiplication equation as a comparison, e.g. interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.
4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Generate and analyze patterns.
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features for the pattern that were not explicit in the rule itself. For example, given the rule "add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Big Ideas:

6) Multiplication is a method of counting by equal groups of a number.
7) A multiplication equation shows a comparative relationship.
8) A multiple can be represented as a product of its factors.
9) Basic multiplication facts can be recalled by using properties of numbers, counting strategies and related facts.
10) Division and multiplication are related operations.

Essential Questions:

5) What is multiplication?
6) How can multiplication be used to compare magnitudes?
7) How can we represent a multiplication problem?
8) What strategies can we use when we can't remember the answer to a basic fact?
9) How are multiplication and division related?

Learning Objectives	**Assessments**	**Learning Activities**
SWBAT define factor, multiple, product, commutative property, identity property, zero property. Tutoring Days: 13, 14, 23	• Pre-test/Post test questions 1 - 5 • Problem strings • Observation/discussion	• Problem strings • In context throughout unit • Label examples in spiral
SWBAT define multiplication as counting by groups of a given number. (ie 7 x 5 is 7 groups of 5) Tutoring Days: 18	• Oral response in discussion on what is multiplication? Identify examples from real world.	• *What comes in 2's, 3's, and 4's?* book • Examples of multiples from real world. • Circles & Stars • Two Colored Counters • Arrays with tiles • Multiplication quilt
SWBAT explain the commutative property, identity property, zero property of multiplication and use them to determine unknown variables in a multiplication equation. Tutoring Days: 13, 14	• Pre-test/Post test questions 12-17 • Problem Strings	• Multiplication quilt • Warm ups

SWBAT represent a contextual problem involving multiplication with a model and an equation. Tutoring Days: 12, 14, 15, 17, 19, 21, 26	• Pre-test/Post test questions 30 - 34 • Contextual Problems	• Models taught through problem solving: arrays, area model, open arrays, tree diagrams, open number lines, groups & members, pictures, repeated addition, arrow math, partial products
SWBAT represent a multiplication equation including 1 digit by 1, 2, or 3 digit numbers with a model. Tutoring Days: 12, 14, 15, 17, 18, 19, 20, 21, 25, 26	• Pre-test/Post test questions 35 - 40 • Warm Ups • Contextual Problems	• Models taught through problem solving: arrays, area model, open arrays, tree diagrams, open number lines, groups & members, pictures, repeated addition, arrow math, partial products, distributive property. • Manipulatives: tiles, pattern blocks. two colored counters
Given a model of a multiplication equation, SWBAT write an equation to match the model. Tutoring Days: 22, 24	• Pre-test/Post test questions 24 - 29 • Problems on board	• Warm ups • Problems on board • Group discussions
SWBAT solve multiplication equations including 1 digit by 1, 2, or 3 digit numbers using symbolic computation. Tutoring Days: 12, 14, 15, 17, 18, 19, 20, 21, 25, 26	• Pre-test/Post test questions 30 - 40 • Problems on board	• Progress from informal models and strategies to formal algorithm.
SWBAT use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12.	• Warm ups • Timed Basic Fact Assessments • Games - observation	• Multiplication Fact Sequence & Strategies • Multiplication Quilts • Warm Ups

Tutoring Days: 13, 14, 16, 18, 19, 21, 22, 24, 26, 27		• Basic Fact Practice Sheets
SWBAT find all factor pairs for a whole number in the range 1-100. Tutoring Days: 15, 16	• Pre-test/Post test question 6 a, b, c • Problems on board	• Multiplication quilt • Making rectangles • Arrays with tiles
SWBAT will show with a model how a multiplication equation or context shows a multiplicative comparison. (ie 7 x 5 = 35 means 35 is 7 times as many as 5) Tutoring Days: 13, 22, 23	• Observation of pattern block baggies • Problems on board • Pre-test/Post test question 7 – 11 and 18 – 23.	• Pattern Block Baggies • Oral dictation – write equation from statement. Example: 5 times more than three equals – write equation with answer.
SWBAT distinguish between a multiplicative comparison from an additive comparison. Tutoring Days: 22, 23	• Shape problems on board. Example: Draw 3 times more shapes • Pre-test/Post test question 7 – 11 and 18 – 23.	• Pattern block baggies • Two colored counters • Draw n times more shapes
SWBAT represent verbal statements of multiplicative comparisons as multiplication equations. Tutoring Days: 22, 23	• Problems on board • Teacher writes or says multiplicative comparison with words and students notate it with symbols	• Baggies of pattern blocks - use multiplication notation to show comparisons between designs after enlarging. • Two colored counters • Draw n times more shapes
SWBAT solve word problems involving multiplicative comparisons. Tutoring Days: 22, 23	• Pre-test/Post test question 32 • Contextual problems	• Models taught through problem solving: arrays, area model, open arrays, tree diagrams, open number lines, groups & members, pictures, repeated addition, arrow math, partial products, distributive property.

SWBAT explain how division and multiplication are related and make a model showing a division context. Tutoring Days: 24, 25	• Show models of simple division problems with two colored counters.	• Problem solving with division context. • Two colored counters

Multiplication Unit Block Plan

Tutoring Day 12	Tutoring Day 13	Tutoring Day 14
Content Objective: -Review from previous unit: Represent a given number with a model **Language Objective:** -Discuss differences between models. **Key Vocabulary:** model, arrow math, open number line, array, area model, tree diagram, bar model, factor, multiple **Warm Up:** Make a model of the number 24 **Lesson:** • Compare and contrast models from warm up • Make a list of models used. Help students create an example of each of the following models in their notebooks: -Arrow math -Open number line -Array -Area -Tree Diagram -Cubes -Bar **Assessment:** Observation on models student used and success with each one. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, unifix cubes	**Content Objective:** -Define factor, multiple, product, commutative property, identity property, zero property -Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. -Model how a multiplication equation or context shows a multiplicative comparison **Language Objective:** Describe the number patterns in the properties of multiplying by zero, one and 10. **Key Vocabulary:** Properties, model, property of zero, property of one, factor, multiple **Warm Up:** List 6 multiples of each number given: 3, 10, 7, 9 Problem String: 8 x 0 9 x 0 3 x 0 3 x 100 8 x 1 9 x 1 3 x 1 8 x 10 9 x 10 3 x 10 **Lesson:** • Discuss property of zero and one and pattern of tens. • Solve together 12 x 10, 6 x 10, 4 x 10, 325 x 10, 3 x 10 • Discuss language "7 times more than 10" is the same as 7 x 10.	**Content Objective:** -Define factor, multiple, product, commutative property, identity property, zero property -Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. - Represent a contextual problem involving multiplication with a model and an equation. **Language Objective:** Describe strategy and model for solving a contextual problem and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** Properties, model, property of zero, property of one, factor, multiple **Warm Up:** Solve these: 8 x 1 3 x 0 3 x 11 4 x 10 5 x 1 10 x 10 9 x 0 7 x 0 8 x 10 1 x 6 2 x 10 6 x 11 **Lesson:** • Discuss patterns we know with 0's, 1's, 10's and 11's. Color on Multiplication Sequence Color Sheet • Discuss pattern of fives and color on sheet • Practice fives with wrap ups and worksheet.

	• Have students show with a model. **Assessment:** What is 6 times more than 10? Show with a model and an equation. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser,	• Solve this problem and show your answer with a model: *Jane wanted to bring five friends bowling for her birthday party. It costs $8 for each kid to bowl. How much money will I need for Jane's bowling party?* **Assessment:** Observe how each student approaches and solves bowling problem. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, fives wrap ups, multiplication sequence worksheet, fives work sheet,

Tutoring Day 15	**Tutoring Day 16**	**Tutoring Day 17**
Content Objective: **-** Represent a contextual problem involving multiplication with a model and an equation. - Find all factor pairs for a whole number in the range 1-100. **Language Objective:** Describe strategy and model for solving a contextual problem and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** model, factor, multiple, array **Warm Up:** Solve and show your answer with a model: *Mrs. Compton gave four children five cans of food for a food drive. How many cans did they have all together?* **Lesson:** • Discuss models and solutions to warm up. Show the problem modeled with tiles. • Have students make an array of the following facts with tiles: 6 x 3 4 x 4 12 x 2 • After they have one, have them arrange each	**Content Objective:** **-** Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. - Find all factor pairs for a whole number in the range 1-100. **Language Objective:** Orally explain the difference between a factor and a multiple. **Key Vocabulary:** number properties, even, odd, factor, multiple, factor rainbow, array **Warm Up:** Problem Strings 5 x 2 5 x 3 5 x 4 5 x 5 5 x 8 5 x 7 5 x 10 5 x 9 Make an array of 5 x 6 **Lesson:** • Discuss differences between multiplying an even versus an odd number by five from warm up. • Give each student a different number: 18, 12, 24, 32 (Differentiate by ability)	**Content Objective:** **-** Represent a contextual problem involving multiplication with a model and an equation. **Language Objective:** -Describe strategy and model for solving a contextual problem and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** Model, factor, multiple **Warm Up:** Solve and show your answer with two different models: *Pudding comes in packs of 4. If I buy 2 packs of vanilla and 3 packs of chocolate how many pudding cups do I have?* **Lesson:** • Discuss strategies and models used to solve warm up problem. Compare and contrast models. • Eat pudding! **Assessment:** Observe how each student approaches and solves pudding problem. **Materials needed:** Spiral notebooks, pencils, dry

number into a different array and say the new fact. • Have them pick a number. Show as two different arrays and write corresponding equations. • Practice 5’s and 2’s with wrap ups. **Assessment:** Observe how each student approaches and solves food drive problem. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, tiles, wrap ups 5’s and 2’s	• Make as many different arrays as they can for their number with tiles • Show how their arrays can turn into factor rainbows – use to define and discuss difference between a factor and a multiple. **Assessment:** Orally explain the difference between a factor and a multiple. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, tiles	erase pens & eraser, pudding cups, spoons, tiles

Tutoring Day 18	Tutoring Day 19	Tutoring Day 20
Content Objective: -Define multiplication as counting by groups of a given number. -Represent a multiplication equation including 1 digit by 1, 2, or 3 digit numbers with a model. -Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. **Language Objective:** Explain an example of something in life that comes in 5’s, 6’s, 7’s or 8’s. **Key Vocabulary:** set model, groups, members, factor, multiple **Warm Up:** Practice fives with dice. Multiply whatever number you roll with 2 dice by 5. Do at least 10 problems. **Lesson:** • Read book what comes in 2’s, 3’s and 4’s • Brainstorm what comes in 5’s, 6’s, 7’s, 8’s • Discuss 5 x 6 as 5 groups	**Content Objective:** -Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. - Represent a contextual problem involving multiplication with a model and an equation. -Represent a multiplication equation including 1 digit by 1, 2, or 3 digit numbers with a model. **Language Objective:** -Describe strategy and model for solving a contextual problem and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** Set model, partial product model **Warm Up:** Problem Strings 3 x 2 6 x 2 4 x 2 8 x 2 3 x 4 6 x 4 4 x 4 8 x 4	**Content Objective:** -Represent a multiplication equation including 1 digit by 1, 2, or 3 digit numbers with a model. **Language Objective:** -Describe strategy and model for solving an equation and discuss comparisons between different solution strategies and models of other students. **Key Vocabulary:** area model, open array **Warm Up:** Make a model of 32 x 3 **Lesson:** • Share and discuss models from warm up • Guide students to make area model with 32 x 3 on graph paper. • Demonstrate how to make an open array with same problem. **Assessment:** Make an open array of 6 x 64 and 324 x 3.

with 6 members – show set model. • Play Circles and Stars game • Discuss how set model can be used for 2 digit numbers. Have students try different two digit numbers and explain models and answers to each other. **Assessment:** Show 3 x 12 with a set model. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, book, dice	**Lesson:** • Discuss doubling relationship of 2's and 4's in warm up. Color on Multiplication Sequence coloring sheet. • Solve the problem and show with a model and an equation: *Zingers come in packs of 12. How many zingers would I have if I buy 7 packs.* • Share models and strategies. Emphasize set model from yesterday. Eat zingers! • Lead to showing partial product method of multiplying 1 by 2 digit with 22 x 6 . **Assessment:** Use partial product method to solve: Make a model if needed. 24 x 5 14 x 6 32 x 4 **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, box of zingers	**Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, graph paper, colored pencils

Tutoring Day 21	**Tutoring Day 22**	**Tutoring Day 23**
Content Objective: **-** Represent a contextual problem involving multiplication with a model and an equation. **-**Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. **-**Represent a multiplication equation including 1 digit by 1, 2, or 3 digit numbers with a model. **Language Objective:** -Describe strategy and model for solving a contextual problem and discuss comparisons between different solution strategies and models of other students.	**Content Objective:** **-**Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. -Model how a multiplication equation or context shows a multiplicative comparison. (ie 7 x 5 = 35 means 35 is 7 times as many as 5) -Distinguish between a multiplicative comparison and an additive comparison. **Language Objective:** Answer the question what is the difference between 7 x 3 and 7 + 3?	**Content Objective:** -Distinguish between a multiplicative comparison and an additive comparison. -Define factor, multiple, product -Model how a multiplication equation or context shows a multiplicative comparison. **Language Objective:** Write an explanation of the difference between a multiple and a factor. **Key Vocabulary:** Multiplicative comparison, multiple, factor, product **Warm Up:** What is 6 times more than 4? What is 10 times more than 7? What is 2 times more than 8?

Key Vocabulary: Distributive property, doubling, partial product, open array **Warm Up:** Problem Strings: 5 x 2 9 x 2 8 x 10 20 x 10 5 x 4 9 x 4 10 x 10 3 x 10 18 x 10 23 x 10 **Lesson:** • Discuss doubling strategy for 2's and 4's from warm up – also works for 3's and 6's. • Relate distributive property and partial product method to problems in warm up. • Solve the problem and show with a model and an equation: *Walmart Math: Bottled water comes in cases of 24. If I buy 4 cases, how many bottles will I have? If each case costs $3.25, how much will I spend?* • Share models and strategies. Emphasize open array and partial product models from yesterday **Assessment:** Solve 48 x 6 and 73 x 2. Show answers as numbers and as a model. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser,	**Key Vocabulary:** multiplicative comparison, number properties, model **Warm Up** Models on board. Write equation to match. **Lesson:** • Practiced all the facts worksheet without timing. • Give each student a different number of two colored counters. Show number with red side. With yellow side showing, help them make 4 times more yellow chips than red chips. • Discuss the difference between adding 3 and multiplying by 3. **Assessment:** With two colored counters, show 6 times larger than 3 **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, models worksheet for warm up, two colored counters	List 4 multiples of 11. List 4 multiples of 3. What are 2 factors of 24? Draw 3 times more squares: □□ **Lesson:** • In notebooks, write an explanation of the difference between a multiple and a factor. • Give students baggies with different amounts of pattern blocks inside. On the baggies, write different numbers 2 – 9. Have the students make a pattern block design that is the number on the bag times larger than the blocks in the bag. **Assessment:** Observation of enlarging pattern block designs. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, pattern blocks, pattern block baggies made

Tutoring Day 24	**Tutoring Day 25**	**Tutoring Day 26**
Content Objective: -Explain how division and multiplication are related and make a model showing a division context. -Given a model of a multiplication equation, write an equation to match the model.	**Content Objective:** -Explain how division and multiplication are related and make a model showing a division contextual problem. -Represent a multiplication equation including 1 digit by 1, 2, or 3 digit numbers with a	**Content Objective:** -Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. - Represent a contextual problem involving multiplication with a model and an equation.

-Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12. **Language Objective:** Participate in a discussion on a division contextual problem and explain the relationship between multiplication and division. **Key Vocabulary:** division, opposite, fact family **Warm Up:** Models on board. Write equation to match. **Lesson:** • Show students a share size package of starburst (from Walmart☺). Estimate and then count number of candies in package. • There are 20 candies in a share size package of starburst. How can I divide these evenly among four students? • Discuss relationship of multiplication and division with this context. Show with equations: 4 x 5 = 20, 20 ÷ 4 = 5 • Do the following with two colored counters: 15 ÷3, 15 ÷5, 12 ÷2, 12 ÷6, 27 ÷9, 27 ÷3 Discuss opposite multiplication fact. **Assessment:** Timed multiplication fact assessment. See how far they get in two minutes, then let them finish without timer. **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, quick quiz, share size Starburst, two colored counters, multiplication fact test	model. **Language Objective:** Participate in a discussion on a division contextual problem and explain the relationship between multiplication and division. **Key Vocabulary:** division, opposite, fact family, names of models **Warm Up:** For my foldable, my paper was 12 inches wide. I needed six sections. How many inches should each section be? Make a model and write an equation showing your solution. **Lesson:** • Make a foldable to review and make examples of the models for multiplication learned in tutoring. Include arrow math, open number line, area model, set model, open array, tree diagram. Show an example with both a one digit by one digit and one digit by two digit equation. **Assessment:** Foldable **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, construction paper, pens, scissors	**Language Objective:** Share a strategy for computing one tricky math fact. **Key Vocabulary:** Tricky fact, mental strategy, related fact, names of models **Warm Up:** Solve the problem and show with a model and an equation: *Walmart Math: I bought 4 packs of Starburst that had 16 candies in each pack. How many candies did I have all together?* **Lesson:** • Discuss warm up problem strategies and models and explain related division problem. • Practice timed multiplication fact assessment – discuss each student's tricky facts and strategies for solving them. • Finish foldable **Assessment:** Multiplication Fact Assessment **Materials needed:** Spiral notebooks, pencils, dry erase pens & eraser, 100's chart quilts, multiplication fact assessment

Tutoring Day 27

Content Objective:
-Use properties of numbers, counting strategies and related facts to compute basic multiplication facts up to 12 x 12.
Language Objective:
Explain orally how a picture they draw represents their feelings about mathematics after participating in tutoring.
Key Vocabulary:
factors, multiple
Warm Up:
Multiplication War Card Game (multiplication fact practice)
Lesson:

- Ask students to draw a new picture of how they feel about math now.
- Ask students to describe how participating in tutoring has helped them. Note answers.

Assessment:
Observation of fact fluency while playing Multiplication War.
Materials needed:
Spiral notebooks, pencils, dry erase pens & eraser, colored pencils, playing cards, 100's chart quilts

References for Activities in Multiplication Unit

Math and Literature Story:

Aker, S. (1990). *What Comes in 2's, 3's, & 4's?* New York, NY: Simon & Schuster.

Circles and Stars game from:

Burns, M. (1991). *Math By All Means Multiplication Grade 3.*Sausalito, CA: Math Solutions.

Multiplication models from:

Idaho State Department of Education (2011). *Mathematical Thinking for Instruction Workbook: 4-8*. Boise, ID: Idaho State Department of Education.

Multiplication sequence coloring sheet based on:

Van de Walle, J., Karp, K., & Bay-Williams, J. (2010) *Elementary and Middle School Mathematics: Teaching Developmentally 7th Edition.* Boston, MA: Allyn and Bacon.

Fives worksheet from:

Stuart, M.W. (1996). *10 Days to Multiplication Mastery.* Ogden, UT: Learning Wrap-ups Inc.

Materials for Activities Multiplication Unit
Multiplication Sequence Coloring Sheet

Multiplication Facts

- ☐ Doubles – Square numbers
- ☐ Addition Doubles (2's)
- ☐ Zeros & Ones
- ☐ Fives
- ☐ Tens
- ☐ Nifty Nines
- ☐ Elevens
- ☐ Use Related Fact Strategies
  - Turn around facts
  - Double and double again
  - Double and one more set
  - Half then double
  - Helping fact – plus or minus one set

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Adapted from:Van De Walle J.A., Karp, K.S., & Bay- Williams, J.M. (2010) *Elementary and Middle School Mathematics: Teaching Developmentally*. Boston: Allyn & Bacon. 177 – 182.

Multiplication War

Objectives:

- Practice basic multiplication facts up to 10 x 10

Materials: Playing cards

Number of players: 2 or 3

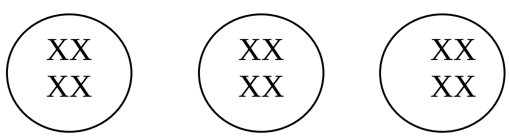
Rules:

- Remove the face cards from a deck of cards and shuffle the deck. Deal all of the cards out among the players
- Each player puts down two cards and says the answer to the fact after multiplying the digits on the two cards together.
- The player with the larger answer collects all the cards.
- If the numbers are equal, players have a war. Each player puts down two more cards and says the product of the two cards. The player who wins on the second round gets all the cards from both turns.
- The player with the most cards after the allotted time for the game wins or the player who wins all of the cards.
- When players use all of their cards, they shuffle their win pile and keep playing.

Day 22 Warm Up

Write the equation that goes with each model:

1.



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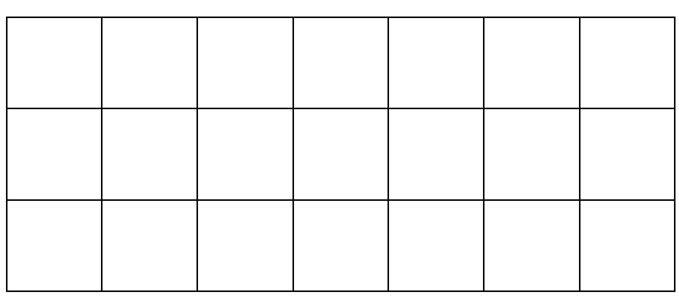
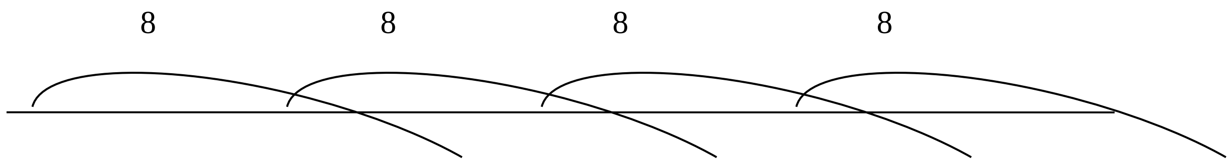


256

3.



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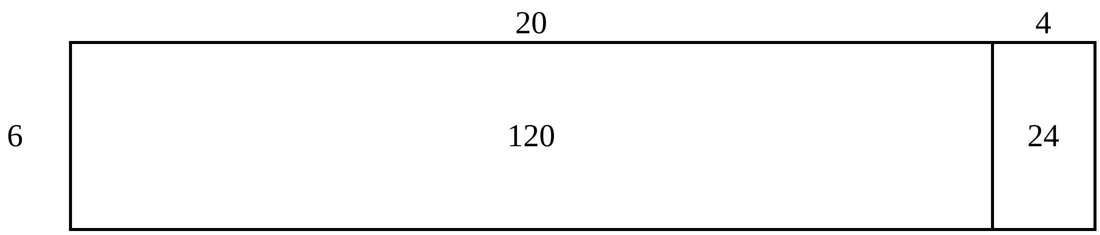
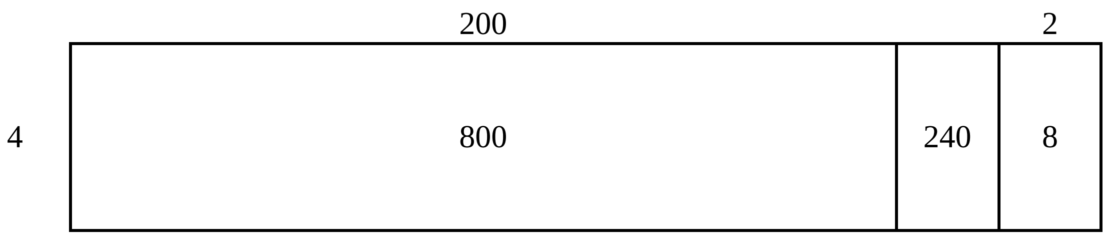
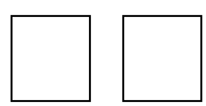


5.

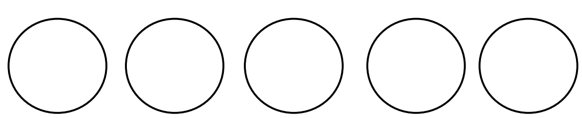


Day 24 Warm Up

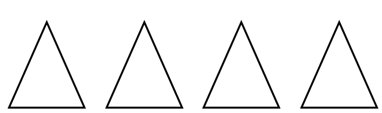
1. Draw 3 times more shapes:



2. Draw 4 times more shapes:

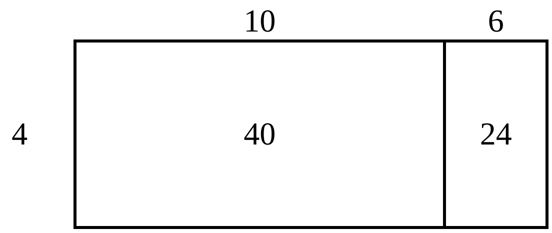


3. Draw 6 times more shapes:

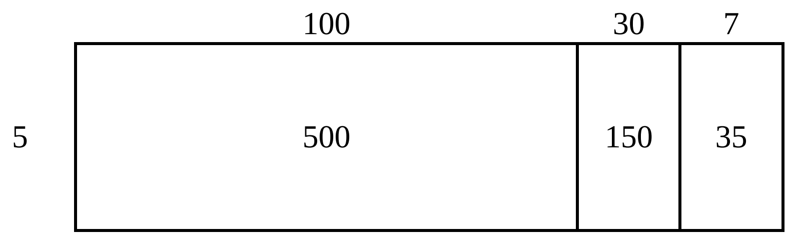


What equation goes with these models?

4.



5.



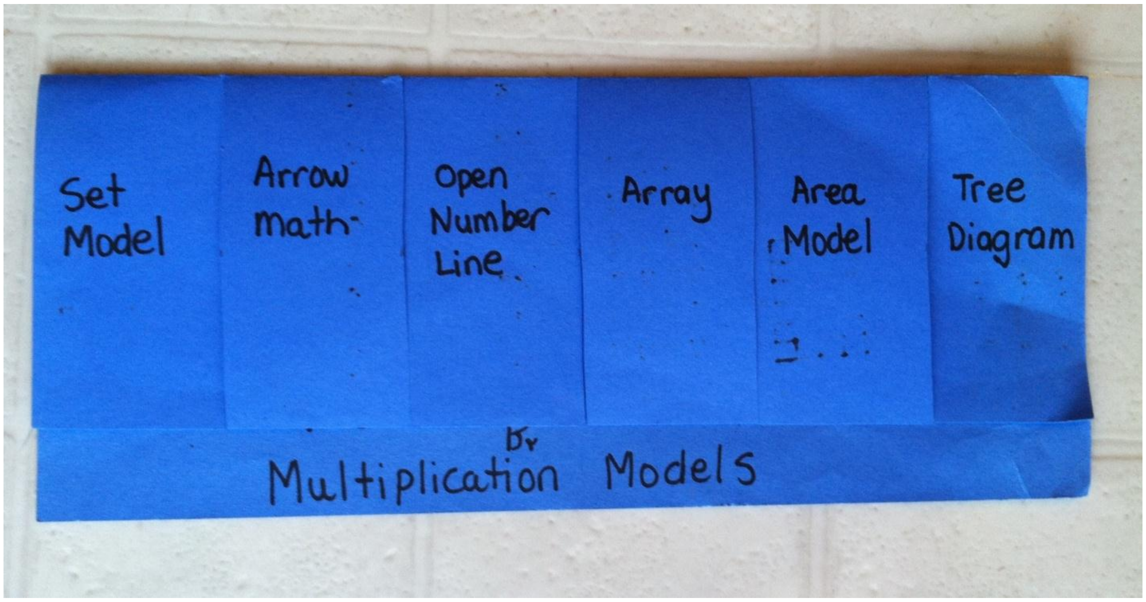
Solve and make a model of the following equations:

6. 62 x 4 =

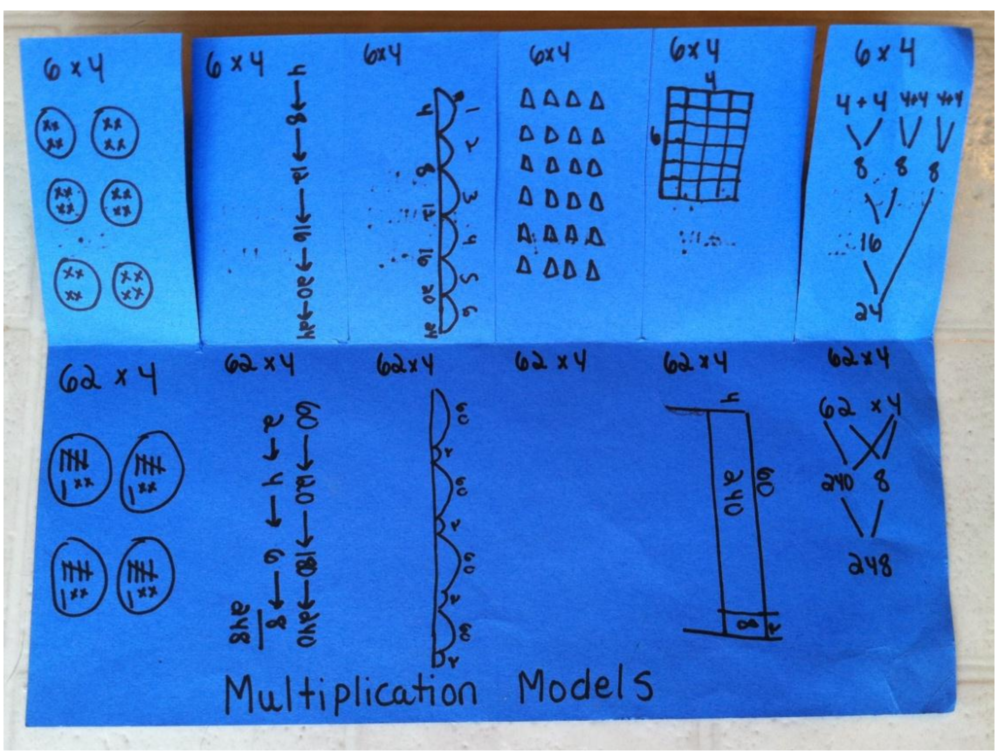
7. 216 x 3 =

Sample Flip Book

Closed



Open



APPENDIX C

Class Assessment Data

Data Collection Sheet - Multiplication Pretest

Student	Skip Counting	Arrow or Open # Line	Models of Number	n times larger with model	Mult Properties	Drawing Shapes n times larger	n times more shapes	Models to Equations 1 digit	Models to Equations 2 digit	Problem Solving	Equations to Models	Average Score	# of skills proficient	Mult Facts
1	4	1	1	4	4	4	3	4	0	1	3	2.636	7	33
Destiny	4	1	2	4	4	2	2	3	2	2	3	2.636	5	32
3	3	3	0	4	4	4	2	2	3	1	3	2.636	7	27
4	4	1	0	4	4	4	4	3	1	3	3	2.818	8	32
5	3	1	3	4	4	4	4	4	1	3	3	3.091	9	45
6	4	1	3	2	4	4	2	2	1	4	3	2.727	6	64
7	4	4	3	4	4	4	2	4	3	4	3	3.545	10	51
8	3	0	3	0	4	3	2	4	3	3	4	2.636	8	47
9	2	4	3	4	4	3	3	3	1	3	3	3.000	9	45
Happy Gilmore	2	0	1	0	2	1	2	1	1	1	1	1.091	0	23
11	3	0	0	4	3	4	2	4	0	2	2	2.182	5	38
12	4	1	1	2	3	4	2	1	2	3	2	2.273	4	46
13	4	1	3	4	4	4	4	4	3	3	3	3.364	10	44
Big Jay	4	1	1	2	4	4	3	2	1	3	1	2.364	5	53
15	4	4	3	4	4	3	4	4	2	4	2	3.455	9	67
16	4	1	3	4	2	4	4	4	2	4	2	3.091	7	41
17	4	0	0	3	4	4	2	4	3	2	2	2.545	6	60
18	3	0	2	4	4	4	3	2	2	4	2	2.727	6	38
19	4	1	3	2	4	1	2	2	0	3	2	2.182	4	25
20	4	1	3	1	4	2	2	4	2	3	4	2.727	6	64
Katarina	4	1	0	3	3	1	2	2	1	2	2	1.909	2	23
Average	3.571	1.286	1.810	3.000	3.667	3.238	2.667	3.000	1.619	2.762	2.524	2.649	6.333	43.750
Stand Dev	0.676	1.309	1.289	1.378	0.658	1.136	0.856	1.095	1.024	0.995	0.814	0.555	2.556	13.198

Scoring Rubric: 4 Full Accomplishment; 3 Substantial Accomplishment; 2 Partial Accomplishment; 1 Little Accomplishment

Data Collection Sheet - Multiplication Post Test

Student	Skip Counting	Arrow or Open # Line	Models of Number	n times larger with model	Mult Properties	Drawing Shapes n times larger	n times more shapes	Models to Equations 1 digit	Models to Equations 2 digit	Problem Solving	Equations to Models	Average Score	# of skills proficient	Mult Facts
1	4	3	3	4	4	4	3	3	0	4	3	3.182	10	46
Destiny	4	3	4	4	4	3	3	3	4	3	4	3.545	11	50
3	3	4	0	4	4	4	2	3	1	4	3	2.909	5	32
4	4	4	0	4	4	4	2	3	1	4	3	3.000	8	46
5	3	3	3	4	4	3	4	4	2	4	3	3.364	10	48
6	4	3	4	2	4	3	4	4	4	4	3	3.545	10	66
7	4	4	3	4	4	3	4	4	3	4	2	3.545	10	56
8	0	0	0	1	4	4	2	4	0	4	4	2.091	5	64
9	4	4	3	4	4	4	2	4	1	3	3	3.273	9	39
Happy Gilmore	4	3	1	2	3	3	2	4	1	2	2	2.455	5	27
11	4	3	0	3	0	3	4	3	0	3	3	2.364	8	32
12	4	3	1	2	4	4	2	2	1	4	3	2.727	6	54
13	4	4	3	2	4	3	2	4	4	4	3	3.364	8	47
Big Jay	4	3	4	4	4	4	4	4	2	3	3	3.545	10	55
15	4	4	3	4	4	4	4	4	1	4	3	3.545	10	65
16	1	3	3	4	4	4	2	4	2	4	3	3.091	8	46
17	4	3	3	4	4	3	2	4	1	3	3	3.000	9	55
18	4	4	3	4	4	3	3	2	1	4	0	3.000	8	41
19	4	3	1	2	4	1	2	4	1	2	4	2.545	5	28
20	2	3	3	1	4	4	4	4	1	4	3	3.000	8	58
Katarina	1	1	0	1	2	1	2	1	1	2	1	1.182	0	23
Average	3.333	3.095	2.143	3.048	3.667	3.286	2.810	3.429	1.524	3.476	2.810	2.965	7.762	47.750
Stand Dev	1.238	0.995	1.493	1.203	0.966	0.902	0.928	0.870	1.250	0.750	0.928	0.590	2.625	11.845

Scoring Rubric: 4 Full Accomplishment; 3 Substantial Accomplishment; 2 Partial Accomplishment; 1 Little Accomplishment

Data Collection Sheet - Division Pretest								
Student	Making Groups	Dividing Shapes	Models to Equatio ns	n times smaller with model	Problem Solving from context	Equatio ns to Models	Average Score	# of skills proficie nt
1	3	4	3	4	3	4	3.500	6
Destiny	2	2	1	1	1	3	1.667	1
3	2	4	1	1	1	0	1.500	1
4	2	4	3	2	3	3	2.833	4
5	3	4	4	1	2	2	2.667	3
6	3	3	4	3	3	4	3.333	6
7	4	4	3	3	4	4	3.667	6
8	3	3	1	3	3	3	2.667	5
9	4	4	2	0	2	2	2.333	2
Happy Gilmore	1	3	2	1	1	1	1.500	1
11	4	4	2	1	1	1	2.167	2
12	3	2	1	1	1	3	1.833	2
13	3	4	3	4	4	3	3.500	6
Big Jay	4	2	0	0	2	1	1.500	1
15	3	4	3	2	4	3	3.167	5
16	4	3	1	2	3	3	2.667	4
17	3	3	3	3	3	3	3.000	6
18	4	4	3	4	3	3	3.500	6
19	1	4	2	2	2	2	2.167	1
20	4	3	2	1	2	3	2.500	3
Katarina	1	3	1	2	1	2	1.667	1
Average	2.905	3.381	2.143	1.952	2.333	2.524	2.540	3.429
Stand Dev	1.044	0.740	1.108	1.244	1.065	1.078	0.741	2.087

Scoring Rubric: 4 Full Accomplishment; 3 Substantial Accomplishment; 2 Partial Accomplishment; 1 Little Accomplishment

Data Collection Sheet - Division Posttest									
Student	Making Groups	Dividing Shapes	Models to Equations	n times smaller with model	Problem Solving from context	Equations to Models	Average Score	# of skills proficient	
1	4	4	2	1	3	3	2.833	4	
Destiny	3	3	2	4	3	3	3.000	5	
3	4	4	2	1	1	0	2.000	2	
4	3	4	1	2	1	0	1.833	2	
5	4	4	2	1	3	2	2.667	3	
6	3	3	3	1	4	4	3.000	5	
7	4	4	4	4	4	4	4.000	6	
8	4	4	1	3	3	3	3.000	5	
9	4	3	2	2	2	3	2.667	3	
Happy Gilmore	2	4	2	2	1	1	2.000	1	
11	3	3	2	2	1	0	1.833	2	
12	3	2	2	3	3	3	2.667	4	
13	4	4	4	4	3	3	3.667	6	
Big Jay	3	4	2	1	3	0	2.167	3	
15	3	4	4	1	4	3	3.167	4	
16	3	4	3	4	4	3	3.500	6	
17	4	3	3	3	3	2	3.000	5	
18	3	4	3	4	3	3	3.333	6	
19	4	4	1	2	2	1	2.333	2	
20	3	3	2	1	3	4	2.667	4	
Katarina	1	1	1	1	1	0	0.833	0	
Average	3.250	3.450	2.300	2.300	2.600	2.100	2.667	3.700	
Stand Dev	0.786	0.826	0.979	1.218	1.095	1.483	0.747	1.809	

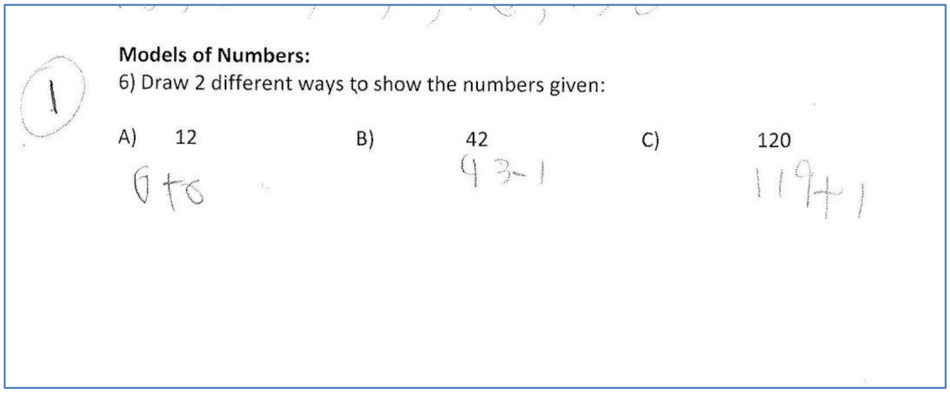
Scoring Rubric: 4 Full Accomplishment; 3 Substantial Accomplishment; 2 Partial Accomplishment; 1 Little Accomplishment

APPENDIX D

Student Work Samples

Big Jay Work Samples

Models of numbers pre-test



1

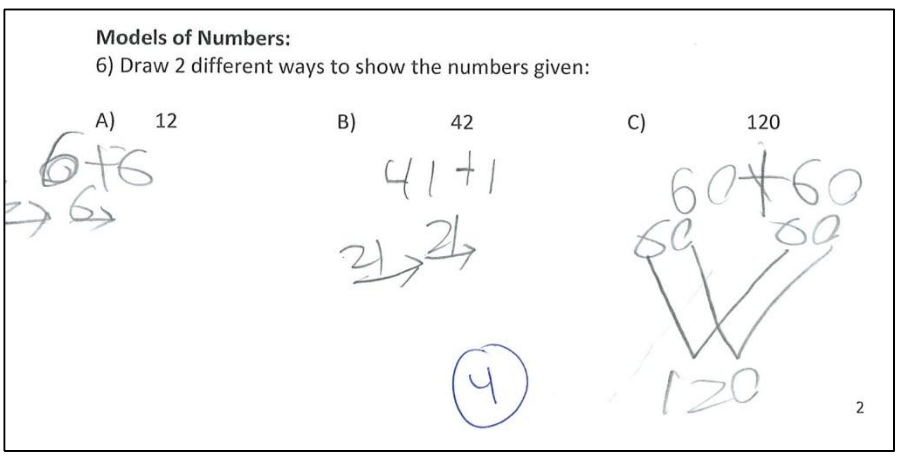
Models of Numbers:

6) Draw 2 different ways to show the numbers given:

A) 12 B) 42 C) 120

6+6 43-1 119+1

Models of numbers post test



Models of Numbers:

6) Draw 2 different ways to show the numbers given:

A) 12 B) 42 C) 120

6+6 41+1 60+60

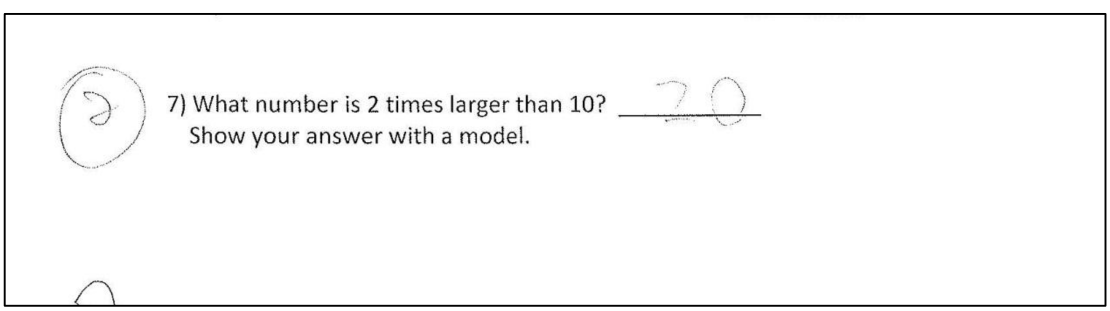
6 21 21 60 60

120

4

2

n times larger pre-test

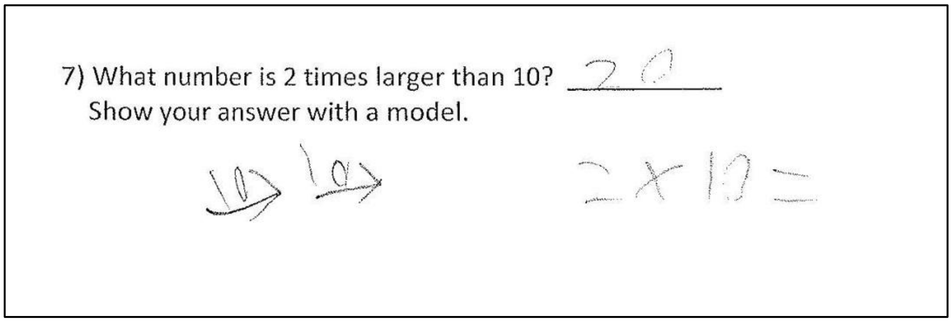


2

7) What number is 2 times larger than 10? 20

Show your answer with a model.

n times larger post test



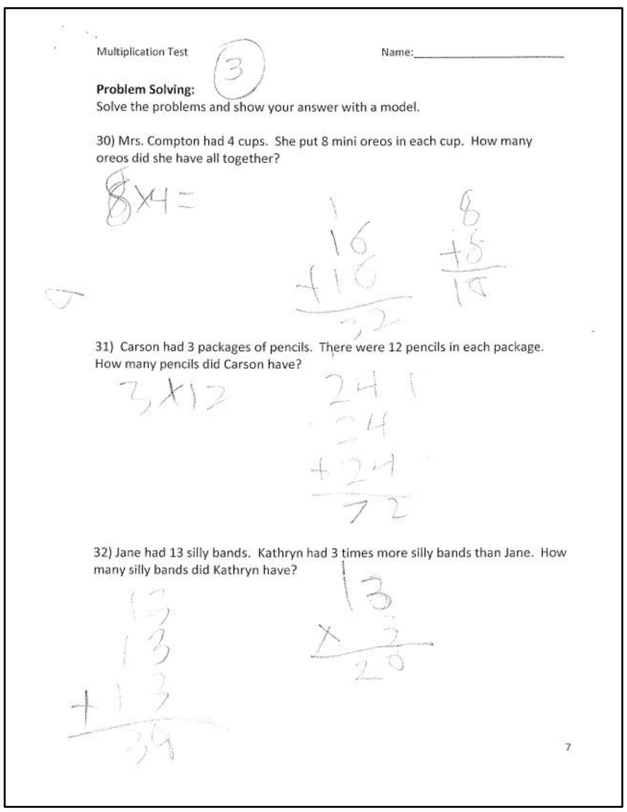
7) What number is 2 times larger than 10? 20

Show your answer with a model.

10 10

2 x 10 =

Multiplication problem solving pre test



Multiplication Test 3 Name:

Problem Solving:

Solve the problems and show your answer with a model.

30) Mrs. Compton had 4 cups. She put 8 mini oreos in each cup. How many oreos did she have all together?

8x4=

16 + 16 = 32

8 + 8 = 16

31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

3x12

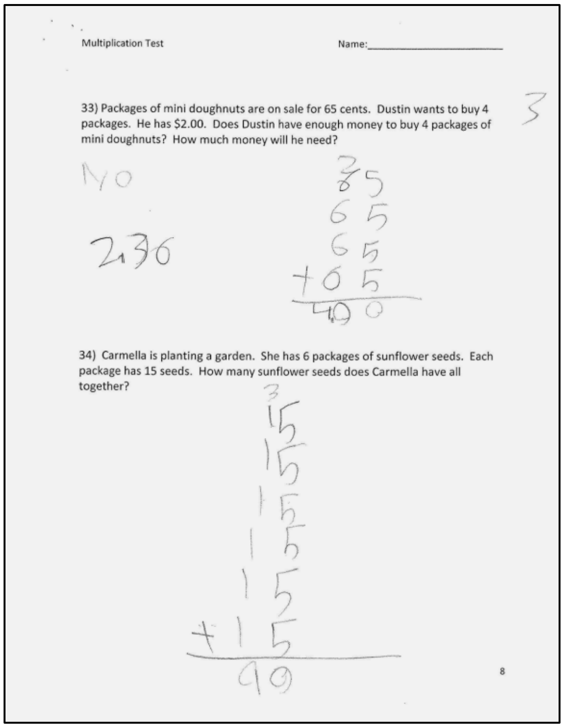
24 + 24 + 24 = 72

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

13 + 13 + 13 = 39

13 x 3 = 20

7



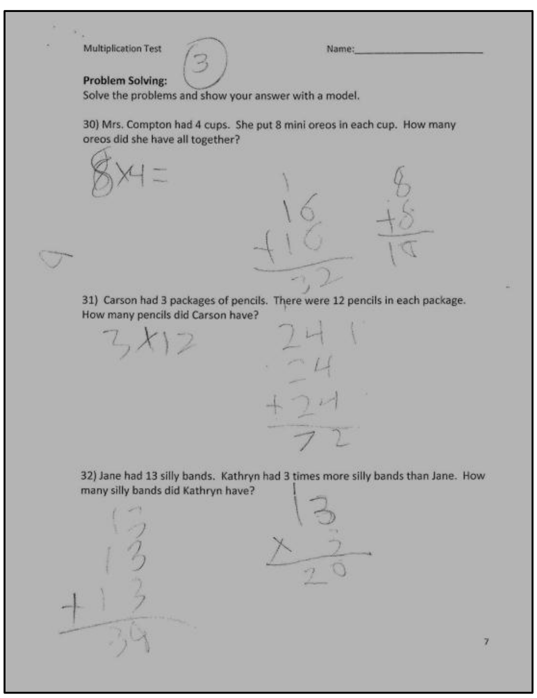
Multiplication Test Name:______________________

33) Packages of mini doughnuts are on sale for 65 cents. Dustin wants to buy 4 packages. He has $2.00. Does Dustin have enough money to buy 4 packages of mini doughnuts? How much money will he need?

34) Carmella is planting a garden. She has 6 packages of sunflower seeds. Each package has 15 seeds. How many sunflower seeds does Carmella have all together?

8

Multiplication problem solving post test



Multiplication Test Name:______________________

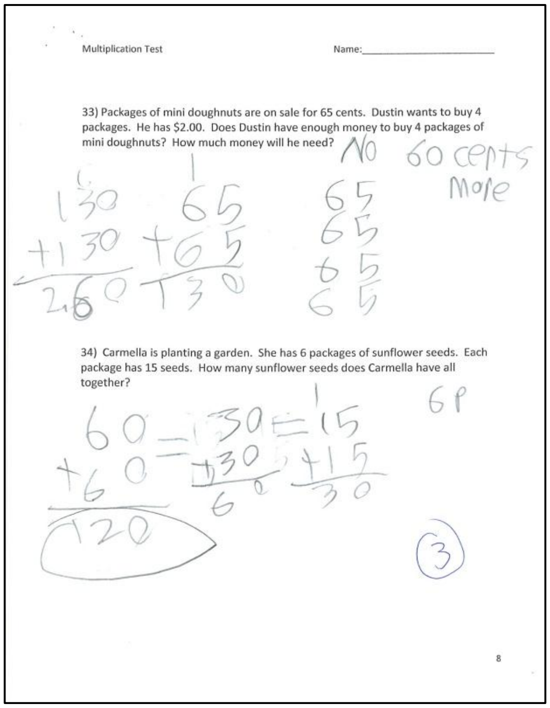
Problem Solving:
Solve the problems and show your answer with a model.

30) Mrs. Compton had 4 cups. She put 8 mini oreos in each cup. How many oreos did she have all together?

31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

7



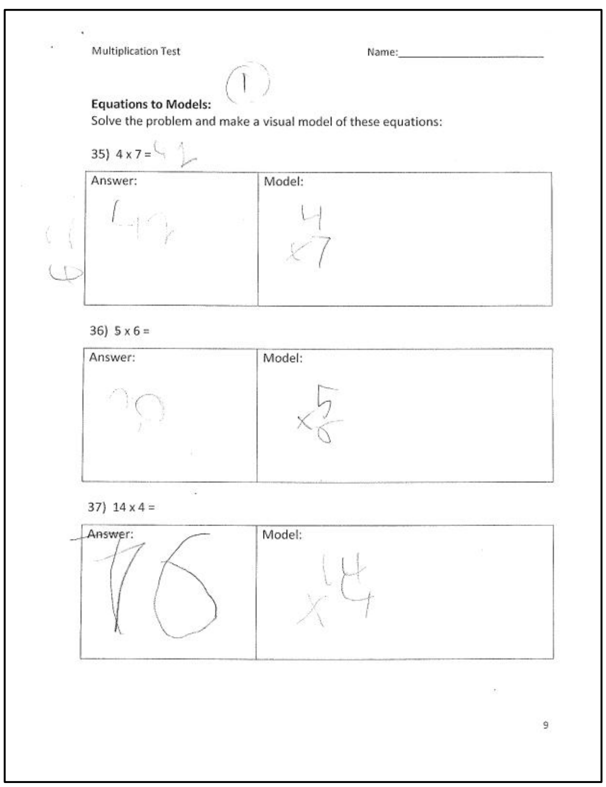
Multiplication Test Name:______________________

33) Packages of mini doughnuts are on sale for 65 cents. Dustin wants to buy 4 packages. He has $2.00. Does Dustin have enough money to buy 4 packages of mini doughnuts? How much money will he need? No 60 cents more

34) Carmella is planting a garden. She has 6 packages of sunflower seeds. Each package has 15 seeds. How many sunflower seeds does Carmella have all together?

8

Multiplication equations to models pre-test



Multiplication Test Name:______________________

Equations to Models:
Solve the problem and make a visual model of these equations:

35) 4 x 7 =

Answer:	Model:

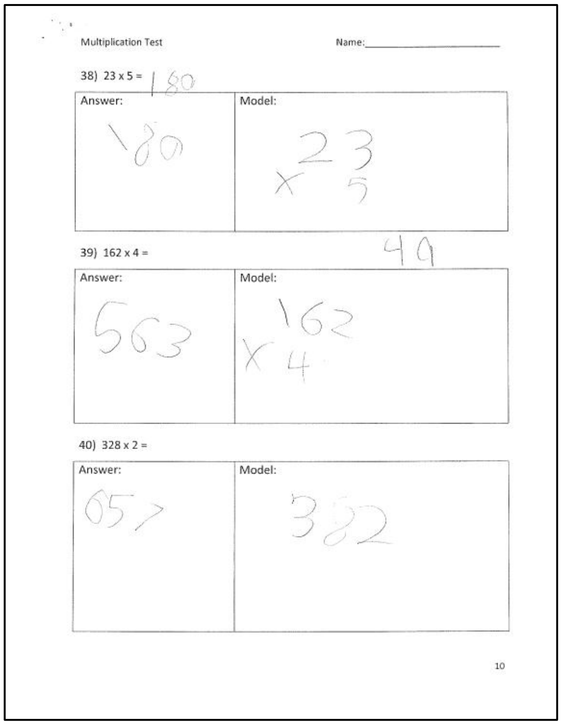
36) 5 x 6 =

Answer:	Model:

37) 14 x 4 =

Answer:	Model:
16	

9



Multiplication Test Name:____________________

38) 23 x 5 = 180

Answer:	Model:
180	23 x 5

39) 162 x 4 = 49

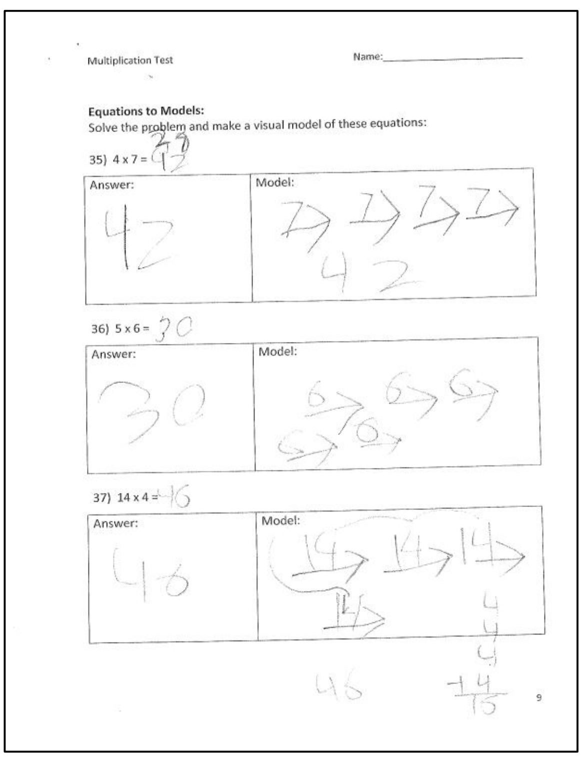
Answer:	Model:
563	162 x 4

40) 328 x 2 =

Answer:	Model:
657	382

10

Multiplication equations to models post test



Multiplication Test Name:____________________

Equations to Models:
Solve the problem and make a visual model of these equations:

35) 4 x 7 = 42

Answer:	Model:
42	7> 7> 7> 7> 42

36) 5 x 6 = 30

Answer:	Model:
30	6> 6> 6> 6> 6>

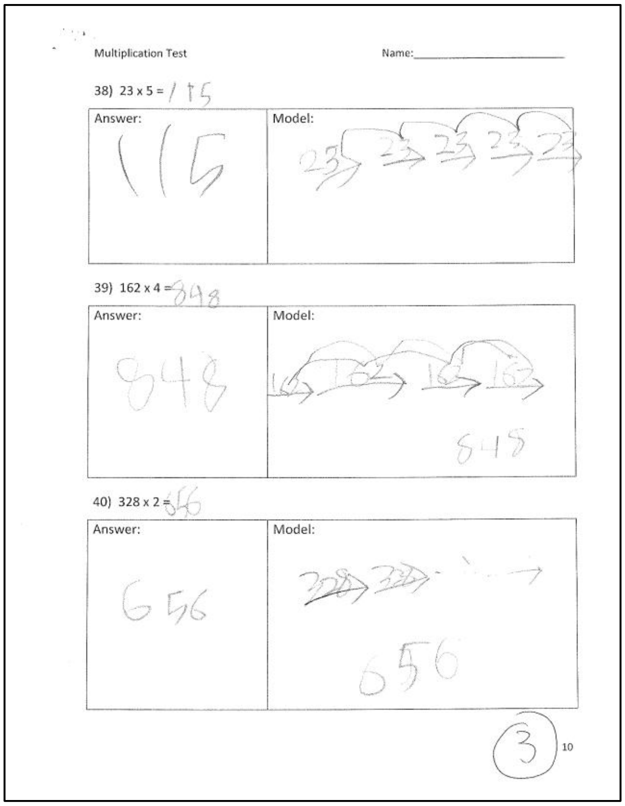
37) 14 x 4 = 46

Answer:	Model:
46	14> 14> 14> 14>

46

4
4
4
+4
15

9


Multiplication Test Name:____________________

38) 23 x 5 = 115

Answer:	Model:
115	23 23 23 23 23

39) 162 x 4 = 848

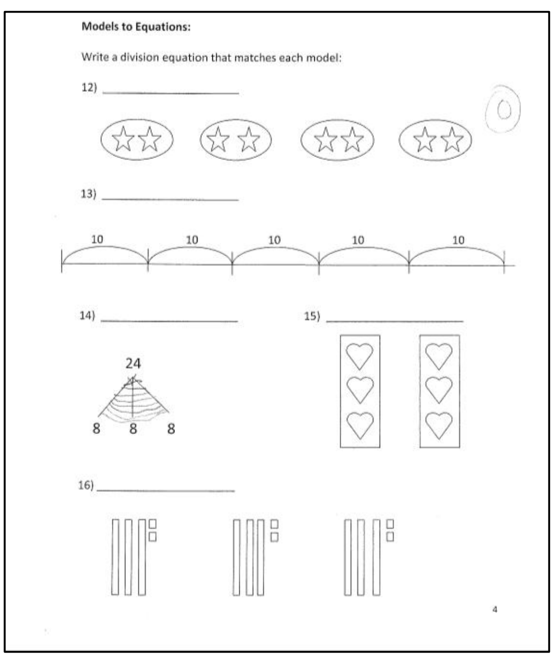
Answer:	Model:
848	162 162 162 162 848

40) 328 x 2 = 656

Answer:	Model:
656	328 328 656

3 10

Division models to equations pre-test


Models to Equations:

Write a division equation that matches each model:

12) __________________

0

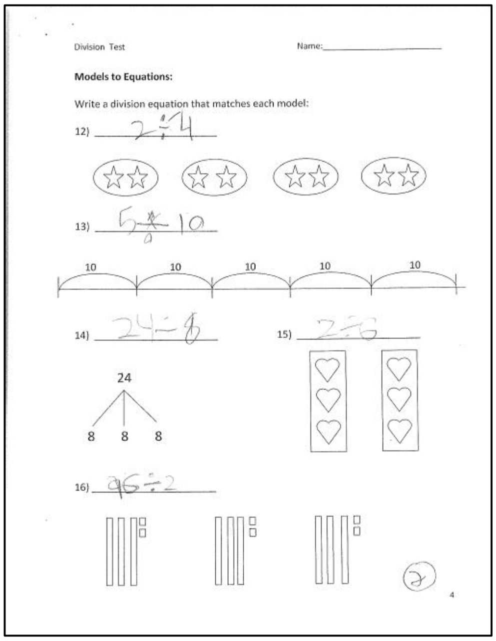
13) __________________

14) __________________ 15) __________________

16) __________________

4

Division models to equations post test


Division Test Name:

Models to Equations:

Write a division equation that matches each model:

12) 2 ÷ 4

13) 5 ÷ 10

10 10 10 10 10

14) 24 ÷ 8

15) 2 ÷ 6

24

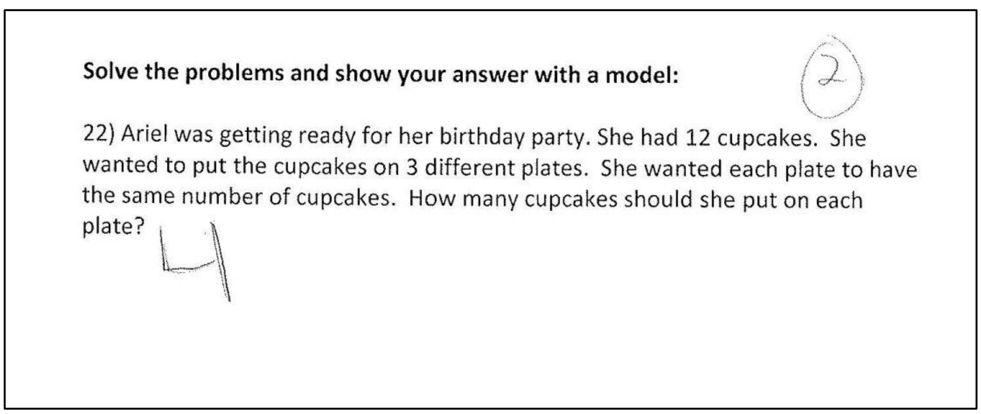
8 8 8

16) 96 ÷ 2

2

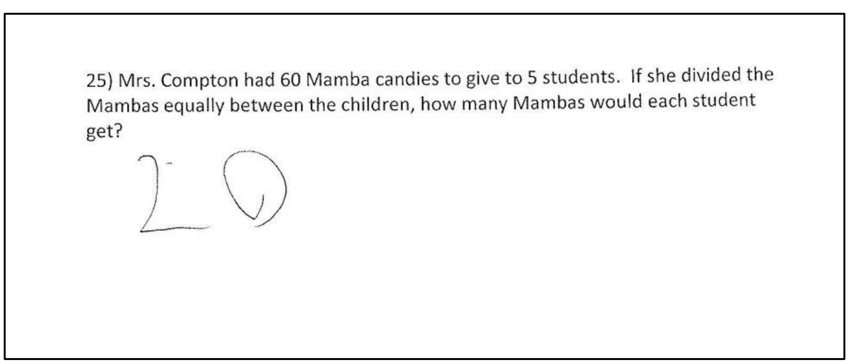
4

Division problem solving pre-test


Solve the problems and show your answer with a model: 2

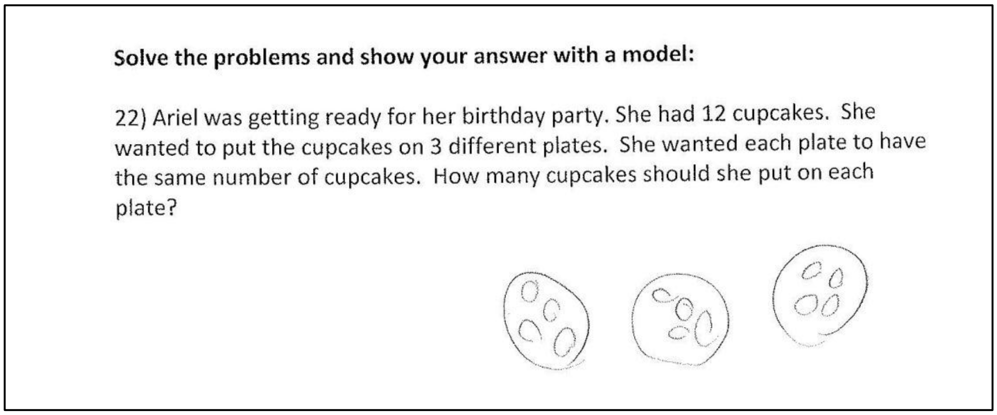
22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?

4

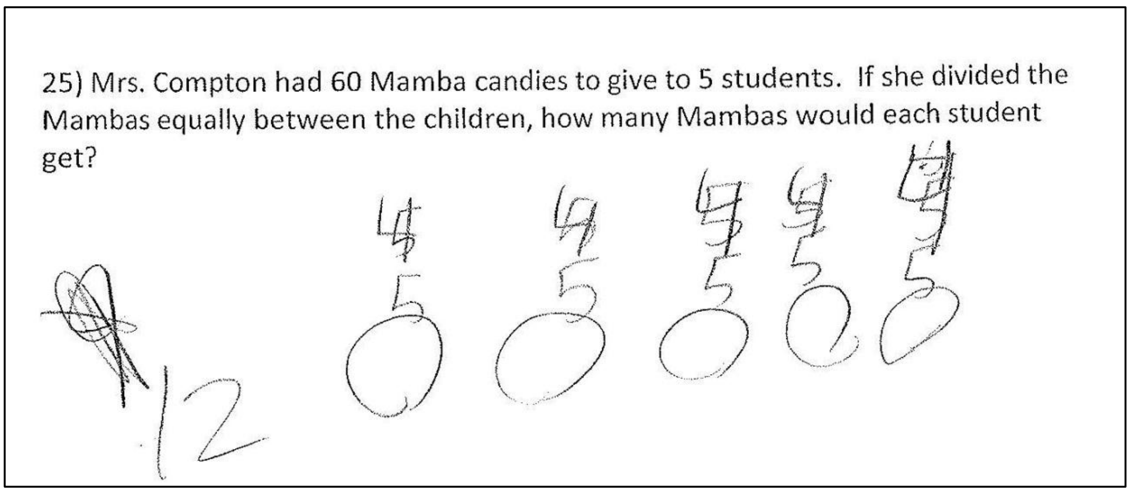

25) Mrs. Compton had 60 Mamba candies to give to 5 students. If she divided the Mambas equally between the children, how many Mambas would each student get?

20

Division problem solving post test


Solve the problems and show your answer with a model:

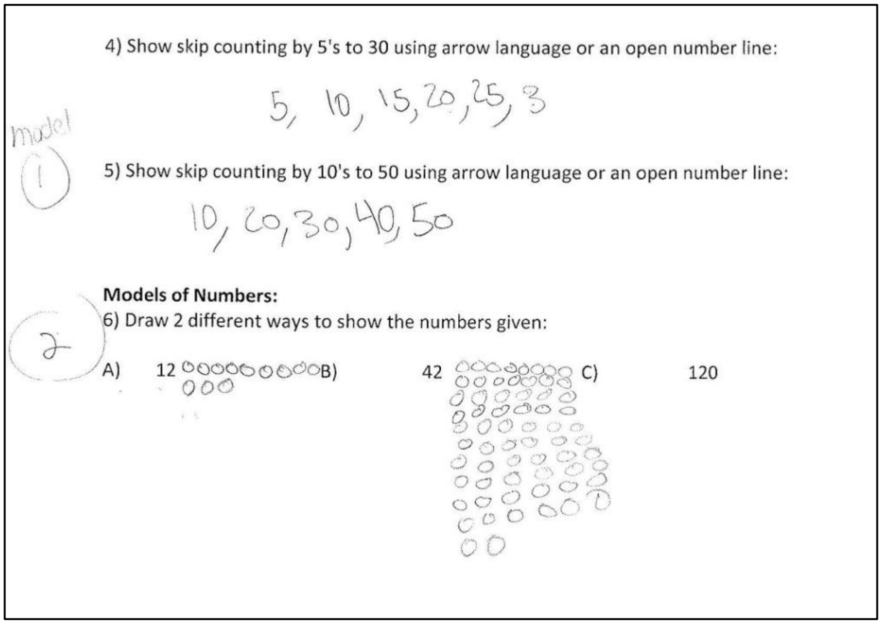
22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?


25) Mrs. Compton had 60 Mamba candies to give to 5 students. If she divided the Mambas equally between the children, how many Mambas would each student get?

12

Destiny Work Samples

Skip counting models and models of numbers pre-test


4) Show skip counting by 5's to 30 using arrow language or an open number line:

5, 10, 15, 20, 25, 3

model

1

5) Show skip counting by 10's to 50 using arrow language or an open number line:

10, 20, 30, 40, 50

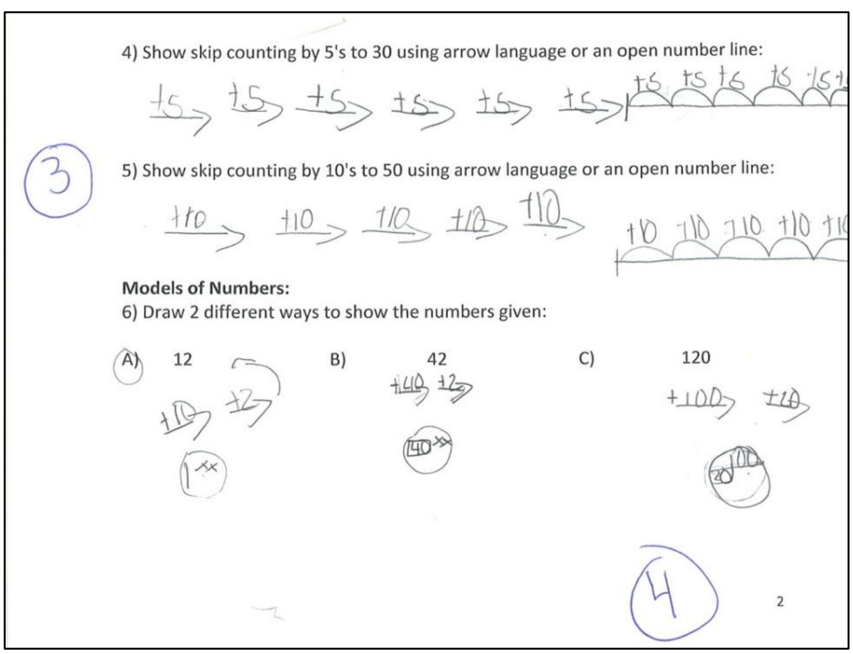
Models of Numbers:

2

6) Draw 2 different ways to show the numbers given:

A) 12 B) 42 C) 120

Skip counting models and models of numbers post test


4) Show skip counting by 5's to 30 using arrow language or an open number line:

+5 +5 +5 +5 +5 +5

3

5) Show skip counting by 10's to 50 using arrow language or an open number line:

+10 +10 +10 +10 +10

Models of Numbers:

6) Draw 2 different ways to show the numbers given:

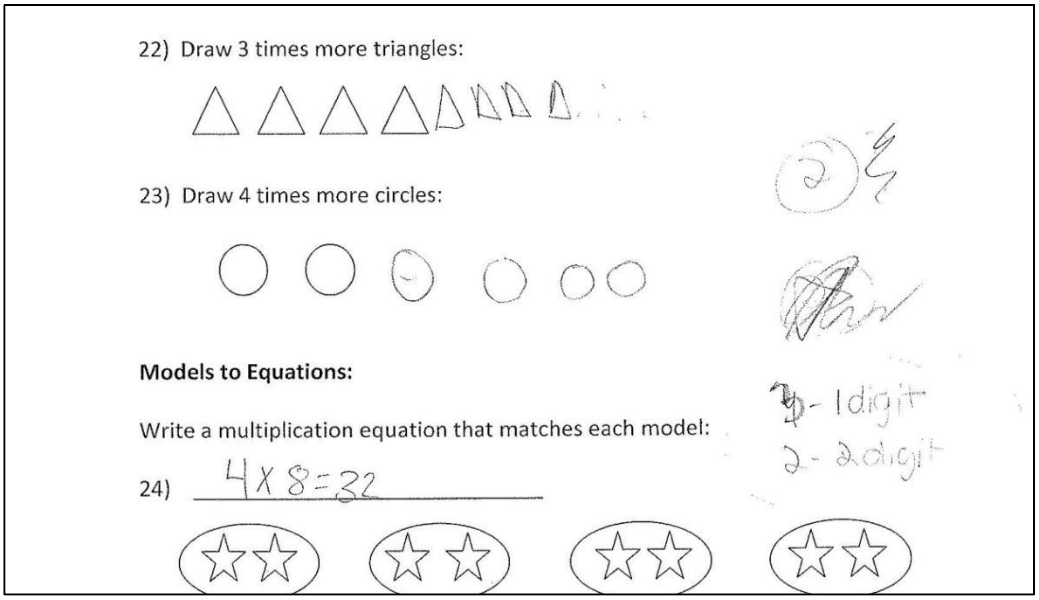
A) 12 B) 42 C) 120

+10 +2 +40 +2 +100 +20

4

2

n times more shapes and models to equations pre-test



22) Draw 3 times more triangles:

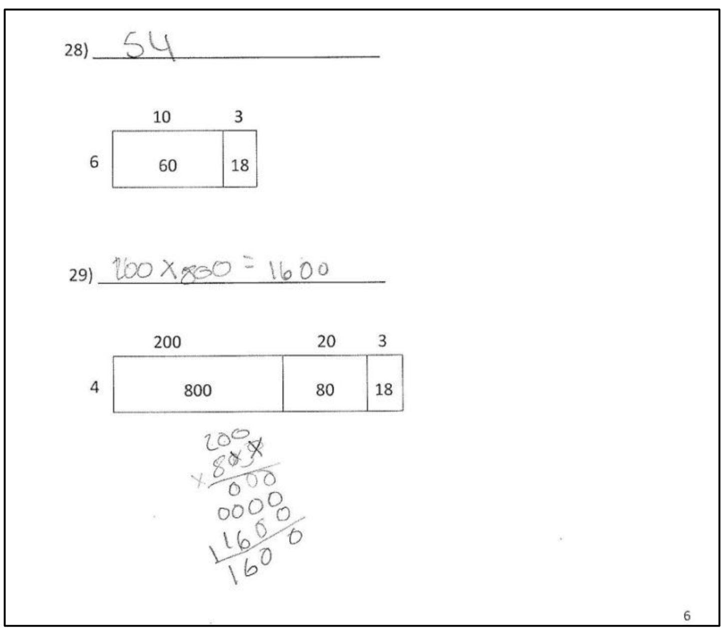
23) Draw 4 times more circles:

Models to Equations:

Write a multiplication equation that matches each model:

24) 4 X 8 = 32

1 - 1 digit
2 - 2 digit



28) 54

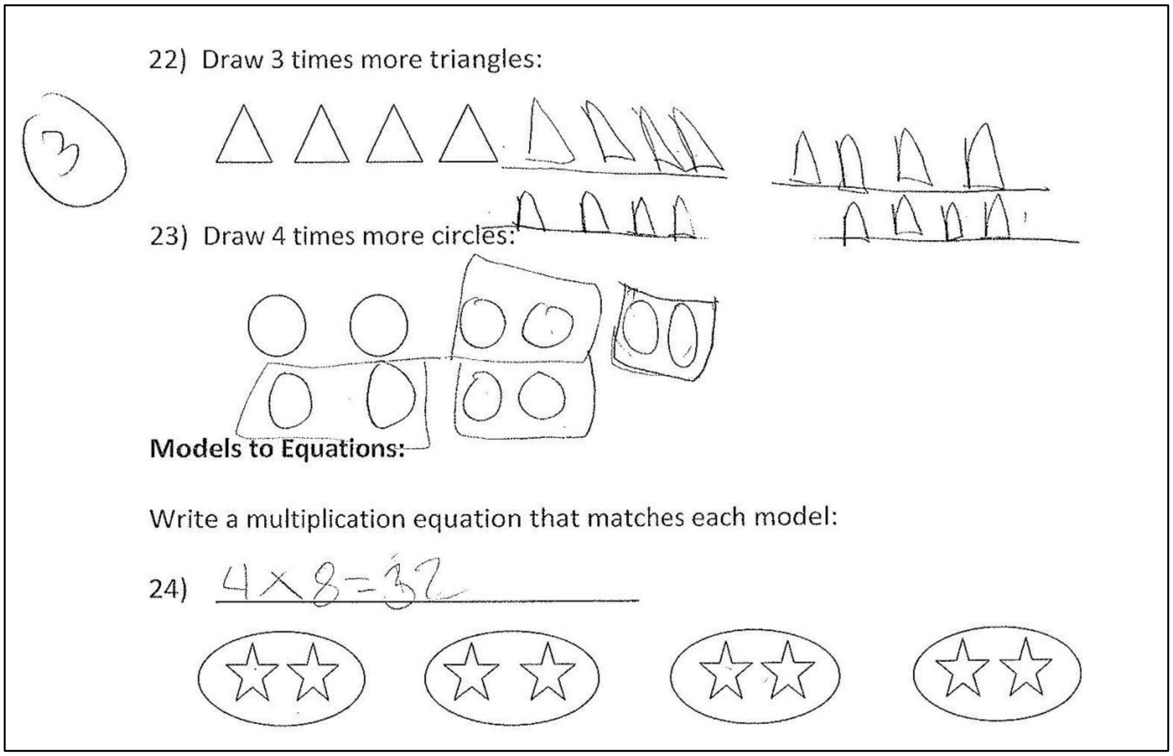
	10	3
6	60	18

29) 200 X 800 = 1600

	200	20	3
4	800	80	18

6

n times more shapes and models to equations post test



3

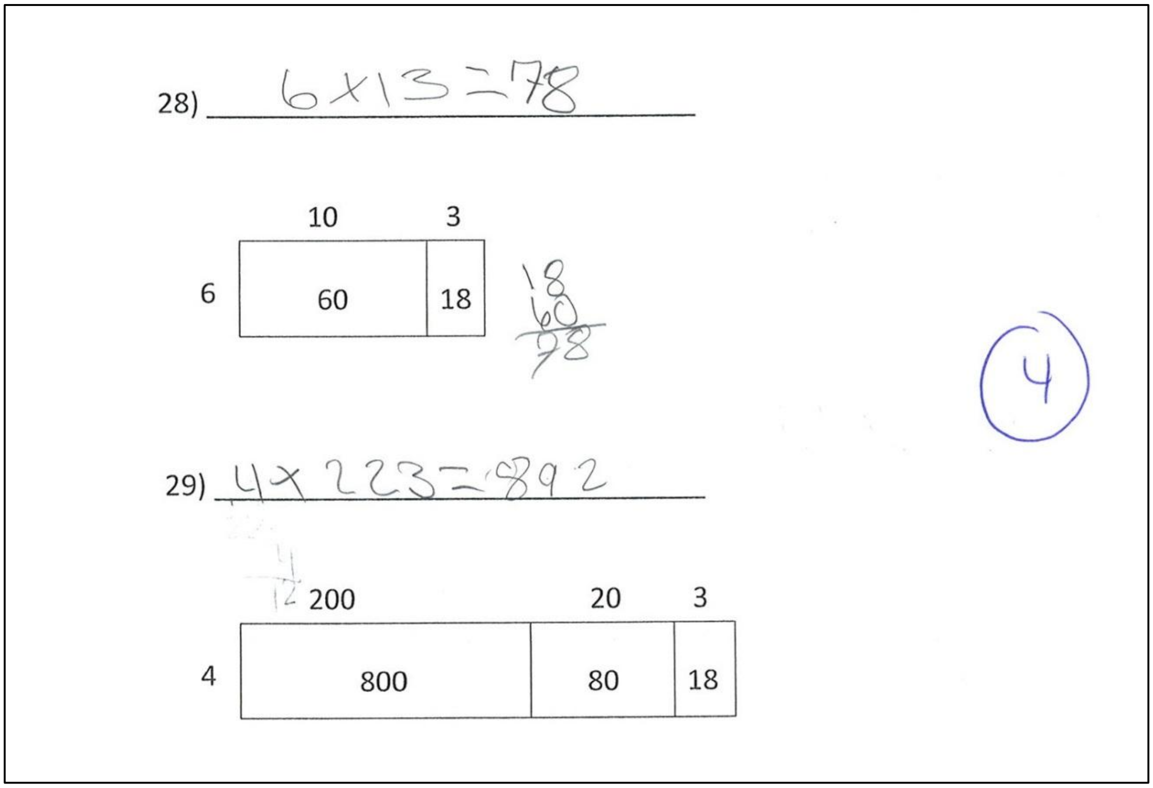
22) Draw 3 times more triangles:

23) Draw 4 times more circles:

Models to Equations:

Write a multiplication equation that matches each model:

24) 4×8=32



28) 6x13=78

	10	3
6	60	18

18
60
78

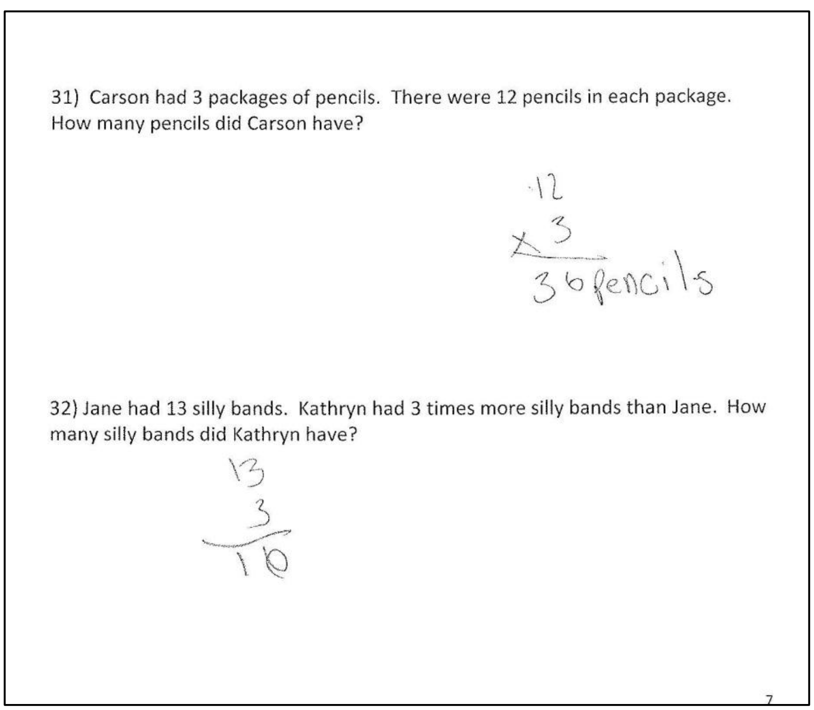
4

29) 4x223=892

4
12

	200	20	3
4	800	80	18

Multiplication problem solving pre-test



31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

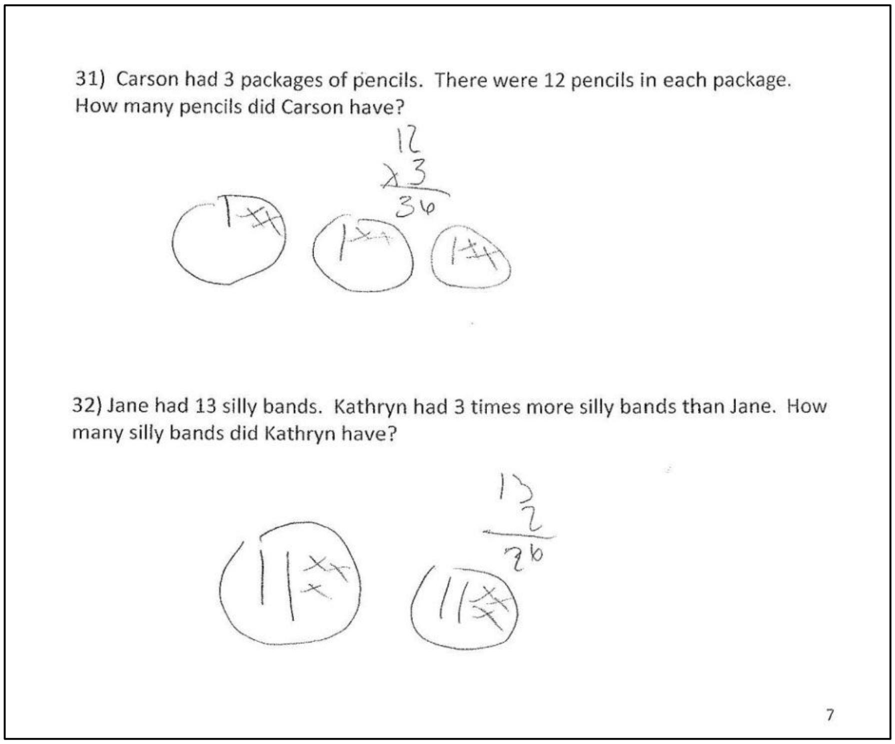
12
x 3
36 pencils

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

13
3
16

7

Multiplication problem solving post test



31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

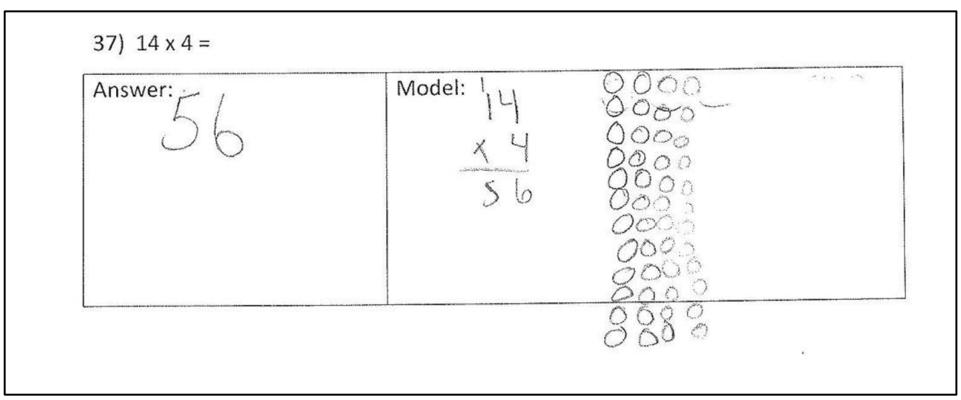
12
x 3
36

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

13
2
26

7

Multiplication equations to models pre-test



37) 14 x 4 =

Answer:	Model:
56	14 x 4 = 56

Multiplication Test Name:________________________

38) 23 x 5 =

Answer:	Model:
115	1 23 x 5 115

39) 162 x 4 =

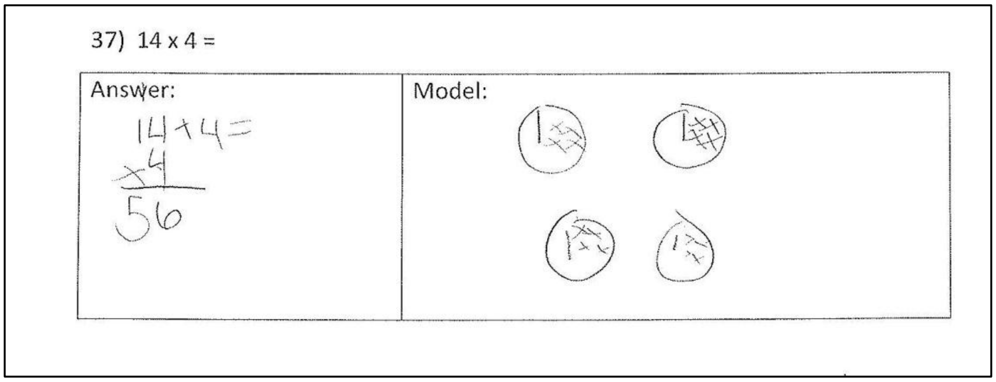
Answer:	Model:
648	2 162 x 4 648

40) 328 x 2 =

Answer:	Model:
650	1 328 x 2 656

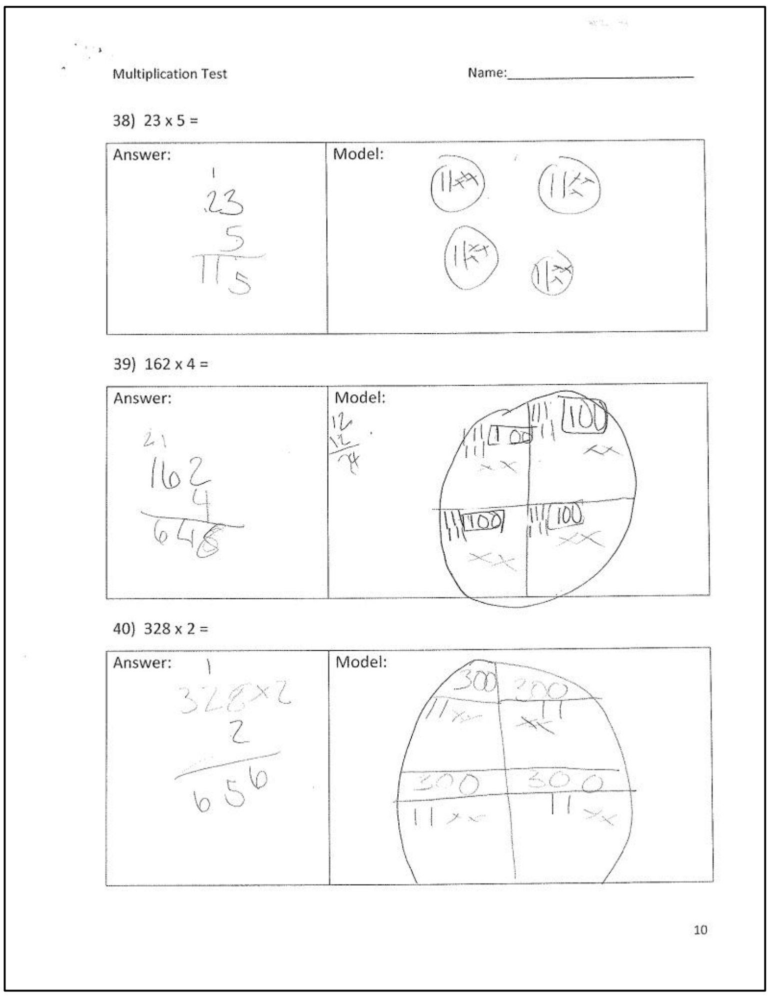
10

Multiplication equations to models post test



37) 14 x 4 =

Answer:	Model:
14 + 4 = x 4 56	



Multiplication Test Name:____________________

38) 23 x 5 =

Answer:	Model:
23 5 115	

39) 162 x 4 =

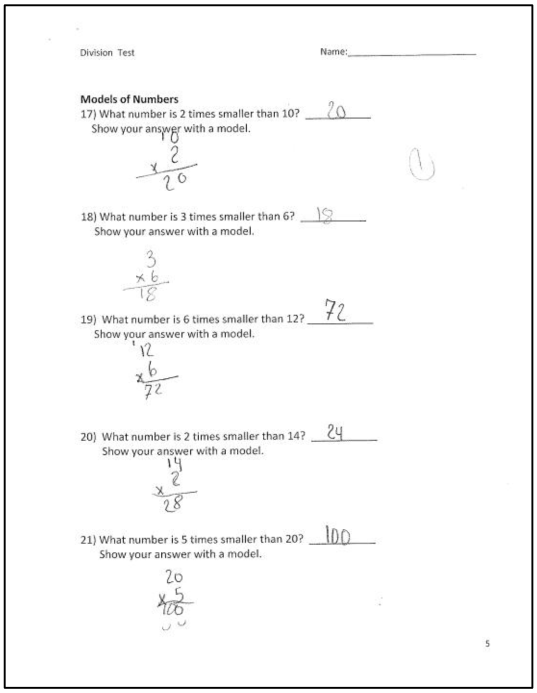
Answer:	Model:
162 4 648	100 100 100 100

40) 328 x 2 =

Answer:	Model:
328 x 2 2 656	300 300 300 300

10

n times smaller pre-test



Division Test Name:___________________

Models of Numbers

17) What number is 2 times smaller than 10? __20__
Show your answer with a model.

10
x 2
20

18) What number is 3 times smaller than 6? __18__
Show your answer with a model.

3
x 6
18

19) What number is 6 times smaller than 12? __72__
Show your answer with a model.

12
x 6
72

20) What number is 2 times smaller than 14? __24__
Show your answer with a model.

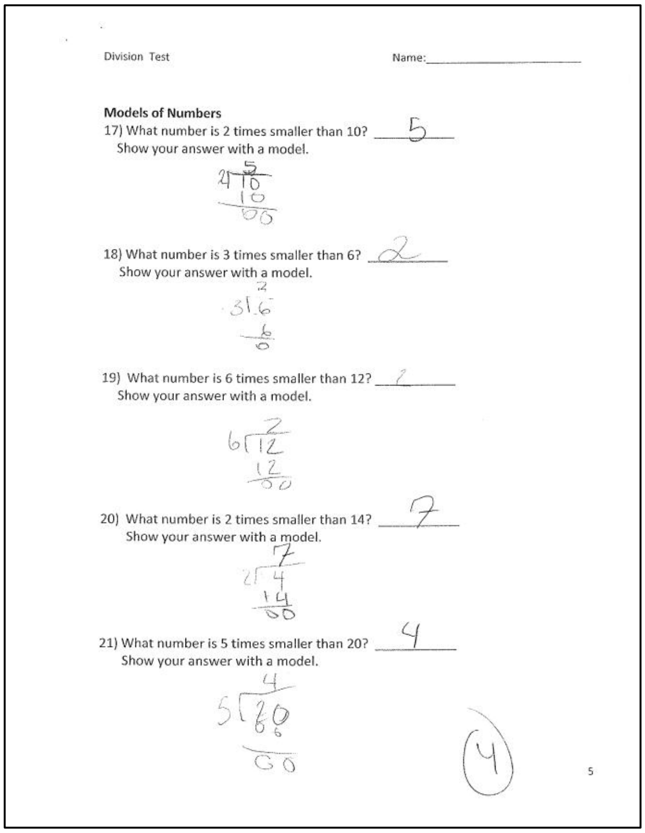
14
x 2
28

21) What number is 5 times smaller than 20? __100__
Show your answer with a model.

20
x 5
100

5

n times smaller post test



Division Test Name:______________________

Models of Numbers

17) What number is 2 times smaller than 10? ___5___
Show your answer with a model.

18) What number is 3 times smaller than 6? ___2___
Show your answer with a model.

19) What number is 6 times smaller than 12? ___2___
Show your answer with a model.

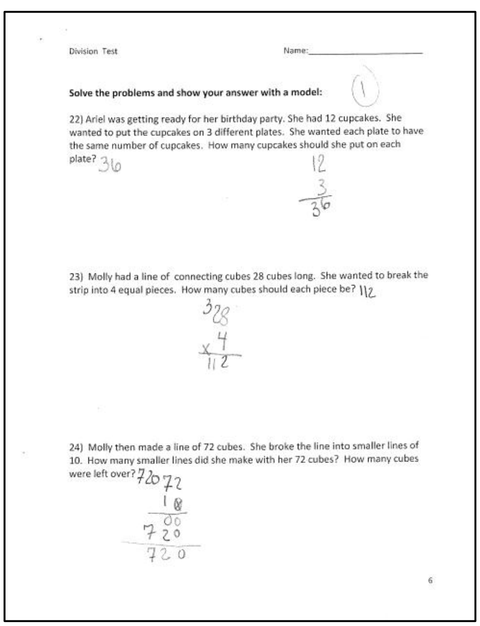
20) What number is 2 times smaller than 14? ___7___
Show your answer with a model.

21) What number is 5 times smaller than 20? ___4___
Show your answer with a model.

4

5

Division problem solving pre-test



Division Test Name:______________________

Solve the problems and show your answer with a model: 1

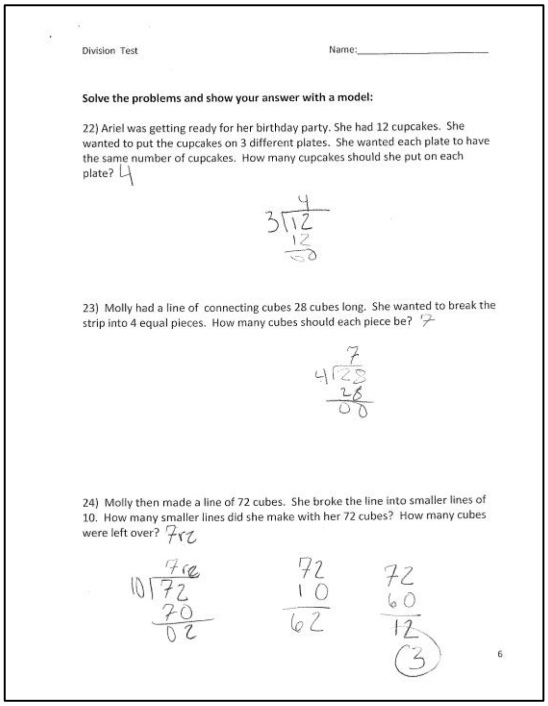
22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate? 36

23) Molly had a line of connecting cubes 28 cubes long. She wanted to break the strip into 4 equal pieces. How many cubes should each piece be? 112

24) Molly then made a line of 72 cubes. She broke the line into smaller lines of 10. How many smaller lines did she make with her 72 cubes? How many cubes were left over? 720 72

6

Division problem solving post test



Division Test Name:____________________

Solve the problems and show your answer with a model:

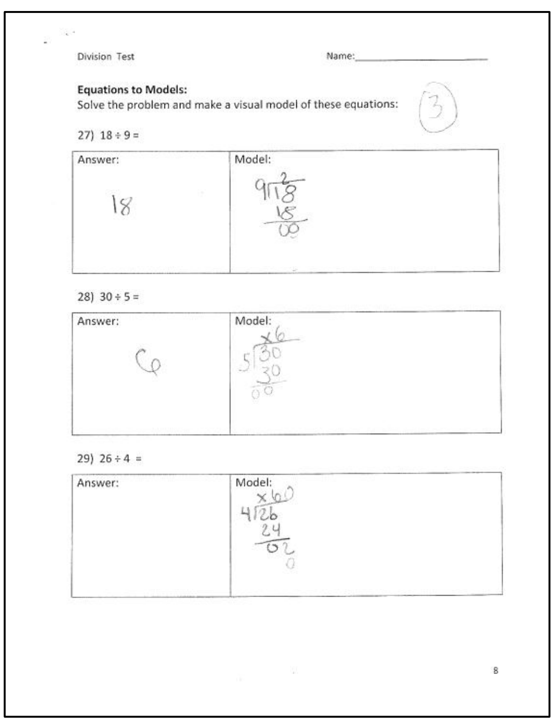
22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate? 4

23) Molly had a line of connecting cubes 28 cubes long. She wanted to break the strip into 4 equal pieces. How many cubes should each piece be? 7

24) Molly then made a line of 72 cubes. She broke the line into smaller lines of 10. How many smaller lines did she make with her 72 cubes? How many cubes were left over? 7r2

6

Division equations to models pre-test



Division Test Name:____________________

Equations to Models:

Solve the problem and make a visual model of these equations:

3

27) 18 ÷ 9 =

Answer:	Model:
18	

28) 30 ÷ 5 =

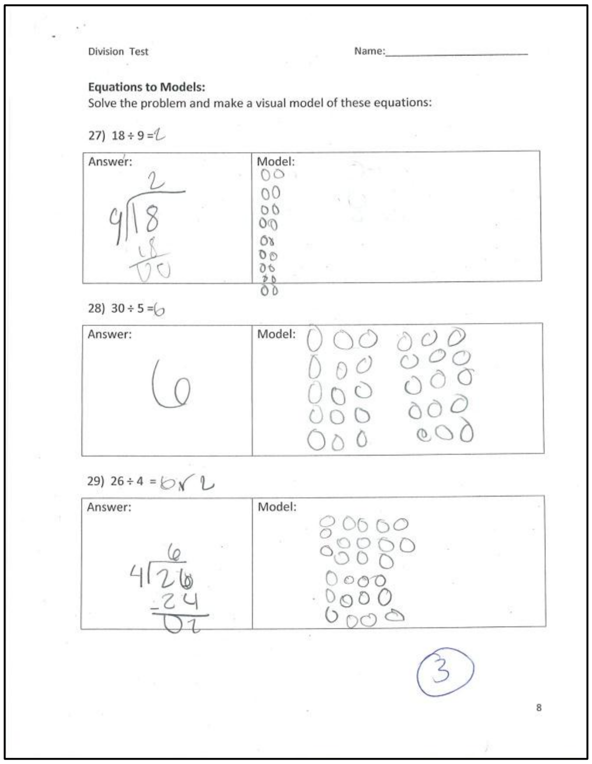
Answer:	Model:
6	

29) 26 ÷ 4 =

Answer:	Model:

8

Division equations to models post test


Division Test Name:____________________

Equations to Models:
Solve the problem and make a visual model of these equations:

27) 18 ÷ 9 = 2

Answer:	Model:
2 9)18 -18 00	

28) 30 ÷ 5 = 6

Answer:	Model:
6	

29) 26 ÷ 4 = 6 r 2

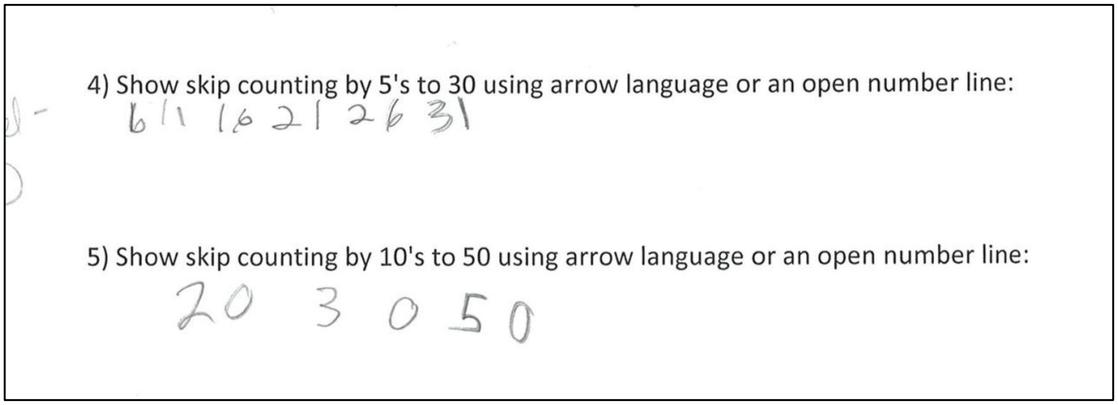
Answer:	Model:
6 4)26 -24 02	

3

8

Happy Gilmore Work Samples

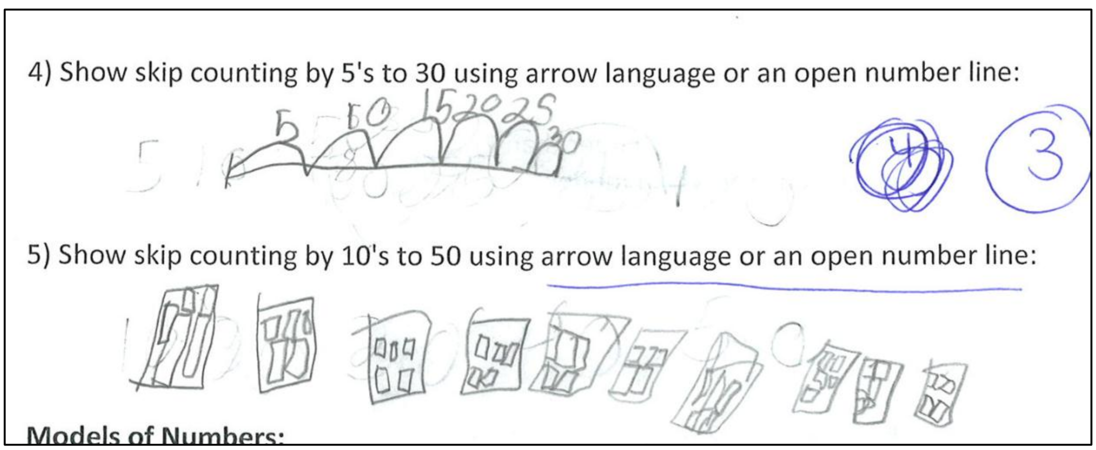
Skip counting model pre-test



4) Show skip counting by 5's to 30 using arrow language or an open number line:
6 11 16 21 26 31

5) Show skip counting by 10's to 50 using arrow language or an open number line:
20 30 50

Skip counting model post test

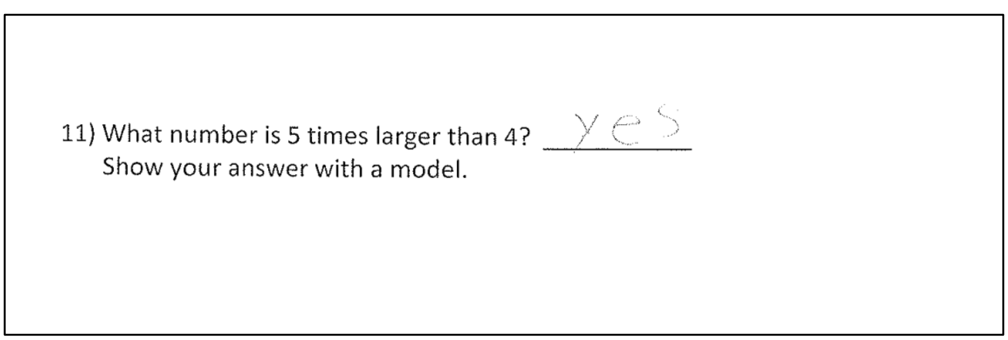


4) Show skip counting by 5's to 30 using arrow language or an open number line:

5) Show skip counting by 10's to 50 using arrow language or an open number line:

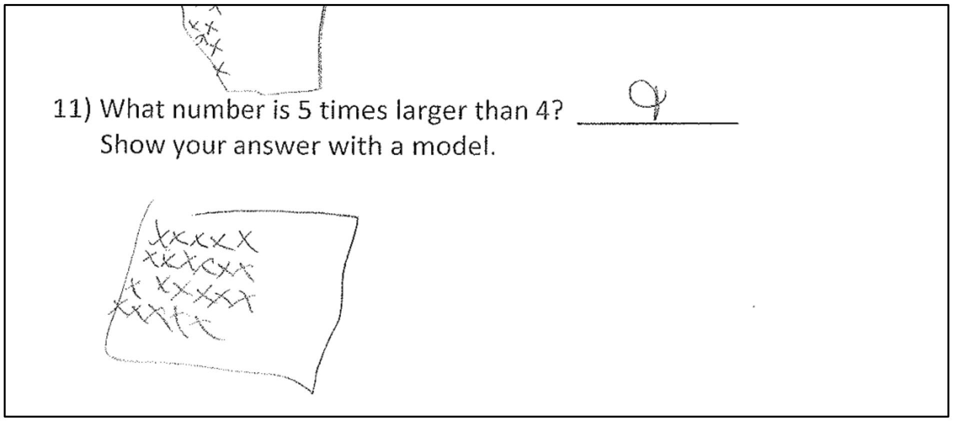
Models of Numbers:

n times larger with model pre-test



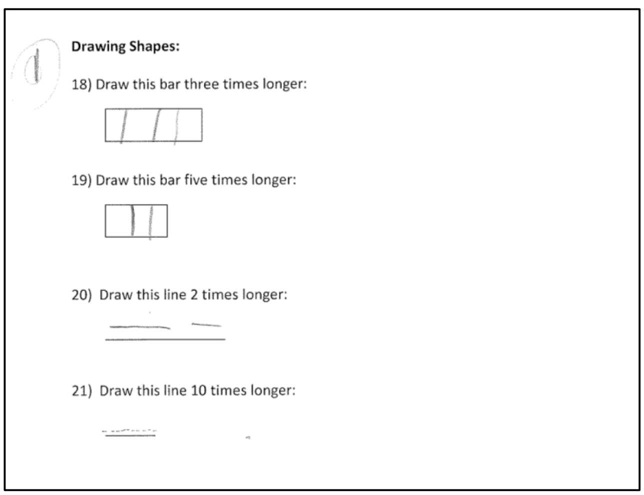
11) What number is 5 times larger than 4? yes
Show your answer with a model.

n times larger with model post test



11) What number is 5 times larger than 4? 9
Show your answer with a model.

Drawing shapes pre-test



Drawing Shapes:

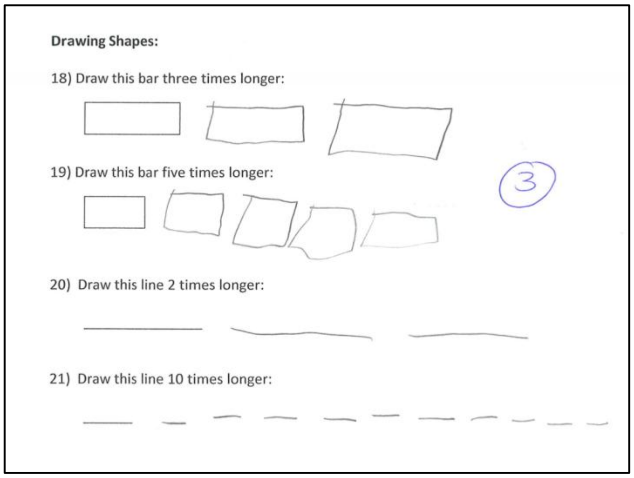
18) Draw this bar three times longer:

19) Draw this bar five times longer:

20) Draw this line 2 times longer:

21) Draw this line 10 times longer:

Drawing shapes post test


Drawing Shapes:

18) Draw this bar three times longer:

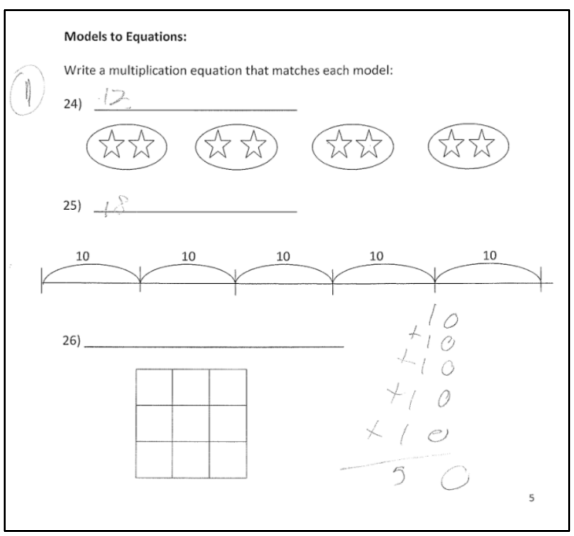
19) Draw this bar five times longer:

3

20) Draw this line 2 times longer:

21) Draw this line 10 times longer:

Multiplication models to equations pre-test


Models to Equations:

Write a multiplication equation that matches each model:

1

24) 12

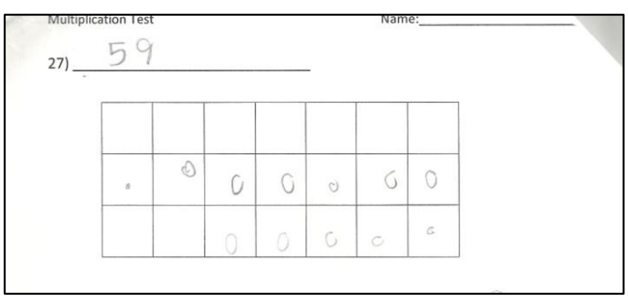
25) 1?

10 10 10 10 10

26)

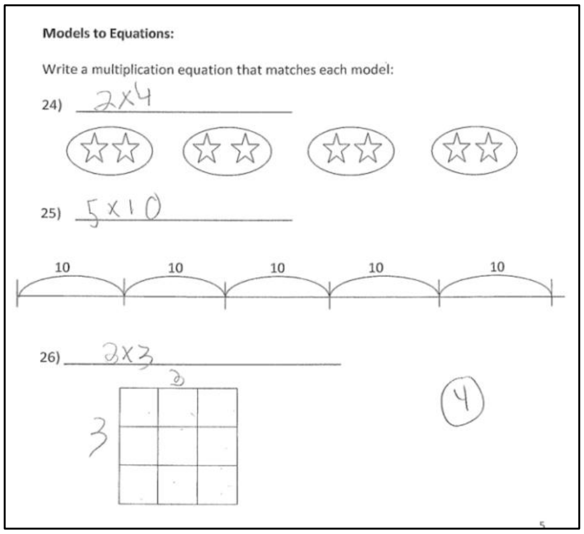
10
+10
+10
+10
+10
50

5


Multiplication Test Name:

27) 59

Multiplication models to equations post test


Models to Equations:

Write a multiplication equation that matches each model:

24) 2x4

25) 5x10

10 10 10 10 10

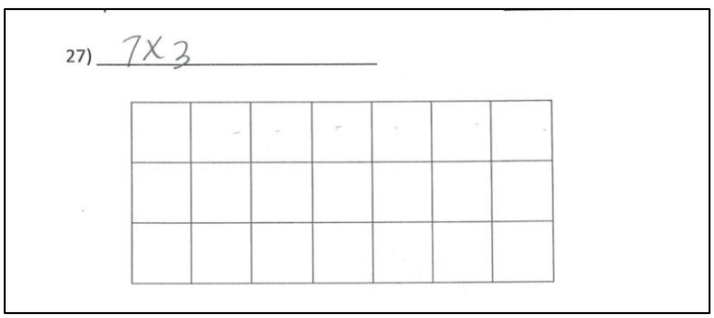
26) 3x3

2

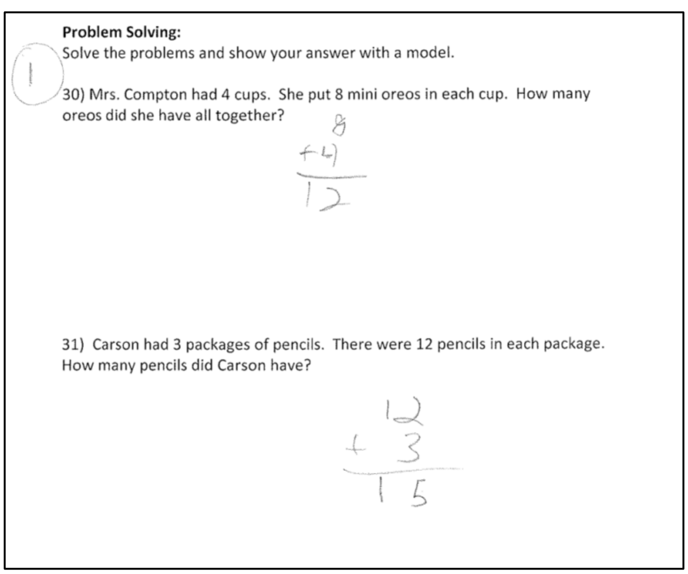
3

4

5


27) 7X3

Multiplication problem solving pre-test


Problem Solving:

Solve the problems and show your answer with a model.

1

30) Mrs. Compton had 4 cups. She put 8 mini oreos in each cup. How many oreos did she have all together?

$$\begin{array}{r} 8 \\ +4 \\ \hline 12 \end{array}$$

31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

$$\begin{array}{r} 12 \\ +\ 3 \\ \hline 15 \end{array}$$

34) Carmella is planting a garden. She has 6 packages of sunflower seeds. Each package has 15 seeds. How many sunflower seeds does Carmella have all together?

$$\begin{array}{r} \overset{1}{1}5 \\ +\ 6 \\ \hline 21 \end{array}$$

Multiplication problem solving post test

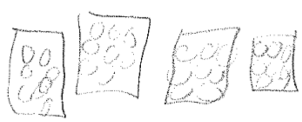
Multiplication Test Name:____________________

2

Problem Solving:

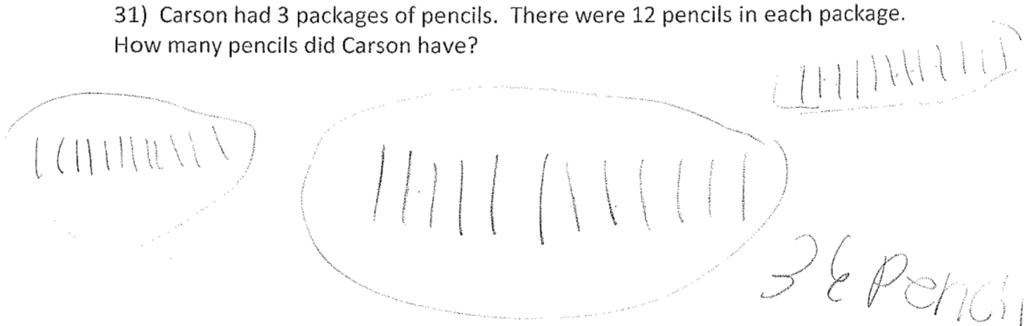
Solve the problems and show your answer with a model.

30) Mrs. Compton had 4 cups. She put 8 mini oreos in each cup. How many oreos did she have all together?



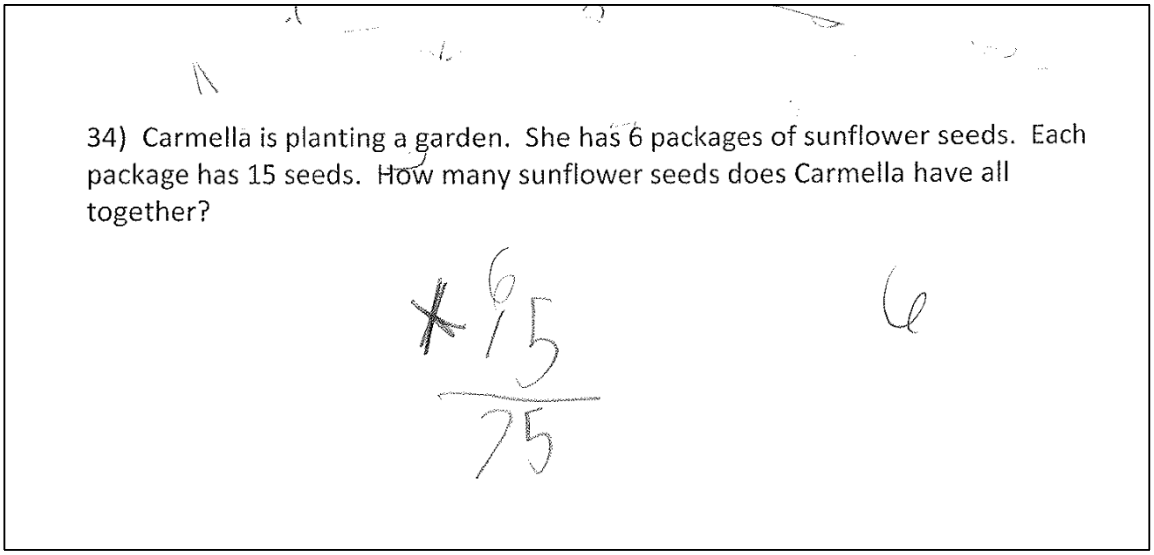
4X8

31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?



36 Pencils

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How

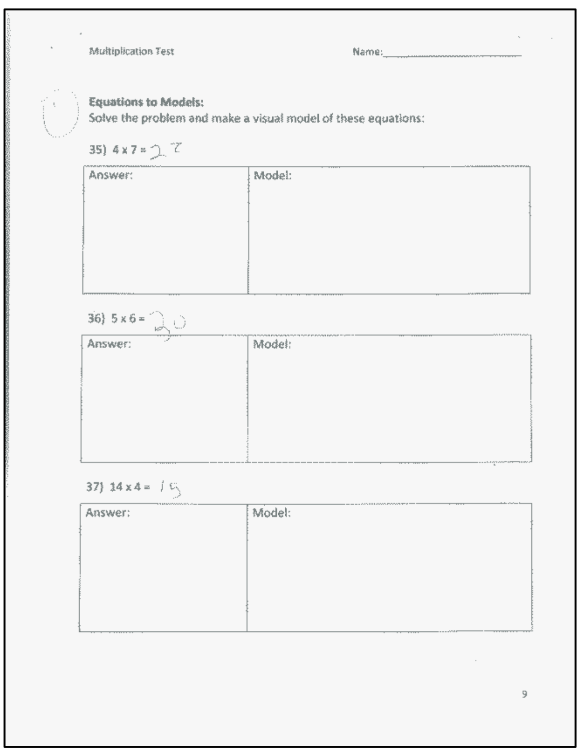


34) Carmella is planting a garden. She has 6 packages of sunflower seeds. Each package has 15 seeds. How many sunflower seeds does Carmella have all together?

$$\begin{array}{r} 6 \\ \times\ 15 \\ \hline 75 \end{array}$$

6

Multiplication equations to models pre-test



Multiplication Test Name:________________

Equations to Models:
Solve the problem and make a visual model of these equations:

35) 4 x 7 = 27

Answer:	Model:

36) 5 x 6 = 30

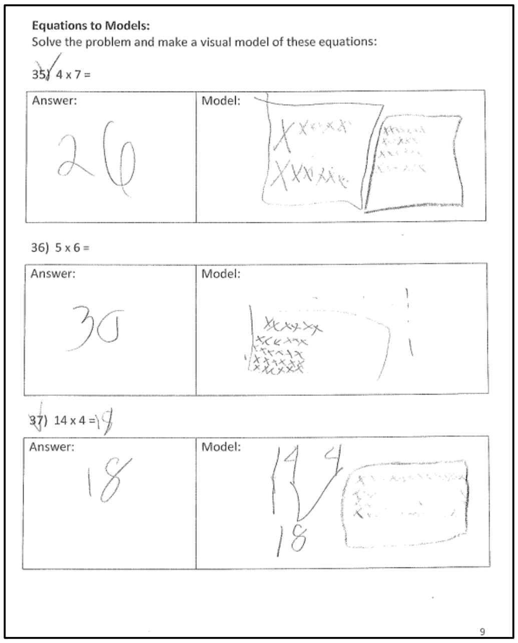
Answer:	Model:

37) 14 x 4 = 15

Answer:	Model:

9

Multiplication equations to models post test



Equations to Models:
Solve the problem and make a visual model of these equations:

35) 4 x 7 =

Answer:	Model:
26	

36) 5 x 6 =

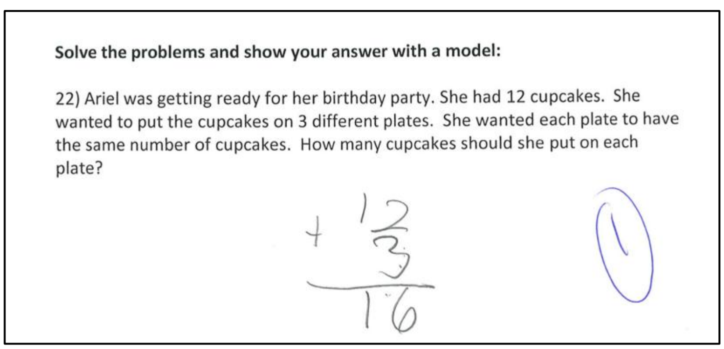
Answer:	Model:
30	

37) 14 x 4 = 18

Answer:	Model:
18	14 4 18

9

Division problem solving pre-test



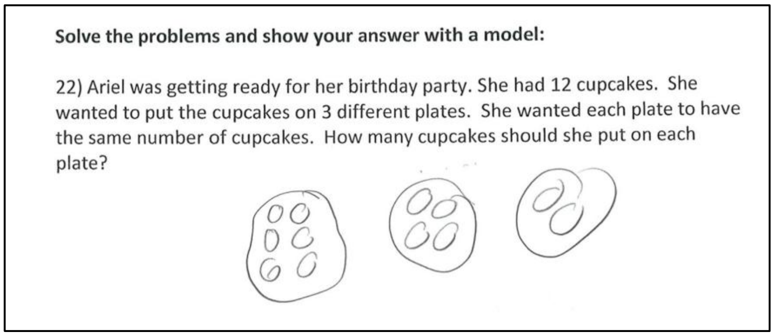
Solve the problems and show your answer with a model:

22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?

$$\begin{array}{r} 12 \\ +\ 3 \\ \hline 16 \end{array}$$

1

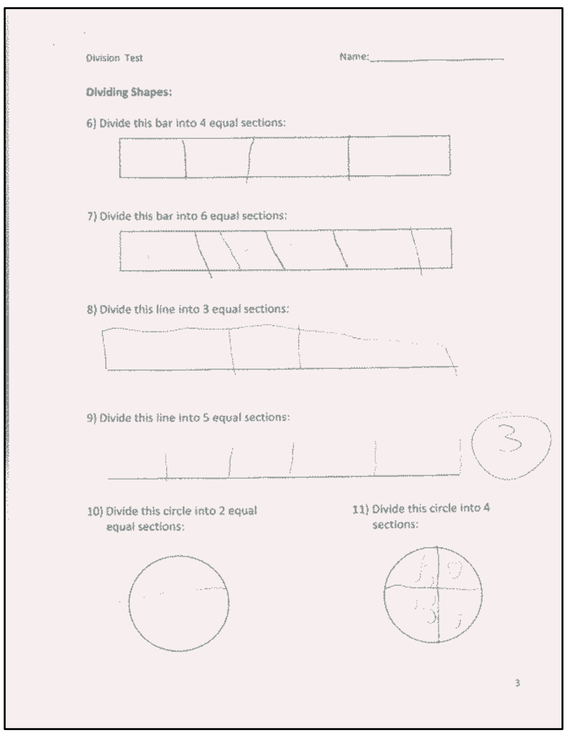
Division problem solving post test



Solve the problems and show your answer with a model:

22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?

Dividing shapes pre-test



Division Test Name:__________________

Dividing Shapes:

6) Divide this bar into 4 equal sections:

7) Divide this bar into 6 equal sections:

8) Divide this line into 3 equal sections:

9) Divide this line into 5 equal sections:

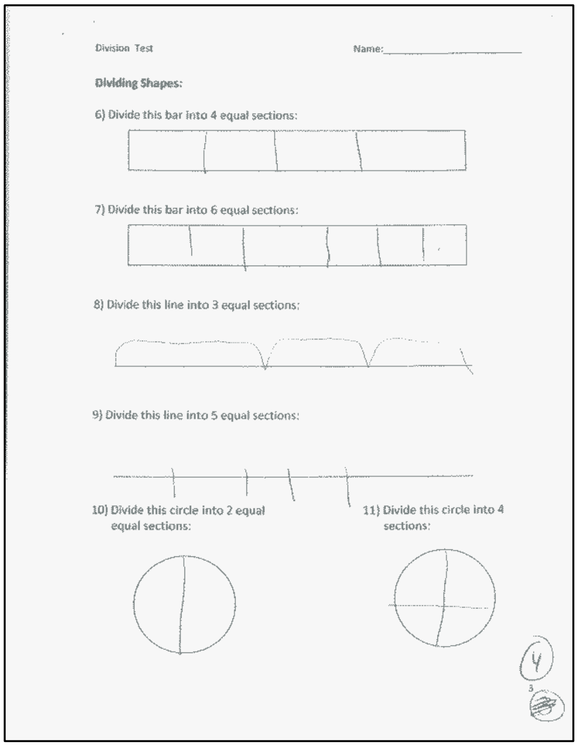
3

10) Divide this circle into 2 equal equal sections:

11) Divide this circle into 4 sections:

3

Dividing shapes post test



Division Test Name:____________

Dividing Shapes:

6) Divide this bar into 4 equal sections:

7) Divide this bar into 6 equal sections:

8) Divide this line into 3 equal sections:

9) Divide this line into 5 equal sections:

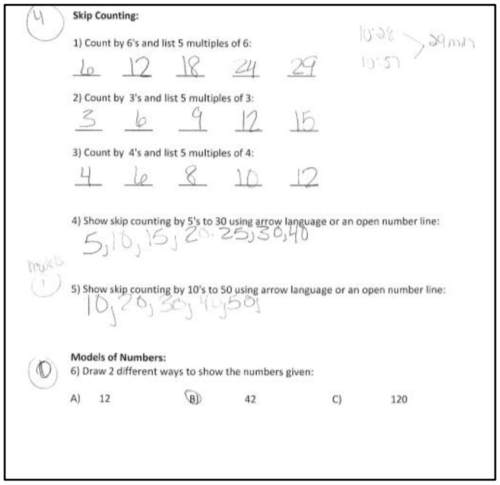
10) Divide this circle into 2 equal equal sections:

11) Divide this circle into 4 sections:

4

Katarina Work Samples

Skip counting and models of numbers pre-test


Skip Counting:

1) Count by 6's and list 5 multiples of 6:
6 12 18 24 29

2) Count by 3's and list 5 multiples of 3:
3 6 9 12 15

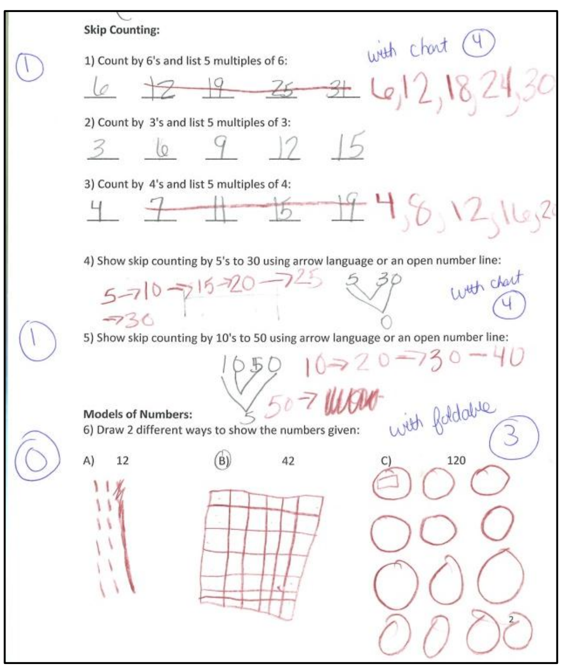
3) Count by 4's and list 5 multiples of 4:
4 6 8 10 12

4) Show skip counting by 5's to 30 using arrow language or an open number line:
5, 10, 15, 20, 25, 30, 40

5) Show skip counting by 10's to 50 using arrow language or an open number line:
10, 20, 30, 40, 50

Models of Numbers:
6) Draw 2 different ways to show the numbers given:
A) 12 B) 42 C) 120

Skip counting and models of numbers post test


Skip Counting:

1) Count by 6's and list 5 multiples of 6:
6 ~~12 19 25 31~~ 6, 12, 18, 24, 30
with chart (4)

2) Count by 3's and list 5 multiples of 3:
3 6 9 12 15

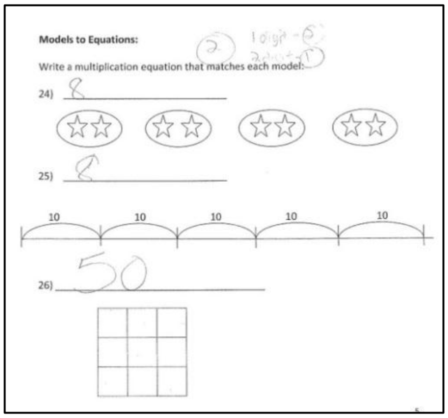
3) Count by 4's and list 5 multiples of 4:
4 ~~7 11 15 19~~ 4, 8, 12, 16, 2

4) Show skip counting by 5's to 30 using arrow language or an open number line:
5 → 10 → 15 → 20 → 25 → 30
with chart (4)

5) Show skip counting by 10's to 50 using arrow language or an open number line:
10 → 20 → 30 → 40 → 50

Models of Numbers:
6) Draw 2 different ways to show the numbers given:
with foldable (3)
A) 12 B) 42 C) 120

Multiplication models to equations pre-test



Models to Equations:

Write a multiplication equation that matches each model:

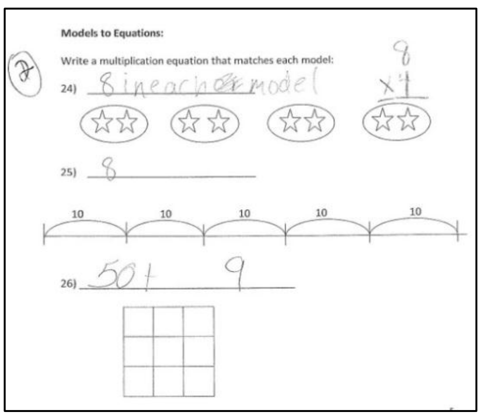
24) 8

25) 8

10 10 10 10 10

26) 50

Multiplication models to equations post test



Models to Equations:

Write a multiplication equation that matches each model:

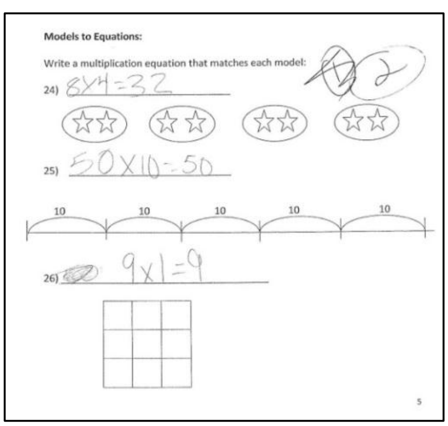
24) 8 in each model

25) 8

10 10 10 10 10

26) 50+ 9

Multiplication models to equations post test with accommodations of test read to her and visual aids available



Models to Equations:

Write a multiplication equation that matches each model:

24) 8x4=32

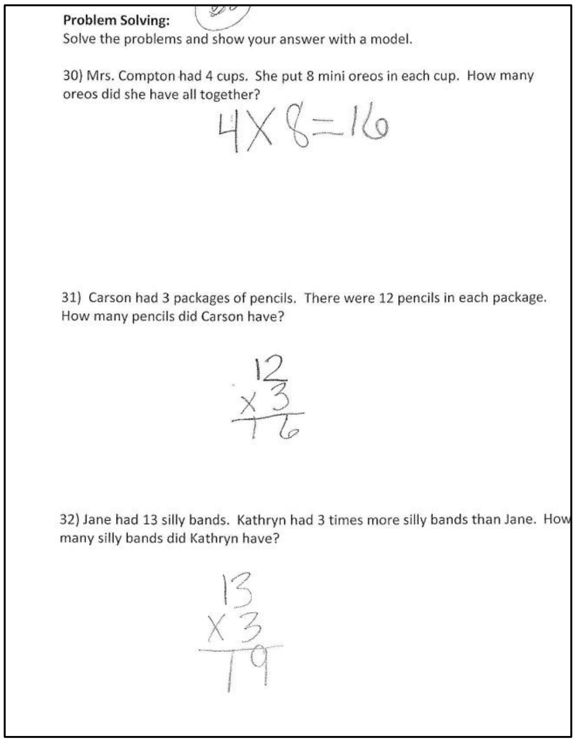
25) 50x10=50

10 10 10 10 10

26) 9x1=9

5

Multiplication problem solving pre test



Problem Solving:
Solve the problems and show your answer with a model.

30) Mrs. Compton had 4 cups. She put 8 mini oreos in each cup. How many oreos did she have all together?

4X8=16

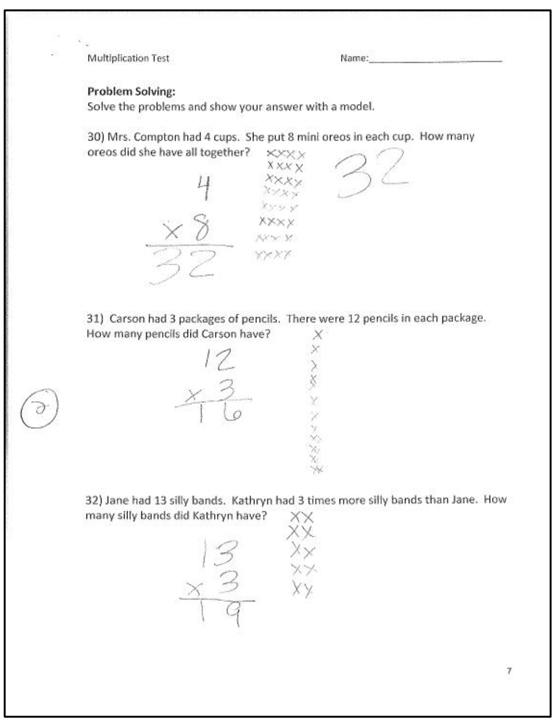
31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

12
x 3
16

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

13
X 3
19

Multiplication problem solving post test



Multiplication Test Name:____________________

Problem Solving:
Solve the problems and show your answer with a model.

30) Mrs. Compton had 4 cups. She put 8 mini oreos in each cup. How many oreos did she have all together?

4
x 8
32

32

31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

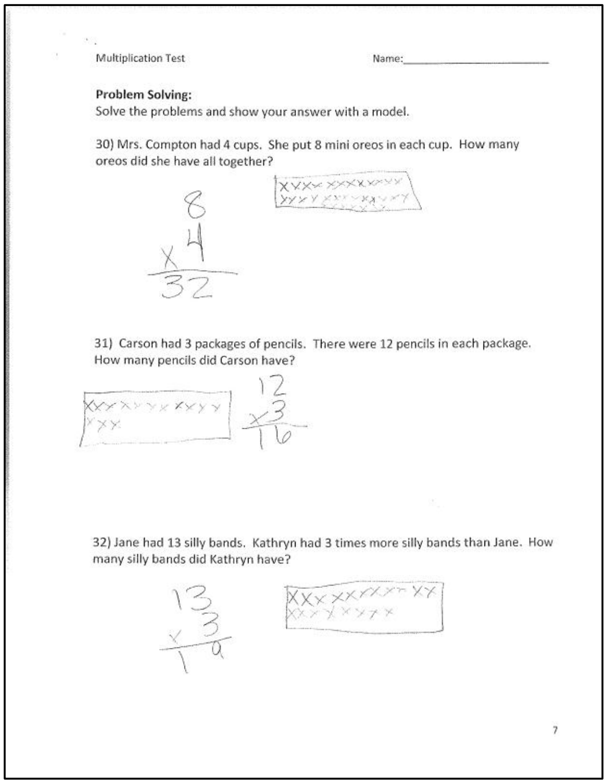
12
x 3
16

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

13
x 3
19

7

Multiplication problem solving with accommodations of test read to her and visual aids available



Multiplication Test Name:________________________

Problem Solving:
Solve the problems and show your answer with a model.

30) Mrs. Compton had 4 cups. She put 8 mini oreos in each cup. How many oreos did she have all together?

8
x 4
32

31) Carson had 3 packages of pencils. There were 12 pencils in each package. How many pencils did Carson have?

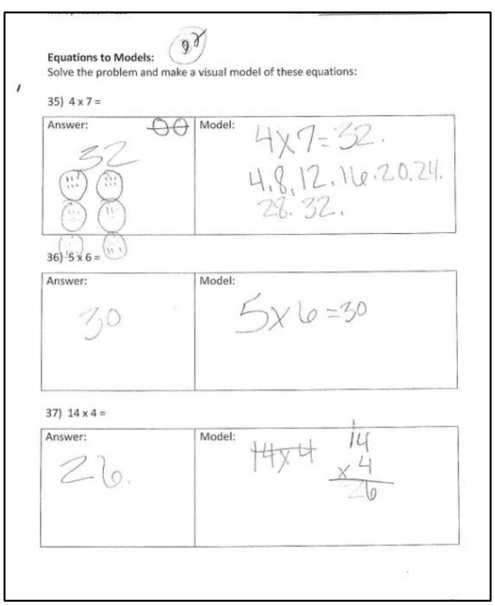
12
x 3
16

32) Jane had 13 silly bands. Kathryn had 3 times more silly bands than Jane. How many silly bands did Kathryn have?

13
x 3
19

7

Multiplication equations to models pre test



Equations to Models:
Solve the problem and make a visual model of these equations:

35) 4 x 7 =

Answer:	Model:
32	4x7=32. 4,8,12,16,20,24. 28. 32.

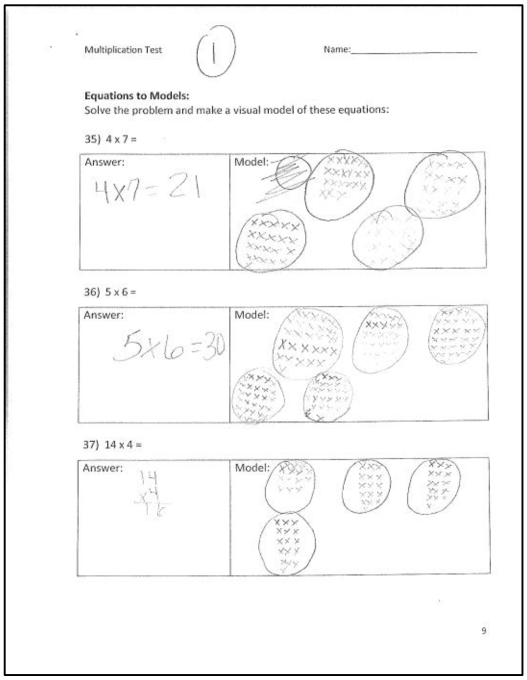
36) 5 x 6 =

Answer:	Model:
30	5x6=30

37) 14 x 4 =

Answer:	Model:
26.	~~14x4~~ 14 x 4 26

Multiplication equations to models post test



Multiplication Test (1) Name:

Equations to Models:
Solve the problem and make a visual model of these equations:

35) 4 x 7 =

Answer: 4x7=21 | Model:

36) 5 x 6 =

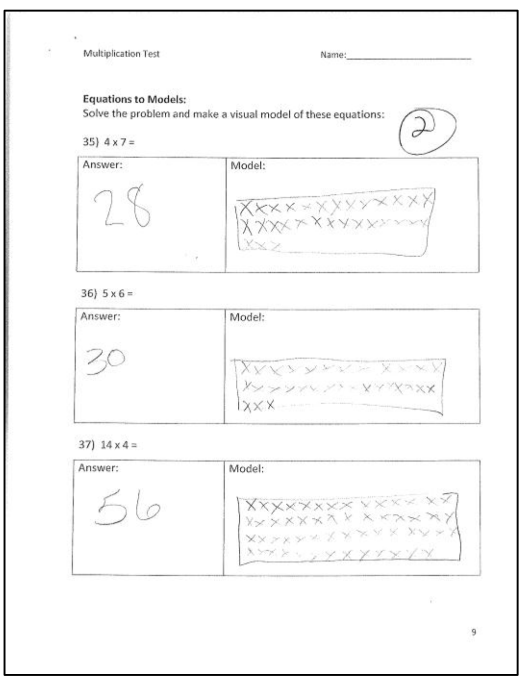
Answer: 5x6=30 | Model:

37) 14 x 4 =

Answer: 14 x4 16 | Model:

9

Multiplication equations to models post test with accommodations of test read to her and visual aids available



Multiplication Test Name:

Equations to Models:
Solve the problem and make a visual model of these equations: (2)

35) 4 x 7 =

Answer: 28 | Model:

36) 5 x 6 =

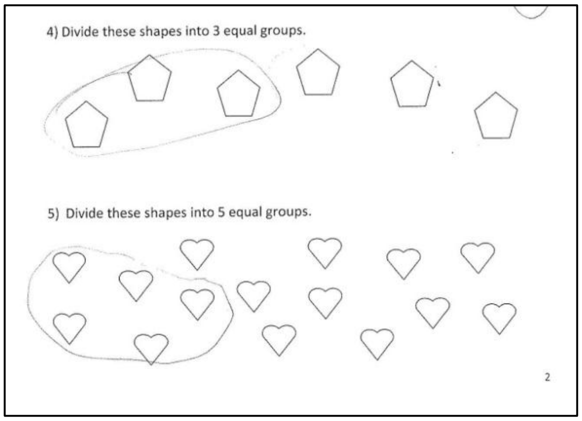
Answer: 30 | Model:

37) 14 x 4 =

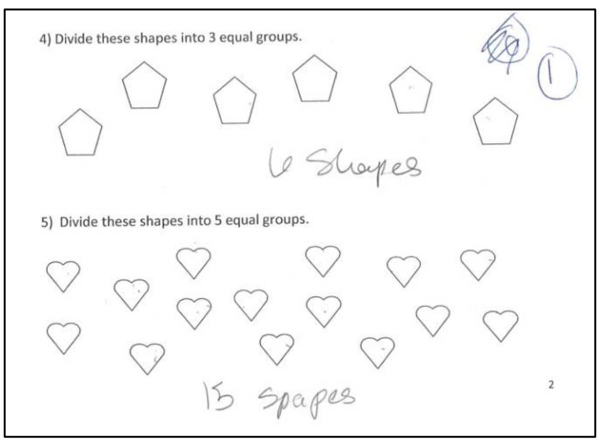
Answer: 56 | Model:

9

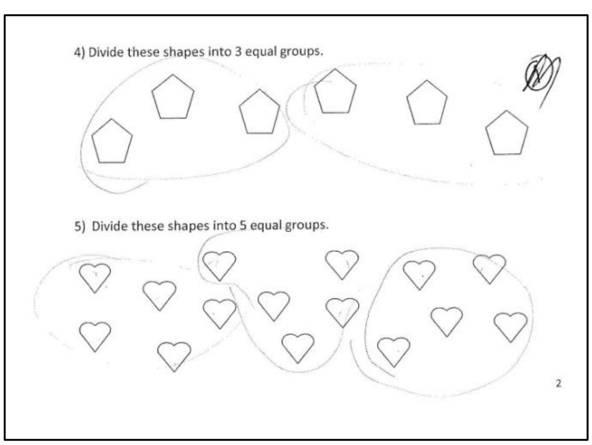
Making groups pre-test



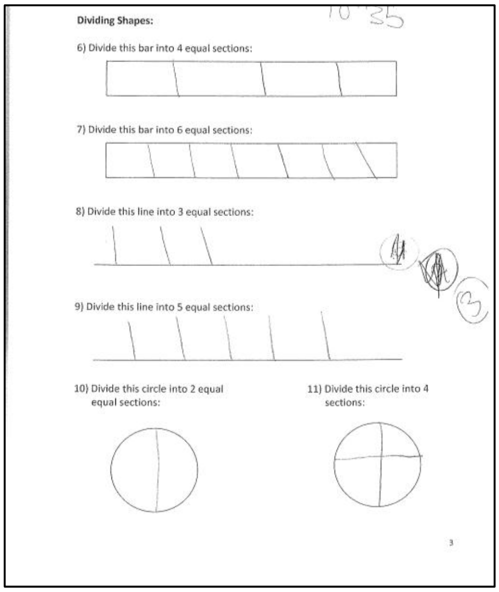
Making groups post test



Making groups post test with accommodations of test read to her and visual aids available



Dividing shapes pre-test


Dividing Shapes:

6) Divide this bar into 4 equal sections:

7) Divide this bar into 6 equal sections:

8) Divide this line into 3 equal sections:

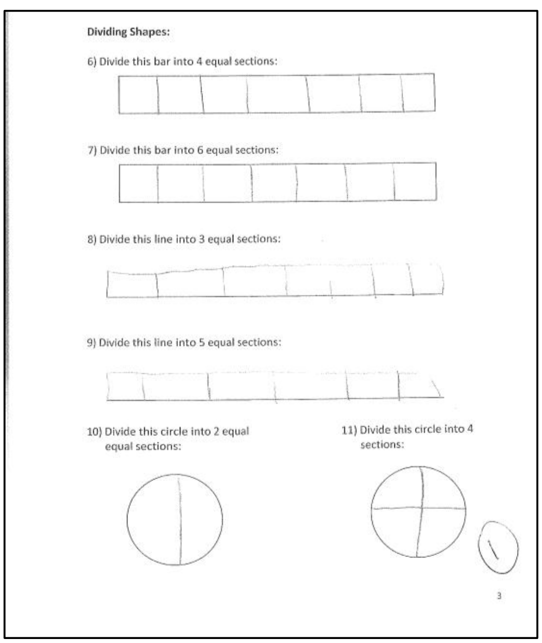
9) Divide this line into 5 equal sections:

10) Divide this circle into 2 equal equal sections:

11) Divide this circle into 4 sections:

3

Dividing shapes post test


Dividing Shapes:

6) Divide this bar into 4 equal sections:

7) Divide this bar into 6 equal sections:

8) Divide this line into 3 equal sections:

9) Divide this line into 5 equal sections:

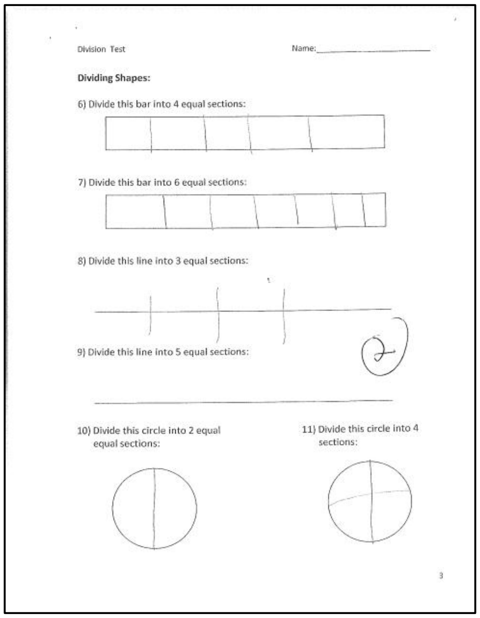
10) Divide this circle into 2 equal equal sections:

11) Divide this circle into 4 sections:

1

3

Dividing shapes post test with accommodations of test read to her and visual aids available



Division Test Name:________________

Dividing Shapes:

6) Divide this bar into 4 equal sections:

7) Divide this bar into 6 equal sections:

8) Divide this line into 3 equal sections:

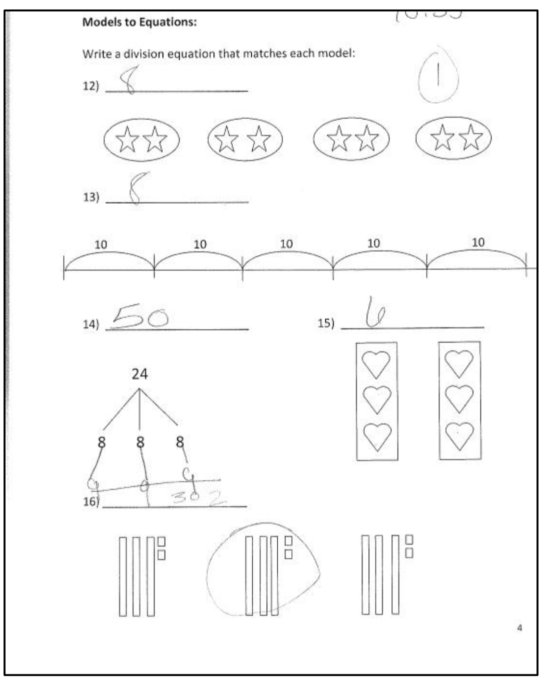
9) Divide this line into 5 equal sections:

10) Divide this circle into 2 equal equal sections:

11) Divide this circle into 4 sections:

3

Division models to equations pre-test



Models to Equations:

Write a division equation that matches each model:

12) 8

13) 8

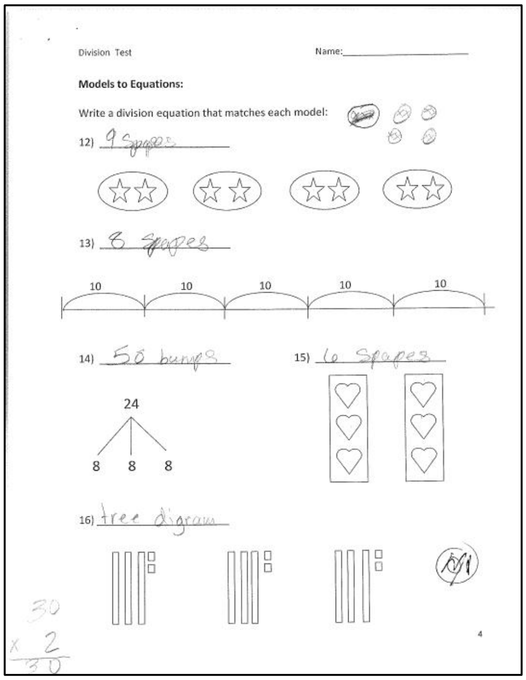
14) 50

15) 6

16)

4

Division models to equations post test


Division Test Name:______________

Models to Equations:

Write a division equation that matches each model:

12) 9 Spapes

13) 8 Spapes

10 10 10 10 10

14) 50 bumps 15) 6 Spapes

24

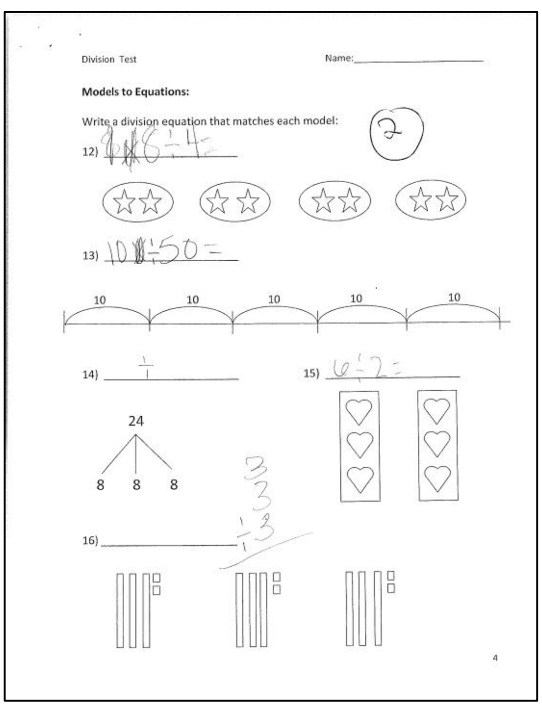
8 8 8

16) tree digram

30
x 2
30

4

Division models to equations post test with accommodations of test read to her and visual aids available


Division Test Name:______________

Models to Equations:

Write a division equation that matches each model:

2

12) 8÷4=

13) 10÷50=

10 10 10 10 10

14) 15) 6÷2=

24

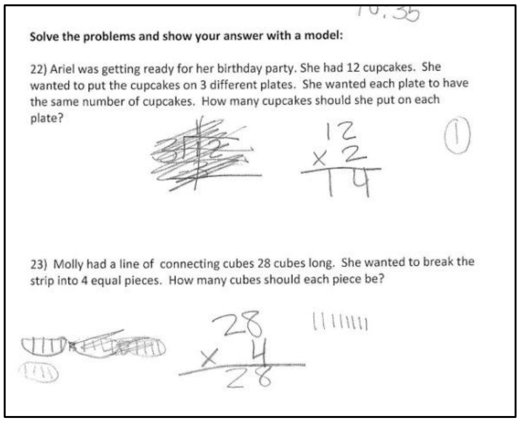
8 8 8

3
3
÷3

16)

4

Division problem solving pre-test


10:35

Solve the problems and show your answer with a model:

22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?

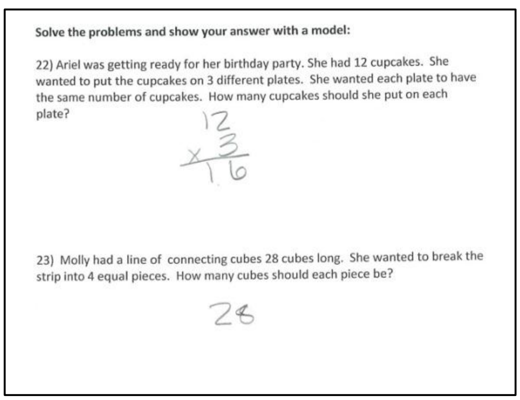
12
× 2
14

1

23) Molly had a line of connecting cubes 28 cubes long. She wanted to break the strip into 4 equal pieces. How many cubes should each piece be?

28
× 4
28

Division problem solving post test


Solve the problems and show your answer with a model:

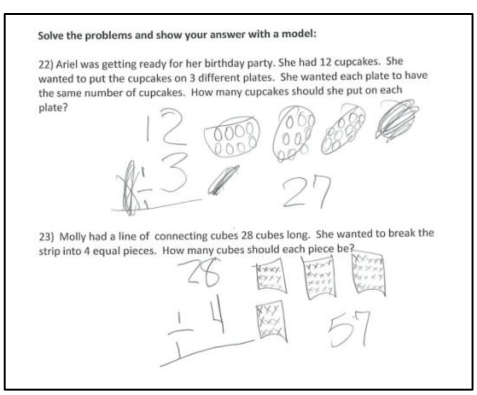
22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?

12
× 3
16

23) Molly had a line of connecting cubes 28 cubes long. She wanted to break the strip into 4 equal pieces. How many cubes should each piece be?

28

Division problem solving post test with accommodations of test read to her and visual aids available


Solve the problems and show your answer with a model:

22) Ariel was getting ready for her birthday party. She had 12 cupcakes. She wanted to put the cupcakes on 3 different plates. She wanted each plate to have the same number of cupcakes. How many cupcakes should she put on each plate?

12
÷ 3

27

23) Molly had a line of connecting cubes 28 cubes long. She wanted to break the strip into 4 equal pieces. How many cubes should each piece be?

28
÷ 4

57

APPENDIX E

Progress Reports for Parents and School

Big Jay

To the parent of Big Jay:

Thank you for allowing Big Jay to participate in mathematics tutoring this spring related to my dissertation study. I greatly enjoyed working with him. He was a pleasure to teach. While I know Big Jay learned a lot about mathematics and improved, I also learned a lot from teaching him.

This letter summarizes some strengths and struggles I observed in Big Jay during the time I tutored him and provides some information about how the tests I gave show his areas of improvement.

Big Jay is a very creative thinker. He is very good at mathematics and problem solving and seems to enjoy the subject. I would describe him as a "big picture" thinker. When solving mathematics problems, Big Jay seems to just see the answer in his head. He is very good at manipulating symbols mentally, which shows up in his good scores on basic math fact tests.

From my experiences with him, I would suggest that Big Jay struggles in mathematics for two reasons. One is that he often wants to solve problems in the way he thinks about them. He usually has an unusual way of going about solving a problem. He does not always look at the details. For example, one day when we were solving a story problem Big Jay knew the problem from the story was 7 x 8. He then said the answer was 42 instead of 56. He even drew a great model to explain his seven eights, but he never took the time to count and check that his sets of eights actually added up to 42. If he had checked his model to his answer he would have realized that he only had 42 instead of 56. This is an example of the type of errors Big Jay often makes. He has very good understanding of mathematical concepts and good number sense. He is good at problem solving and mental math. He has the kind of brain that can be an engineer or scientist someday.

In order to succeed in school math, I suggest that Dillon remember three things.

1) If he wants to solve a problem "his way" check with his teacher and explain what he is doing. Draw a picture or model to show how you are thinking about the problem. My concern for Big Jay is that he will get low grades in math if he does not show his work solving problems the way his teacher shows him. Big Jay's way is usually faster and makes more sense to him, but he may get frustrated when teachers want him to do it their way.
2) Double check your mental math. When an answer pops into your head, slow down and double check your work to make sure the answer you first thought is the correct answer. Big Jay often understands what he is doing but has a lot of small computational errors.

3) Ask for help. Don’t be afraid to tell your teacher when you don’t understand something. You are very good at math. If you don’t understand something, other kids in the class are probably also confused. Remember that it is okay to ask questions.

Big Jay is very visual and kinesthetic. He loved the activities we did with blocks. He would do well with math teachers who use a lot of games, concrete materials, pictures and models to teach mathematics. He also benefits from discussions that help him clarify the meaning of vocabulary words used in mathematics class.

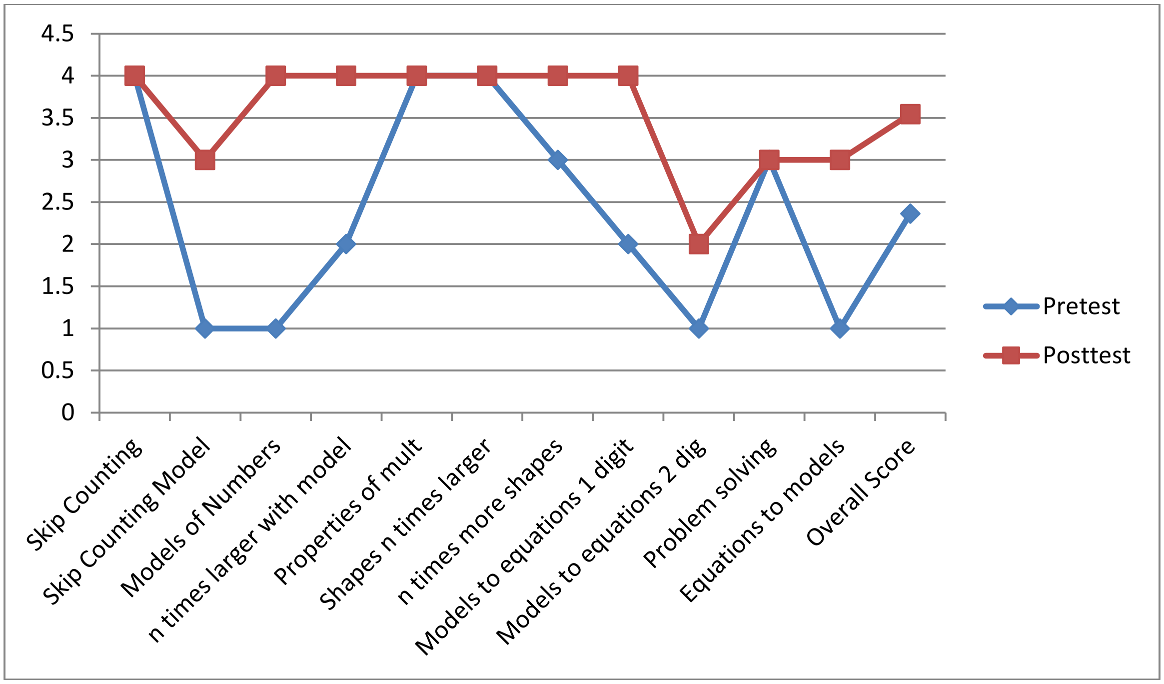
I hope this information is helpful to you. If I can assist you and Big Jay in any way to help him be successful in school, please do not hesitate to call me at 467-8250. I am also willing to conference with you about this report if you would like to.

Best wishes for a wonderful summer.

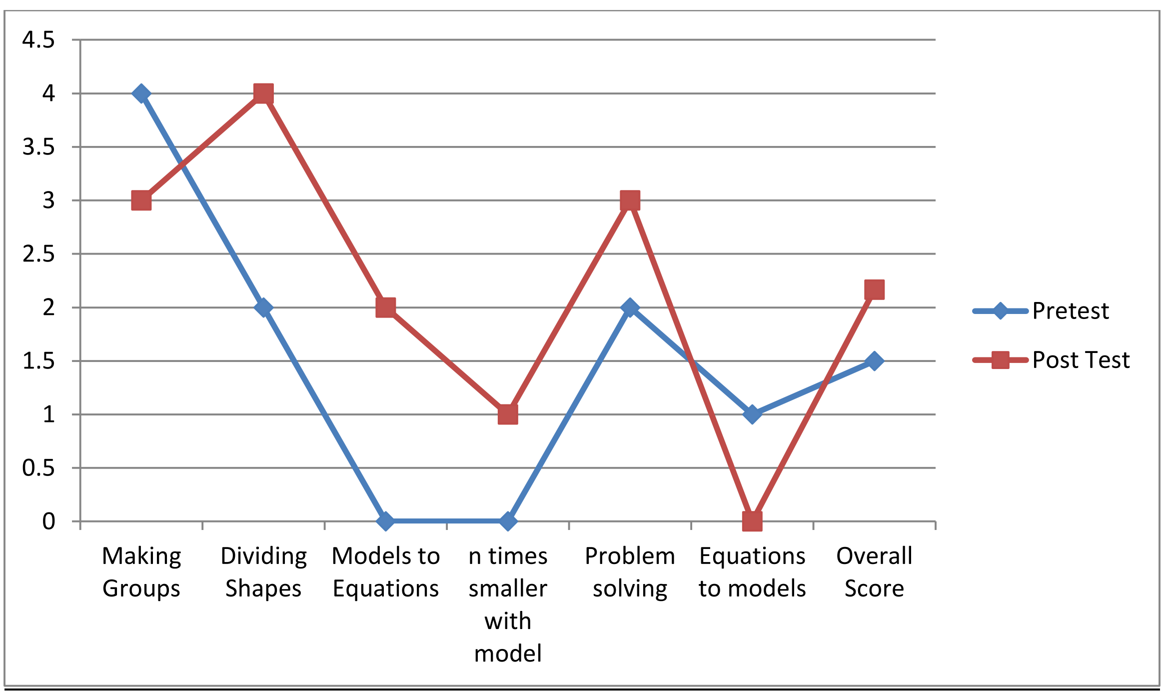
Sincerely,

Catherine Beals, EdS
Assistant Professor of Education

Big Jay - Multiplication Subtests



Big Jay - Division Subtests



Note: We did not spend very much of the tutoring time on division. We mostly studied multiplication and number patterns.

Destiny

To the parent of Destiny:

Thank you for allowing Destiny to participate in mathematics tutoring this spring related to my dissertation study. I greatly enjoyed working with her. She was a pleasure to teach. While I know Destiny earned a lot about mathematics and improved, I also learned a lot from teaching her.

This letter summarizes some strengths and struggles I observed in Destiny during the time I tutored her and provides some information about how the tests I gave show her areas of improvement.

Destiny is a very hard worker. She never gives up. She is very good at mathematics and problem solving and seems to enjoy the subject. She loves color and drawing. If she can learn to draw pictures and models to represent the math problems, that helps her a lot. Destiny is also very good at explaining how to solve problems to other students. When she does this, she also understands it better herself. Encourage her to talk about how she is solving problems and what she is thinking.

From my experiences with her, I would suggest that Destiny struggles in mathematics for two reasons. One is that she sometimes does not understand an English mathematics word like “multiple” or “opposite.” As she is getting into upper grades, there will be more math words in English that she may not know the Spanish word for. I have encouraged Destiny to not be embarrassed to ask a teacher to explain what a word means. I also suggest that she make a personal dictionary of important math words she needs to remember and she can make a picture or model to go with the word.

Destiny also needs to practice her basic facts for addition, subtraction, multiplication and division. She can do this with flashcards, playing cards and games. There are great games available at funbrain.com and coolmath4kids.com. If you need internet access, there are computers at the public library that she can use.

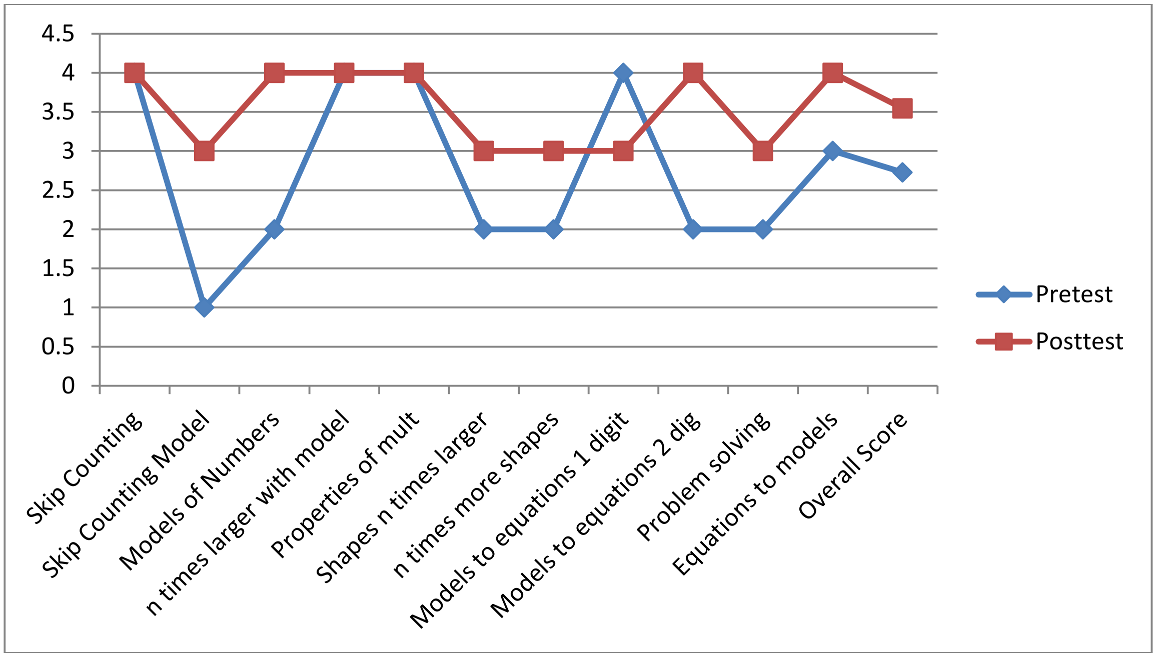
I hope this information is helpful to you. If I can assist you and Destiny in any way to help her be successful in school, please do not hesitate to call me at 467-8250. I am also willing to conference with you about this report if you would like to.

Best wishes for a wonderful summer.

Sincerely,

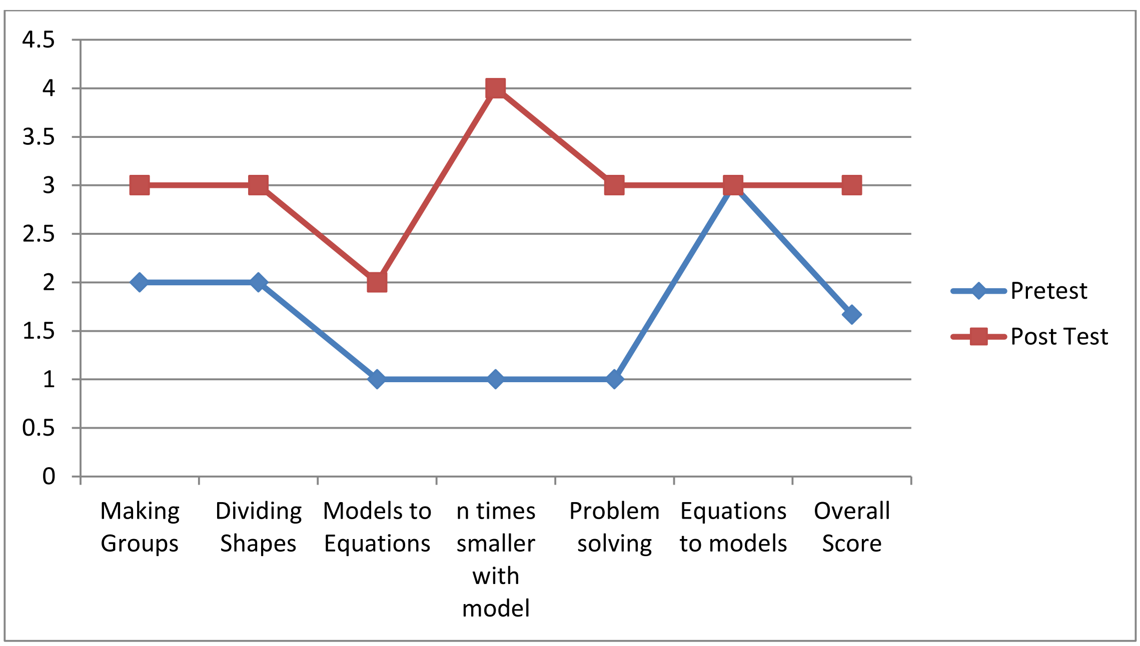
Catherine Beals, EdS
Assistant Professor of Education

Destiny - Multiplication Subtests



Destiny improved her multiplication facts from 32 to 50 in two minutes. Great job!

Destiny - Division Subtests



Note: We did not spend very much of the tutoring time on division. We mostly studied multiplication and number patterns.

Happy Gilmore

To the parent of Happy Gilmore:

Thank you for allowing Happy Gilmore to participate in mathematics tutoring this spring related to my dissertation study. I greatly enjoyed working with him. He was a pleasure to teach. While I know Happy Gilmore learned a lot about mathematics and improved, I also learned a lot from teaching him.

This letter summarizes some strengths and struggles I observed in Happy Gilmore during the time I tutored him and provides some information about how the tests I gave show his areas of improvement.

Happy Gilmore worked hard in tutoring and had wonderful behavior. He showed a lot of improvement in his understanding of multiplication and was getting very good at solving word problems at the end of the tutoring. He learned to draw models and make sense of numbers by drawing models to show what is happening in a problem. Happy Gilmore also enjoyed talking with the other students about problems and did a great job explaining how to solve a problem to the other students. He could explain his answers and work in both English and Spanish and taught me many Spanish words.

From my experiences with Happy Gilmore, I was able to get some ideas about why he is struggling in mathematics. He is still much lower in his abilities than other students in his grade level but he did make good progress in the tutoring classes. Happy Gilmore does sometimes struggle with the meaning of English mathematics words but that does not seem to be the main cause of his errors in mathematics. He seems to have a hard time making meaning from symbols and using mathematical symbols correctly. He does not recall basic math facts but can get answers quickly with the use of a 100's board we made. When Happy Gilmore had a model or picture to refer to, he could usually figure out the answer to a problem. He seems to have a hard time visualizing two and three digit numbers.

I have some suggestions to help Happy Gilmore succeed in math in school:

1) As he is getting into upper grades, there will be more math words in English that he may not know the Spanish word for. I suggest that he make a personal dictionary of important math words he needs to remember and make a picture or model to go with the word. He can refer to this during math class when he needs to.
2) Happy Gilmore needs to practice his basic facts for addition, subtraction, multiplication and division. He can do this with flashcards, playing cards and games. There are great games available at funbrain.com and coolmath4kids.com. If you need internet access, there are computers at the public library that he can use. Happy Gilmore may need to use his 100's board

for numbers over 5 with basic facts or he may find with practice he won't use it as much.

3) Happy Gilmore should continue drawing models and pictures to help him understand math concepts. He would also benefit from additional tutoring that reviews place value and addition and subtraction.

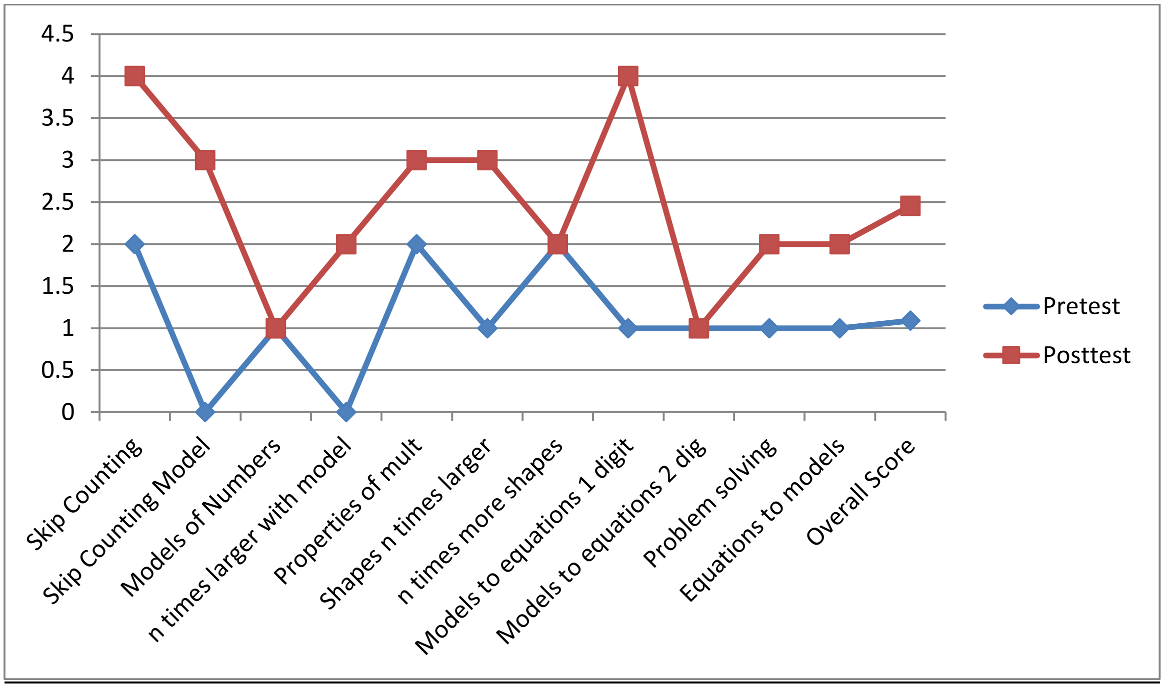
I hope this information is helpful to you. If I can assist you and Happy Gilmore in any way to help him be successful in school, please do not hesitate to call me at 467-8250. I am also willing to conference with you about this report if you would like to.

Best wishes for a wonderful summer.

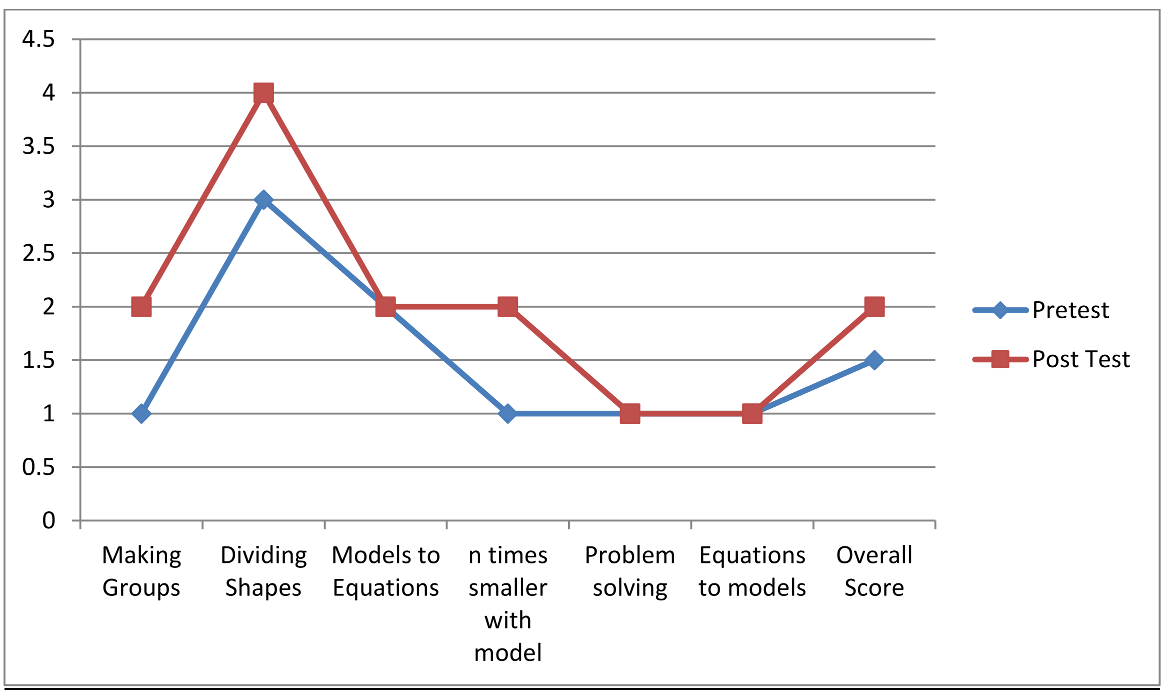
Sincerely,

Catherine Beals, EdS
Assistant Professor of Education

Happy Gilmore - Multiplication Subtests



Happy Gilmore - Division Subtests



Note: We did not spend very much of the tutoring time on division. We mostly studied multiplication and number patterns.

Katarina

To the parent of Katarina:

Thank you for allowing Katarina to participate in mathematics tutoring this spring related to my dissertation study. I greatly enjoyed working with her. She was a pleasure to teach. While I know Katarina learned a lot about mathematics and improved, I also learned a lot from teaching her. This letter summarizes some strengths and struggles I observed in Katarina during the time I tutored her. It also provides some information from the tests I gave.

Katarina worked hard and had good behavior during math tutoring. She enjoyed solving problems with the other students and loved to draw models of the problems on the board. She learned a lot of different ways to draw models of numbers and problems. That seemed to help her understand the mathematics better. Katarina learned to use a 100's board to figure out the answer to multiplication facts. Having a visual aid to remember math concepts and patterns helped her a lot.

From my experiences with her, I can see that Katarina struggles to understand mathematics. She does not understand key concepts that students usually learn in first and second grade. For example, she does not understand how to break a number like 92 into 9 tens and 2 ones. She did not know what a rectangle was. It is possible that she missed learning these concepts due to the fact that she has changed schools several times or because she is not understanding all English math instruction. Katarina did say she did better in mathematics when it was taught in Spanish in first and second grade.

Even when considering the language barrier and her changing schools, Katarina does seem to have a very difficult time visualizing numbers in her head. She cannot recall basic facts and struggles to convert a story problem to a model or picture and then to an equation even with very explicit instruction. Several times, she would seem to understand a skill or concept one day but not remember how to do it a few days later. As she progresses to middle school, the frustration she feels in math may continue to grow. I will provide some additional information to the principal at Sherman so that the school can consider how to best help Katarina continue getting better in mathematics. I recommend she be retaught addition, subtraction and place value.

I also suggest that Katarina practice her basic facts for addition, subtraction and multiplication over the summer. She can do this with flashcards, playing cards and games. There are great games available at funbrain.com and coolmath4kids.com. If you need internet access, there are computers at the public library that she can use.

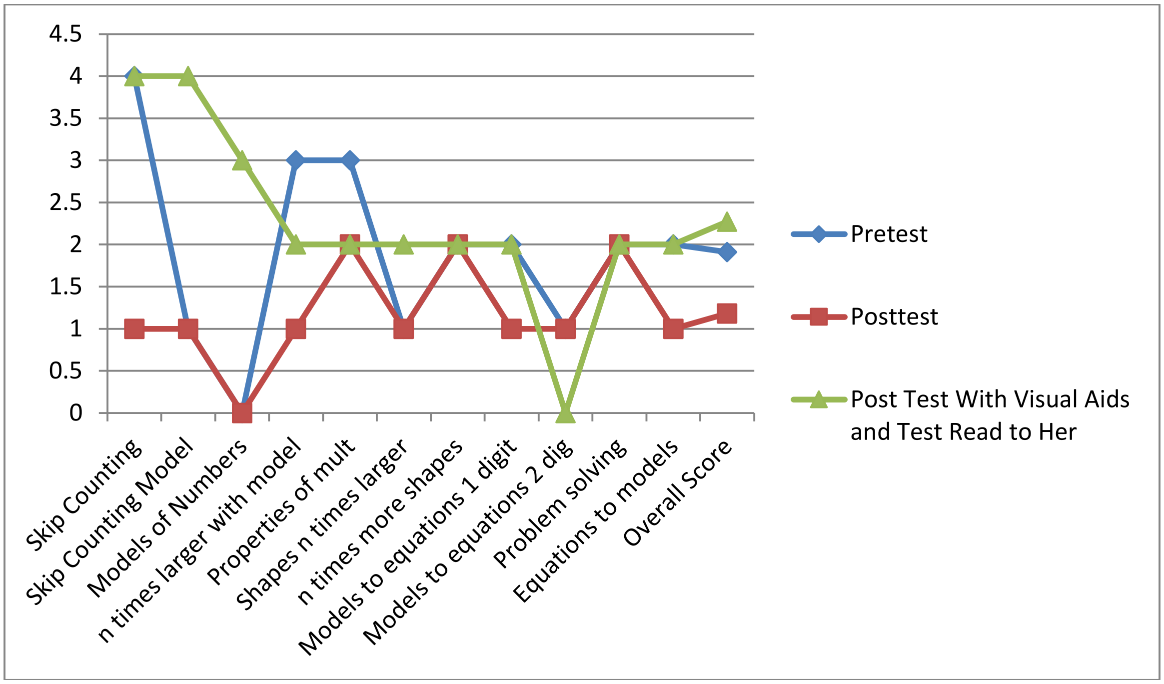
I hope this information is helpful to you. If I can assist you and Katarina in any way to help her be successful in school, please do not hesitate to call me at 467-8250. I am also willing to conference with you about this report if you would like to.

Best wishes for a wonderful summer.

Sincerely,

Catherine Beals, EdS
Assistant Professor of Education

Katarina - Multiplication Subtests



Note: This test shows that Katarina actually scored lower on the post test than the pre-test in some areas. She did better when she was allowed to use the visual aids we had made her and I read the test to her. Katarina missed a few weeks of schools around spring break. This may have impacted her improvement from pre-test to post test.

Katarina scored exactly the same on the pre-test and post test of multiplication facts. She had 23 correct facts in two minutes on both the pre-test and the post test.