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Resonant Frequency Derived from the Rayleigh Wave Dispersion Image: The High Impedance Boundary Problem

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Abstract

We present a simple and automated approach to estimate primary site response resonance, layer thickness, and shear wave velocity directly from a dispersion image for a layer over half-space problem. We demonstrate this for high impedance boundary conditions that lie in the upper tens of meters. Our approach eliminates the need for time-consuming dispersion curve picking and one-dimensional shear-wave velocity inversion for large data volumes that can capture velocity structure in profile. We highlight important relationships between dispersion characteristics and resonance parameters through synthetic modeling and field data acquired over Atlantic Coastal Plain sediments. In this environment, shallow soil conditions are critical to accurately estimate earthquake site response. We suggest that this image processing approach can be applied to a range of high impedance conditions at a range of scales, or can provide model constraints for more complex velocity structures.

Introduction

In the presence of a shallow, high-impedance boundary, seismic wave amplitudes can increase substantially by trapping energy in the shallow layer (e.g., Haskell, 1960; Borchardt, 1970; Shearer and Orcutt, 1987; Singh et al., 1988). These reverberations can increase the amplitude and duration of ground shaking at a resonant frequency (F_n). For primary resonance, F_n is defined by the layer thickness (h) and shear wave velocity (β_1) of the upper layer as

$$F_n = \beta_1/4h. \quad (1)$$

Rayleigh wave velocities and amplitudes depend primarily on the propagation of vertically polarized S waves through each layer in the subsurface and, secondarily on each layer's P-wave velocity (α) and density (ρ). Given an adequate range of frequencies and offsets, such that a Rayleigh wave propagates through more than one layer, the wave becomes dispersive. This dispersion is best observed by measuring phase velocity (c) using some kind of velocity-frequency coherence or semblance image. This analysis approach is common with the spectral analysis of surface waves (SASW) method (Stokoe and Nazarian, 1985), the multichannel analysis of surface wave (MASW) method (Park et al., 1999) and the refraction microtremor (ReMi) method (Louie, 2001) for characterizing β in the upper tens of meters. The results from these approaches tend to produce smoothed β - h profiles due to the nature of the multi-layer least-squares inversion algorithm that is often used.

Because a semblance plot produces simple curves for a layer over a stiffer half-space model, and because the secondary properties (α and ρ) have established empirical relationships for most soil types, high impedance two-layer scenarios present a straightforward relationship between the fundamental dispersion curve shape and F_n (Figure 1). Given a reasonable estimate of the upper layer's Poisson's ratio (ν), we can accurately relate the high-frequency semblance of c to β_1 . To obtain estimates of h and the lower layer shear wave velocity (β_2), we examine the low-frequency semblance of c . While an estimate of β through a two-layer inversion of fundamental mode dispersion picks can produce a robust solution, we show that we can directly estimate β_1 , F_n , and h from field data without the time-consuming steps of

picking dispersion curves and inverting these curves. This approach is most useful for examining large data volumes (e.g. seismic data in profile) with shallow hard boundary conditions, while eliminating uncertainties related to dispersion picking and inverse methods.

In this paper, we show synthetic and field-based active source seismic data examples where Rayleigh wave dispersion patterns provide a direct estimate of F_n in the presence of a shallow, high-impedance boundary. We focus on a non-gradational upper layer and a bedrock boundary that lies in the upper tens of meters by exploring model sensitivities to dispersion image features. We first show a forward modeling grid-search approach to obtain an accurate β -h profile through fundamental mode dispersion picks for a high impedance constant β sediment over rock model. This geologic setting is consistent with Atlantic Coastal Plain (ACP) strata that lie throughout the southeastern United States. This high impedance boundary has been noted as the most important site response input parameter for the region (Chapman et al., 2006).

We explore the data limits of our grid search method, then explore dispersion characteristics for a range of high impedance synthetic models to show how β_1 , F_n , and h can be directly extracted from the dispersion image without the effort of picking peak dispersion values and inverting these curves. We test this approach using an active-source seismic land streamer dataset with both surface and body wave targets. With this dataset, we pick dispersion curves and generate a best-fit model using our grid-search approach. Then, we compare these results to those derived directly and automatically from the dispersion image. We show that this image extraction approach can provide F_n estimates below the frequencies recorded with an active source with no picking or inversion uncertainties. We suggest that this approach can be used to rapidly assess the spatial variability of site response or liquefaction potential, can be used to generate starting models or place constraints for surface wave inversion of picked data, can provide constraints to complementary data such as horizontal-to-vertical (H/V) spectral ratio data, and can potentially be applied to more complex velocity models (i.e. more than two layers or layers with a velocity gradient) through a machine learning approach.

Rayleigh Wave Dispersion

A standard approach to estimate β from active source seismic data is through Rayleigh wave dispersion curve picks and inversion. With this approach, the peak semblance of c is measured over a range of frequencies and the data are used to estimate a β -h model after making assumptions about α and ρ . For a layer over half space model, most of the Rayleigh wave dispersion signal is contained in the fundamental mode and the dispersion curve fits a predictable pattern. To highlight the c - β_1 relationship, we show synthetic dispersion curves (Figure 1a) for models with properties typical of ACP strata (Andrus et al., 2006; Chapman et al., 2006; Heidari and Andrus, 2012). We model unsaturated ($\alpha=400$ m/s, $\beta=180$ m/s, $\rho=1.55$ g/cc) and saturated ($\alpha=1500$ m/s, $\beta=180$ m/s, $\rho=1.80$ g/cc) sediments for the upper layer over semi-consolidated materials for the lower layer ($\alpha=1600$ m/s, $\beta=400$ m/s, $\rho=1.85$ g/cc). We generate fundamental mode dispersion curves (Michaels and Smith, 1997) for models where $h=5, 15$, and 25 m to show that 1) the horizontal asymptote of these curves directly relates to β_1 , and 2) the dispersion curve inflection and the high-semblance vertical asymptote directly relates to F_n . With equation 1, these relationships provide a direct estimation of h from the dispersion image. We note that while theoretical dispersion curves will approach β_2 at low frequencies, this low frequency coherence is not typically observed on field records due to limited resolution or higher mode interference (e.g., Park et al., 1999; Gao et al., 2016).

For a layer over half-space model, the dispersion curve will flatten to a near constant c above frequencies where Rayleigh wavelengths only probe the upper layer. This high-frequency asymptote does not appear in the presence a velocity gradient for the upper layer, as all wavelengths probe different velocities. The horizontal asymptote of c relates to β_1 and v_1 (or α_1/β_1). For example, dispersion curves shift to a slower c for dry sediments when compared to saturated sediments because of the drastic reduction in α compared to a near constant β (Figure 1a).

The c - β relationship can be directly derived from the Rayleigh wave equations for a layer over a half space (e.g., Aki and Richards, 2002)

$$(2 - c^2/\beta^2)^2 = 4\sqrt{(1 - c^2/\alpha^2)(1 - c^2/\beta^2)}. \quad (2)$$

Equation 2 is simplified for a Poisson solid where α/β is about 1.8 and $c \approx 92\% * \beta$. While a Poisson solid is a reasonable approximation for hard rock, it is inappropriate for unconsolidated, high-porosity sediments in a fluid-supported suspension (e.g., Mavko et al., 2009). For example, α/β for unconsolidated saturated marine sediments can exceed 13 at the sea floor (e.g., Hamilton, 1979) and α/β for saturated ACP sediments measures about seven (e.g., Odum et al., 2003; Andrus et al., 2006). For similar dry soils at low confining pressures, α/β approaches a Poisson solid (e.g., Han et al., 1986; Prasad et al., 2004). Figure 1a shows that β_1 and v_1 controls the asymptote of c . Because of the high α - β ratio, the c - β ratio for saturated soils will always be greater than that of a Poisson solid and that of the same unsaturated soils.

To extract a reasonable estimate of β_1 from the asymptote of c , we use the Castagna et al. (1985) empirical “mud line” relationship that ties β to α for saturated sediments. This relationship

$$\alpha = 1.16\beta + 1360, \quad (3)$$

where α and β are in units of m/s, was established for clay-rich sediments (similar to ACP sediments) and has been shown to work for a range of low-velocity unconsolidated sediments (Brocher, 2005). From Equations 2 and 3, we develop a relationship between the c - β ratio and α/β or v (Figure 2). For simplicity, we use the National Earthquake Hazard Reduction Program (NEHRP) site classifications for soil type (Building Seismic Safety Council [BSSC], 2009). For soft, saturated NEHRP site class E soils ($\beta < 180$ m/s), a high α/β leads to a c - β ratio of about 95.5%. For stiffer NEHRP site class C saturated soils (β ranges from 360 to 760 m/s), the c - β ratio decreases to about 95%.

For unsaturated sediments, we highlight the empirical relationships of Han et al. (1986) and Prasad et al. (2004) at low confining pressures. For dry conditions, the mud line relationship is not valid and β is driven primarily by confining pressure, but also by soil type and porosity (e.g., Mavko et al., 2009). Prasad et al. (2004) established a general relationship of

$$\beta = 436 * (P^{0.243}), \quad (4)$$

and

$$\alpha = 774 * (P^{0.227}), \quad (5)$$

for dry unconsolidated sands and clays, where P represents confining pressure (MPa) and where α and β are in units of m/s. They showed that for typical dry sediments at depths of tens of meters, α/β ranged from 1.6 to 2.5 and v ranged from 0.21 to 0.41. From their laboratory measurements, they showed that increasing clay content and porosity increases this ratio. From these equations, Figure 2 shows that for typical dry soils in the upper tens of meters, c ranges from about 92% to 94% of β .

While the range of c - β ratios can only vary by a few percent for a reasonable range of soil properties, this relationship can be refined for saturated or unsaturated conditions. Thus, we conclude that one key site response parameter (β_1) can be directly estimated from the dispersion image by summing the peak semblance for high frequencies (Figure 1b) and adjusting for inferred soil properties and saturation (Figure 2). The asymptote of c can provide a reasonable estimate of soil stiffness (NEHRP site class) and saturation can be inferred from first arrival times or hydrologic measurements. Using equations (3) through (5), we can simplify equation (2) to provide direct relationships between α and β . Table 1 summarizes values of α , v and c for a range of β for both saturated and unsaturated conditions.

To estimate F_n from the dispersion image, we analyze the dispersion curve inflection and the high-semblance vertical asymptote. Specifically, we focus on the relationship between F_n and the fundamental Rayleigh wave period. Figure 1a shows that with an increase in boundary depth, the dispersion curve inflection shifts toward lower frequencies. In fact, this figure suggests that the sensitivity of the vertical asymptote closely matches one-half of the wavelength of the upper layer, or twice F_n as defined in equation (1). Consistent with this observation, Pina-Flores et al (2020) recently showed that the Rayleigh wave vertical asymptote matches an H/V spectral ratio minimum that relates to upper layer properties.

We will explore the β_1/F_n relationship and sensitivities through a range of two-layer velocity/depth models to reinforce this relationship with the vertical asymptote of the dispersion curve. We will first solve for velocity-depth models through a dispersion pick and grid search approach that identifies a best-fit model. We will then estimate β_1 , F_n , and h directly from the dispersion image and compare the two approaches.

Grid-Search Estimation

To match dispersion properties to model parameters, we generate 39 synthetic “observed” fundamental mode Rayleigh wave dispersion curves using code derived from Michaels and Smith (1997). The models have the same α , β and ρ as our saturated model described in Figure 1 and we vary the thickness of the upper layer between 2 and 40 m in one meter increments. To recover the model parameters from these dispersion curves, we compute dispersion curves for all possible combinations of velocity and depth parameters in steps of 10 m/s for β and 0.4 m for h . We measure the RMS misfit between observed and calculated picks at 1 Hz intervals between 4 to 40 Hz (Figure 3). For the final recovered model, we choose the model that minimizes the RMS misfit between the observed and calculated data. We choose this frequency range because they match the band limitations of many field datasets and because for our range of depths, these frequencies define the limits of our dispersion curve models.

Our grid-search approach accurately estimates β_1 below four meters depth and h to depths upward of 18 m (Figure 3). For shallower depths, frequencies above 40 Hz are needed to accurately estimate β_1 . From 18 to 40 m depth, the minimum misfit occurs at the correct depth to within one meter depth (less than 3% error). This suggests that F_n can be estimated from the dispersion curve picks themselves at most sampling depths for this model and within the select frequency range. This numerical exercise also demonstrates that β_2 is sensitive to the dispersion curve shape, but this parameter loses sensitivity below about 30 m depth because we probe mostly the upper layer at low frequencies ($\beta_1=180$ m/s at 4 Hz or 45 m wavelength). To obtain accurate estimates of β_2 at greater depths, lower frequency signals are needed but often are poorly resolved. We note that β_2 is not needed to estimate F_n , but it is needed to estimate impedance at the layer boundary. Thus, while the impedance can be estimated by using low frequency dispersion picks, β_2 is less well constrained when compared to β_1 and h for the studied range of frequencies and depths.

Sensitivity of Dispersion Image Derived Parameters

Figure 1 shows that, for our initial model, summation over a range of 4 to 50 Hz in the dispersion image produces a velocity semblance peak between 94% to 95.5% of β_1 for saturated sediments and 92% to 94% of β_1 for unsaturated sediments. We also show that summation over all phase velocities produces a peak in frequency that approximates $\beta_1/2h$ or twice F_n . Although this relationship may slightly vary with saturation or α , β_2 , and ρ , we suggest that these variables do not significantly influence the vertical or horizontal asymptote of the dispersion image. To test this hypothesis, we present a range of synthetic models for 5 to 40 m thick NEHRP Class E, D, and C soils (180 m/s to 480 m/s) in a top layer that lies above a hard boundary. We explore models where the β_2/β_1 impedance ranges from 2 to 4 for both saturated and unsaturated conditions in the upper layer. For saturated models, we use equation (3) to provide estimates of α and the empirical relationship of Boore et al. (2016) to compute ρ values from β . For dry models, we use $\alpha/\beta = 2.2$ (Prasad et al., 2004) and a soil $\rho=1.5$ g/cc. We limit β_2 to less than 1500 m/s or NEHRP Class A rock and assume the Castagna et al. (1985) and Boore et al. (2016) relationships are valid for the lower layer half space.

To produce dispersion images, we use the elastic finite difference Seismic Unix forward modeling program *suea2df* (Cohen, 2015) to generate over 1,000 1-D vertical component synthetic shot records using a vertical force source and a 50 Hz Ricker wavelet. While signal attenuation can impact dispersion, we assume no dispersion effects from amplitude attenuation. We use a receiver spacing of 0.5 m from 0-100 m offset (201 receivers) to produce unaliased and smooth dispersion images for our range of frequencies. We transform each shot record into the dispersion domain using the Seismic Unix program *suphasevel*. From the dispersion image, we estimate β_1 by summing the semblance values between 50% of the true β_1 value to β_2 along the horizontal axis. This summation produces a peak value of c that probes mostly the upper layer (Figure 1). We estimate F_n by summing the semblance values along the vertical axis. We only sum values of c to twice β_1 to capture the dispersion curve inflection. We smooth the summed semblance values using a five point running average and then pick the peak value. We highlight dispersion image sensitivities to both β_1 and F_n for a range of models in Figure 4. Note from our Figure 1 example that higher modes do not produce large coherence on our two-layer dispersion images.

Figure 4 shows that by summing semblance image values along each axis, we can closely match β_1 and F_n . We show that the maximum semblance amplitude along the horizontal axis ranges from 93% to 95% of β_1 for unsaturated sediments and between 95% and 97% of β_1 for saturated sediments. If we use the peak semblance value and the site class relationship that ties c to β (Figure 2 and Table 1), we can approximate β_1 . We observe that some models with boundary depths less than about 10 m overestimate c . We relate this misfit to the 50 Hz frequency limitation of our dispersion image, as higher frequencies are needed to fully capture the horizontal asymptote. In other words, we need a range of frequencies that only probe the upper layer. We suggest a visual inspection of the dispersion curve will easily determine whether the asymptote of c has been reached for coherent frequencies within a dataset.

If we sum the dispersion image along the vertical axis over the range of c that captures fundamental mode dispersion, we obtain a peak semblance value that is close to twice F_n (Figure 4). We observe that the image-derived F_n and true F_n matches best for high-impedance, low F_n models as compared to low-impedance, high F_n models which do not match as well. This is important as high impedance boundaries produce higher site amplifications (e.g., Boore and Joyner, 1997) and are often the focus of site response studies. For our saturated models where $\beta_2/\beta_1=4$, we measure a coefficient of determination (R^2) of 0.987 between observed and calculated values of F_n . For saturated models where $\beta_2/\beta_1=2$ we measure $R^2=0.876$ for F_n . For our $\beta_2/\beta_1=4$ unsaturated models, we measure $R^2=0.982$ and for our unsaturated models where $\beta_2/\beta_1=2$, we measure $R^2=.879$.

The increasing misfit of lower impedance models directly relates to β_2 or to the steepness of the dispersion curve inflection. This parameter cannot be easily extracted directly from two dispersion features that we highlight. Also, our cross plot shows increasing divergence between observed and calculated values with increasing F_n . We underestimate F_n for almost all models with an observed F_n above 20 Hz. While these higher frequencies are often not of concern when considering damage from earthquake ground motions (e.g., Boore and Joyner, 1997), we suggest that with the extraction of more image features, we may be able to recover more accurate and more robust models (e.g., estimating β_2 or β_2/β_1 impedance).

Field Example.

We now compare field-based dispersion picking and dispersion image-based estimates of β_1 and F_n . We utilize a one km-long active-source seismic profile located about 25 km west of Charleston, South Carolina. Regionally, ACP deposits place late Quaternary saturated soils unconformably over Tertiary sedimentary rocks (Andrus et al., 2006; Chapman et al., 2006). This seismic land streamer dataset contains 433 72-channel shot records, with 1.25 m spaced contact coupled 10 Hz geophones embedded in a fire hose (Liberty, 2018). We acquired shots every 2.5 m using a 40 kg hitch-mounted accelerated weight-drop source, consistent with an MASW survey. The profile was acquired along a dirt road that lies adjacent to the Stono River, only a few meters elevation above sea level. For the same lithology, Andrus et al. (2006) observed that the top layer Quaternary strata had no clear vertical gradient and suggested the large impedance between this layer and the underlying Tertiary rocks was responsible for high frequency (2-5 Hz) site amplification related to the 1886 Charleston earthquake. Heidari and Andrus (2012) also identified high liquefaction potential with the Quaternary sediments, in part due to their low β in the upper few meters. The depth to top of Tertiary along this profile was confirmed to be 10 m depth or less from a nearby boreholes (Weems et al., 1987). Borehole RA-16 lies adjacent to the profile.

The shot records show coherent surface wave energy from about seven to 40 Hz (Figure 5a) and filtered shot records show clear body wave energy where first arrivals travel about 1500 m/s (Figure 5b). This observation is consistent with saturated unconsolidated sediments within the upper few meters of the ground surface. We transform the shot gathers into the dispersion domain and we fit each of the manually picked dispersion curves to a one-dimensional, layer over half-space model using our grid-search approach. We minimize the misfit between observed and predicted values for a range of synthetic models to derive our final solution. The grid-search solution indicates that β_1 ranges from about 130 m/s to 170 m/s across the profile (Figure 5d). Grid-search estimates of h range from 5 to 8 m depth (Figure 5f). From these two values and equation (1), we estimate that F_n ranges from five to seven Hz (Figure 5e).

For the image processing approach, we sum each shot record dispersion image along horizontal and vertical axes. The dispersion image derived from our example shot record shows that the peak semblance summed over all frequencies at about 150 m/s and the peak semblance summed over all values of c is about 9.5 Hz (Figure 5c). Because the horizontal summation appears as a normal distribution on most shots (Figure 5d), this supports the no-gradient upper layer observation of Andrus et al. (2006). Had we observed asymmetry or skew in the horizontal summation toward higher c , we would have suggested the upper layer was best represented by a velocity gradient. The horizontal

summation shows the high frequency values of c directly relate to β_1 derived from dispersion picks. Based on the known saturated conditions at this site and values of c that generally fall under NEHRP site class E soils, we can convert the asymptote of c to β_1 with a correction factor of 95.4% (Table 1).

A comparison between the two approaches shows an overestimate of β_1 of 4.2 m/s or 3% using the image processing approach compared to our grid search solution. We suggest that this misfit between the two approaches could result from picking uncertainties, grid search derived model uncertainties, from an incorrect assumption of a constant β_1 , incorrect estimates of α or ρ , or from the peak semblance not representing the c asymptote. Assuming we relate the misfit to our image processing approach, we relate the source of error to the inconsistency related to the c asymptote and that of peak semblance for the selected range of frequencies. When we apply a 97% correction factor of c to estimate β_1 we reduce the misfit between the two approaches to 1.7 m/s or a 1.4% difference. This correction factor is consistent with the synthetic results with similar model inputs (Figure 4), suggesting that higher frequencies are needed to fully capture the wavelengths that only probe the upper layer. While our estimate of β_1 may contain a slight error from the limited frequency content of the field data, both approaches show consistent lateral variability along the profile and we conclude that the image processing approach provides a robust estimate of β_1 . The benefit to the dispersion image processing procedure is that it does not allow dispersion pick uncertainties to be mapped into the solution space. Likewise, uncertainties related to either a grid search approach or similar inversion do not enter our image processing solution.

When we sum the dispersion image along the vertical axis for each shot record, we can compare the peak semblance to estimates of F_n derived from the grid-search solution along the entire profile (Figure 5e). Because peak semblance and F_n differ by a factor of two, we halve the image frequencies for display (Figure 5e). The results suggest that we consistently underestimate F_n by, on average, 0.5 Hz using the dispersion image processing approach. This is not surprising considering observations highlighted on Figure 4 for β_2/β_1 impedance that ranges from 2 to 3 from nearby downhole measurements (Andrus et al., 2006). However, we emphasize here that we have captured the lateral variability of F_n with the dispersion image processing approach without the effort of picking dispersion values. Given a reasonable impedance estimate, a correction factor could be included to account for local field conditions. We also emphasize that the peak semblance in the vertical summation represents $\beta_1/2h$, allowing estimates of F_n to half the lowest recorded frequency for each gather.

Because we estimate β_1 and F_n directly from the dispersion image, we can compare estimates of h from the grid-search solution and from the dispersion image processing (Figure 5f). With the use of equation (1) to estimate h from our picking solution, we observe that the image processing approach overestimates our grid-search derived h by, on average, 1.1 m for all shots. While we recognize uncertainties with both of our β - h solutions, the general topography of the Quaternary base is similar with both approaches. If we recalculate our grid-search models using dispersion picks of ± 10 m/s for each gather (Figure 5c), we can provide a measure of uncertainty from our grid search approach. Using this metric, we see that our image-derived estimates for h all lie within our uncertainty range. Borehole RA-16 from Weems et al. (1987) places the top of Tertiary strata at 6.7 m depth, consistent with estimates of h from both approaches.

Discussion

The image processing approach works because of the direct relationship of the phase velocity to probing depths for a two-layer system and because of empirical relationships between ρ , α and β for most soil types. While we focus our study on site response using an example from ACP strata that occur throughout the southeastern United States, we suggest that the parameters estimated directly from the dispersion image can be used for other site response locations and for other geologic or engineering studies at a range of scales. This assumes that a layer over half-space model is sufficient to represent subsurface conditions.

Our observation that the dispersion curve shape provides a direct assessment of subsurface properties is not new. We note that crustal thickness estimates have been obtained from global surface wave datasets by the shape of the dispersion curve (e.g., Lebedev et al., 2013), and that maximum curvature in Rayleigh and Love-wave phase-velocity dispersion curves has been tied to SH-wave resonances and H/V spectral ratio characteristics (e.g., van der Baan, 2009; Pina-Flores et al., 2020). We suggest that our F_n estimation approach may be appropriate for ambient noise, ReMi, or Love-wave surveys that can be processed in a similar manner to MASW data, often with a lower frequency content to probe high impedance boundaries at greater depths. Beyond site response, estimates of β_1 and h are used

with liquefaction susceptibility studies within the ACP (e.g., Heidari and Andrus, 2012) and with the broader hydrogeophysical and critical zone communities (e.g., Hayes et al., 2019; Mi et al., 2020), suggesting this image processing approach may have broad appeal. Direct estimates of β_1 and h could also provide key inputs to inform reflection processing, such as static constraints that require near surface velocity estimates and boundary depths, or optimal filter designs to best attenuate high-amplitude surface wave signals. We suggest that two layer models are valid for many geologic characterizations where Rayleigh wave dispersion images can be directly utilized to assess site conditions.

One strength of the image processing approach is the elimination of dispersion curve picking uncertainties and non-unique inversions to provide site response estimates. While some may be motivated to pick dispersion curves to extract β_2 or for other properties beyond estimates of β_1 , F_n or h , we suggest that the image processing solutions could provide independent support for inverse solutions to constrain model uncertainties. These constraints could be derived rapidly and automatically from the dispersion image.

We suggest that through more complex image processing approaches and with the use of different image features, a characterization of β with gradient conditions or more than two layers may be possible. While we don't explore gradient or multi-layer models here, these models are worthy of exploration with image processing approaches. Extraction of β_2 directly from the dispersion image is also possible, as we show the sensitivity of this parameter to the dispersion curve. This constraint may provide insights into whether high frequency resonance should be considered or this value may provide the amplitude response at F_n .

One weakness of this approach is the bias of image derived F_n values for some physical properties, mostly for low impedance models (Figure 4). This bias should be considered in uncertainty estimates. Ultimately, a range of synthetic models could be constructed to provide a training dataset for a machine learning approach to image derived β - h models.

Conclusions

We show that, through a simple and automated dispersion image processing approach, we can directly estimate β_1 and F_n from the dispersion image in the presence of a shallow, hard boundary. From these values, we can estimate h through equation (1) and constrain β_2 . We highlight these relationships through synthetic modeling and field data. We compare these results with those derived from the time consuming effort of dispersion curve picking and inversion with a two-layer model. Through a numerical study, we show that with increasing impedance, F_n estimates approach the true value. While we recognize that not all field conditions are best described as a constant velocity upper layer over a large impedance boundary, we suggest that the features extracted from the dispersion image processing approach could be expanded to address a range of velocity models. For example, one could exploit the skewness (deviation from a normal distribution) of the horizontal axis summation to estimate a β_1 gradient where high frequency c continues to sample a range of β . Similarly, one could explore the boundary conditions (e.g. impedance) by exploring the character of the frequency summation. We suggest that our approach is a step toward a deep learning estimate of β for a range of depths.

Data and Resources

Field records are available through the Incorporated Research Institutions for Seismology (IRIS) as assembled data set 20-029, https://doi.org/10.7914/SN/3N_2018.

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Table 1. Relationships between β , α , v and c for upper layer saturated conditions using equation (3) and for unsaturated conditions using equations (4) and (5) of Prasad et al (2004) for confining pressures of 0.2 MPa or about 10 m depth.

β_1^*	α_1^*	v_1	α_1/β_1	$c-\beta_1$ ratio	c asymptote [†]	saturation
180	1568.8	0.493	8.72	95.4%	171.8	saturated
240	1638.4	0.489	6.83	95.4%	228.9	saturated
360	1777.6	0.479	4.94	95.3%	342.9	saturated
480	1916.8	0.467	3.99	95.1%	456.5	saturated
760	2241.6	0.435	2.95	94.7%	719.6	saturated
180	338.7	0.303	1.88	92.8%	167.0	unsaturated
240	443.1	0.292	1.85	92.6%	222.3	unsaturated
360	647.2	0.276	1.80	92.4%	332.5	unsaturated
480	846.7	0.263	1.76	92.1%	442.3	unsaturated
760	1300.7	0.241	1.71	91.8%	697.5	unsaturated

* m/s; † represents the high frequency fundamental mode peak dispersion curve semblance.

Figure 1. (a) Fundamental dispersion curves for different layer over half-space models. These models represent 5, 15, and 25 m of Quaternary sediments over Tertiary sedimentary bedrock. Velocities and densities are shown on the figure using m/s and g/cc units, respectively. Dashed lines represent dry soil over bedrock models, while solid lines represent saturated soils over bedrock models. Note the horizontal asymptote for each model ranges from about 93% to 95.5% of β_1 . The vertical asymptote of each dispersion curve approximates twice the resonant frequency (F_n) or $\beta_1/2h$, as shown by the bold vertical lines. (b) Frequency-velocity dispersion image for a 10 m saturated upper layer using physical properties from (a). Horizontal and vertical image semblance summation curves that lie to the right and top of this figure show peaks that relate to β_1 and F_n .

Figure 2. C/β versus α/β (solid black) and c/β versus v (solid gray) using the mudline equation (3) for a range of saturated and unsaturated soils. Dashed lines represent transition from dry to saturated conditions for the α/β (dashed black) and v (dashed gray). Asterisk on each curve represents the global rock average for α/β or v . Porosity increases with increasing v for saturated models and clay content and porosity increases with increasing v for unsaturated models. NEHRP site class range for saturated models are indicated by thin horizontal black lines.

Figure 3. Grid search best fit solutions for a range of velocities and h using Figure 1 layer parameters for saturated soils. Black lines show true model parameters, gray lines and circles show best-fit grid-search results.

Figure 4. (a) Dispersion image summation along the phase velocity axis for models where β_1 ranges from 180 m/s to 480 m/s and the β_2/β_1 impedance ranges from 2 to 4. Note that for a shallow boundary, the frequency summation overestimates β_1 because of our 40 Hz upper frequency limit. (b) Cross plot between predicted and dispersion image-derived estimates of F_n derived from summation of the dispersion image along the frequency axis. Note the deviation of the image derived and predicted models for the high F_n shallow boundary models.

Figure 5. a) Unprocessed field record 8420 showing that low velocity surface waves dominate the wavefield. b) Filtered (30 to 200 Hz) field record 8420, confirming soil saturation from a first arrival velocity of about 1500 m/s. c) Dispersion image from (a). This shot lies adjacent to borehole RA-16 (Weems et al., 1987); Frequency- and velocity-axis summations are included along each margin. d) Frequency axis summation from all shot records plotted in profile view. Red points represent the $c = \beta_1 * 95.5\%$ (saturated E-Class soils) grid-search solution and blue points represent the peak frequency axis summations. e) Phase-velocity semblance summations for all shot records plotted in profile view. Red points represent F_n derived from our grid-search approach, and blue points represent F_n estimated from the peak phase velocity semblance. The image-derived F_n represents $\frac{1}{2}$ the value derived from (c) for each field record. f) Quaternary strata thickness derived from our grid-search approach (red circles) and from our dispersion image processing approach (blue circles). The red dots represent grid-search models derived from ± 10 m/s of each dispersion pick for all gathers. Borehole RA-16, drilled to the top of Tertiary strata, provides a direct depth comparison for the two approaches.