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Student Perception of Mathematical Modeling Before and After Completing a Two Joint Robot Computer Simulation Task (RTP)

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Abstract

Engineers frequently utilize computer simulation as part of their design processes to model and understand the behavior of complex systems. Simulation is also an important tool for developing students' understanding of modeling and strengthening their intuition for problem solving in complex domains. This project uses a two-joint robot arm problem and accompanying computer simulation to demonstrate to AP BC Calculus students how and why we would use calculus concepts simultaneously in Cartesian and polar coordinate systems. We developed the simulation in a way that allows students to experience mathematical modeling in an applications-based engineering context. A small cohort of students in AP BC Calculus completed an open-response survey of their perceptions on mathematical modeling before and after completing our simulation. Analyzing these data using direct content analysis showed that students seemed to increase their understanding of mathematical modeling as an iterative process, although some students narrowed their description to focus on computer simulation. This study supports the role of simulation in developing students' understanding of mathematical modeling and developing specific content knowledge, and how engineering can provide a valuable context for the application of mathematical modeling.

Introduction

Mathematical modeling is a critical component of math, science, and engineering education [1]–[7]. Both the Common Core State Standards for Mathematics (CCSSM) and the Next Generation Science Standards (NGSS) emphasize the importance of mathematical modeling [1]. Mathematical modeling in the classroom helps to develop the critical thinking and math skills required for engineering [2]. It allows students to "revise their preconceptions and… understand the underlying principle[s] of mathematics" [8] and integrate topics similar to professionals in the field [1]. Students are expected to engage in modeling throughout engineering, math, and science curricula [3].

One way to bring mathematical modeling into the classroom is to use a simulation task with engineering applications. In this study, researchers investigated how completing such a task influences student perceptions of mathematical modeling. Using a simulation provides quick and efficient feedback in a cost-effective manner [4]. Simulations also allow students to explore cause and effect relationships between variables [5], test a large number of different models [6], and develop intuition about difficult concepts [7].

Researchers selected the two-joint robot arm as the simulation task for students enrolled in AP BC Calculus (BC Calculus) classes at a public high school in the intermountain west region of the United States. BC Calculus is an Advanced Placement course that is roughly equivalent to the second semester of college calculus. The two-joint robot arm is often used in upper-level engineering courses or modules with a prerequisite of differential equations and linear algebra (e.g., [9], [10], [11]), but it has also been used earlier in the curriculum as a real-world application of trigonometry and calculus-based physics [12],[13].

Sultan designed an activity for students in a precalculus class who have the geometric understanding to determine the position of the end effector, but do not yet understand the concepts of differentiation and rates of change necessary to calculate the velocity [12]. Berkove & Marchand [13] described a robot arm in space, and students calculated forces that the robot arm exerted. Their activity required students to have a strong background in physics, and high school students have varying levels of physics understanding. After more than a year of introductory calculus, though, high school students in BC Calculus are well versed in the differential relationship between position and velocity. This knowledge allows students to explore the two-joint robot arm from the perspective of the motion of the end effector without the need to introduce new physics concepts, differential equations, or complex matrices.

The two-joint robot arm simulation addresses the exploration of parametric equations and working between polar and Cartesian coordinate spaces. These skills are part of the BC Calculus curriculum, and tested on the exam [14]. To address student perceptions of mathematical modeling, researchers designed the task to maximize student engagement in as many aspects of the GAIMME modeling process [15] as possible within the time constraints of the course.

Literature review

There are many definitions for mathematical modeling and across the STEM spectrum [15]. In this study, mathematical modeling was defined using the GAIMME modeling process [15] which includes six interrelated steps:

- Identify and specify the problem to be solved
- Make assumptions and define essential variables
- Do the math: get a solution
- Implement the model and report the results
- Iterate as needed to refine and extend the model
- Analyze and assess the model and the solutions

Many studies investigate how students engage in mathematical modeling or simulation (e.g. [1], [7], [16]), but not how students define the mathematical modeling process. McKenna and Carberry [3] focus on a broader definition of modeling in the engineering design process that includes “any representation of some physical phenomena”. They assessed modeling because it is prolific across math, science, and engineering courses. They found that while students consistently responded about physical aspects of the design process (e.g. prototypes, drawings, charts), students mentioned mathematical modeling significantly less frequently than professors.

The category of mathematical modeling included “ideas represented by mathematical equations and calculations” [3]. Students were also less likely to mention other abstract models, specifically theoretical/conceptual and verbal models. McKenna and Carberry [3] concluded that, while students engage in mathematical and other abstract modeling activities throughout the engineering curriculum, they do not necessarily recognize the importance of these tools in the design process.

Instructors have introduced engineering tasks in calculus classes as a way to increase students’ problem solving skills [17] and improve critical thinking [2]. When students engage in mathematical modeling in the engineering curriculum, it is often in the form of simulation [5]. Simulation may help students develop modeling skills while also deepening their intuition of complicated math topics [5]–[7]. Dickerson and Clark [7] researched the role of SPICE (an electronics circuit simulation computer program) in university microelectronics courses. They explored the difference between teaching a course using an interactive simulation in-class versus teaching the course without. Students reported that engaging in the simulation helped them with test and quiz problems, and that they felt they understood something from the simulation that they would not have learned without it. These students scored higher on the final exam than students who did not take the course with interactive simulation [7].

Modeling in the classroom, including the mathematical modeling task for this project, is often different from modeling in a professional context [4]. Develaki [4] points out the difference between modeling by scientists and modeling in an educational context. Scientists use modeling and simulation in conjunction to develop new, unproven theories that they then test and modify [4]. Similarly, engineers use modeling and simulation in the design process to develop new and innovative solutions to problems [3]. Students, on the other hand, engage in “educational modeling”, where they change specific parameters and initial conditions to develop their understanding of a system that is already well-understood [4].

In this study, researchers designed a simulation to engage BC Calculus students in educational modeling [4] of an engineering problem that illustrates how parametric functions and their derivatives in a polar reference frame (angular joint motion and arm length) to describe straight-line horizontal and vertical motion. Development of the simulation included careful attention to the steps of the GAIMME modeling process [15], particularly assessment and analysis. The researchers used pre- and post-survey data to compare student perceptions of mathematical modeling before and after completing this simulation activity to address the research question: how do student perceptions of mathematical modeling change before and after completing an engineering simulation activity?

Methods

Simulation development and implementation

The two-joint robot simulation was created using Unity [18]. The structure of Unity as a gaming engine allowed for simplification of the code and easier implementation of graphics. Unity also includes the ability to compile and run in WebGL, which allows students to access the program using Chromebooks. Equations for horizontal and vertical motion were pre-programmed in the simulation, and students could vary coefficients within these constraints. Students could also vary angle values, the rate of change of θ_1, and the maximum value of θ_1 (Figure 1). As the end effector (end of the robot arm) moves, it drops a series of game objects to trace the path for student observation.

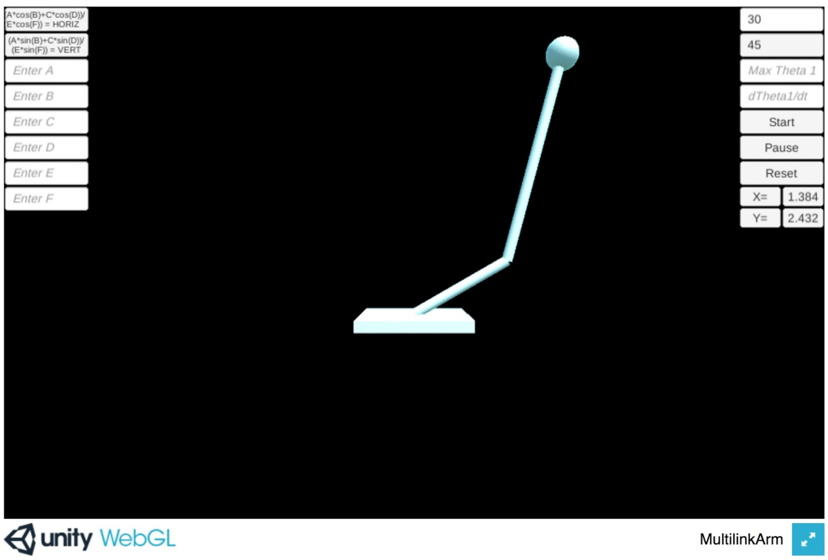


Figure 1. Two-joint robot arm simulation.

Classroom implementation of the simulation occurred over three consecutive days. Students had a total of 120 minutes to work on the simulation activity. Prior to completing the activity, students received introductory instruction on polar and parametric equations and vector calculus. They also watched a video of the Harris T7 Explosive Ordnance Detection (EOD) robot investigating "suspicious" packages in a car [19]. Students discussed the motion of the EOD robot and how the extension of its arm in a linear path is driven by rotation at a number of joints.

Students received a worksheet to scaffold their use of the simulator, as opposed to exploratory engagement where students freely choose which parameters to change. Scaffolding is important for successful implementation of simulation activities [6]. The student worksheet and is included in Appendix A. Students were not shown a diagram that specifically labeled which angle was which or where it was measured from. They used the simulation to determine whether they chose the correct naming convention for each angle. A summary of the simulation activity is shown in Table 1.

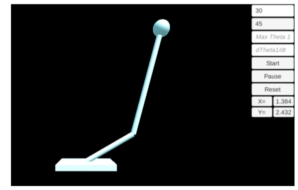
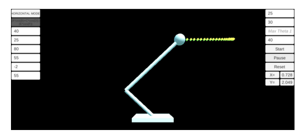
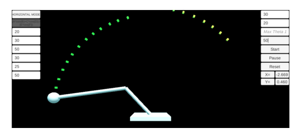
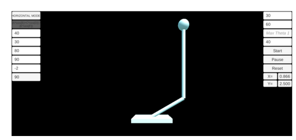
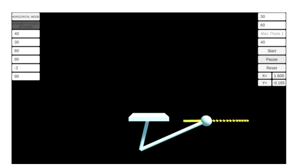
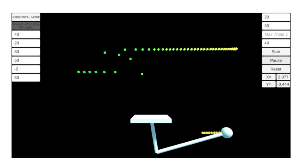
Student Task	Description	Sample Image(s)
Determine position equations	Students created equations to represent the position of the end effector using the angles between the joint arms and the length of each arm. Students selected values for θ_1 and θ_2 and observed changes in the position of the end effector to determine whether their predictions matched the actual position.	
Determine horizontal and vertical motion equations	Students were instructed to solve for the rate of change of θ_2 ($\frac{d\theta_2}{dt}$) if the rate of change of θ_1 ($\frac{d\theta_1}{dt}$) remained constant for horizontal and vertical motion.	
Test horizontal and vertical motion equations	Students rearranged their equations to fit the format of the model, then entered coefficients for the pre-programmed equation. Images show results for correct coefficients (top) and incorrect coefficients (bottom).	 
Explore and propose explanations for anomalies: robot arm "jumps" from its starting position	When the end effector is at a singularity, it will not move smoothly in a horizontal line. If students entered the correct equation, the end effector jumped to a different point and then began horizontal motion. Images show starting position (top) and resulting movement (bottom).	 
Explore and propose explanations for anomalies: robot arm "jumps" after successful	In the simulation, $\frac{d\theta_1}{dt}$ is constant, so once the robot arm is at full extension, it jumps to a new position. Both anomalies could have been fixed with code in Unity, but they were left in so students could hypothesize about the causes of these problems.	

Table 1. Summary of two-joint robot simulation activity.

Data collection and analysis

To assess the effectiveness of this simulation activity, pre-and post-surveys were administered to participating students.The participants were juniors and seniors enrolled in BC Calculus at a high school in a mid-sized city in the intermountain west of the United States. A total of 17 students participated in filling out each survey, although four students only participated in one portion of the data collection.

The pre-activity survey and part 1 of the post-activity survey asked students to define, describe, and diagram how they think mathematicians/scientists/engineers create a mathematical model (see Appendix B for survey questions). The analysis of these questions involved a directed content analysis approach [20]. One researcher used the six steps of the GAIMME modeling process [15] for the theoretical framework. Student responses were divided into phrases (subsections of responses separated by punctuation, bullet points, arrows, or conjunctions).

In the first read through, the researcher coded each phrase of student responses using one of the six steps or noted the phrase as being outside of the framework. After the first read through, the researcher determined that two steps of the GAIMME modeling process [15] (make assumptions/define essential variables and implement the model/report the results) needed to be split into two categories in order to distinguish between student answers. Three additional categories also appeared throughout the data and were added to the code options, as shown in Table 2.

To increase the trustworthiness of the directed content analysis, a second researcher independently coded a random sample of the surveys [20]. The questions for this portion of the survey were carefully designed to be open-ended (e.g. asking about scientists/mathematicians/engineers instead of highlighting a particular profession) in order to prevent leading students to a particular answer [20]. Students were asked to draw a diagram or flow chart of their procedure as a clarifying step. This facilitated comparison to the GAIMME modeling process [15], and it helped to capture the individual steps students considered as part of their description of modeling.

In part 2 of the survey, students were asked to look at the diagram of the GAIMME modeling process [15] and determine whether or not they engaged in each part of the procedure when they completed the simulation. This was included in order to determine whether there were differences between student and researcher perceptions of the modeling process. These questions were classified based on how many of the characteristics the students thought were included.

The final question asked students to comment on whether their description matched the "formal" GAIMME diagram [15]. The intent was for students to expand on areas of their description that were different, which would allow researchers to effectively cross-check their codes with student self-analysis. However, student responses were not specific enough to analyze the data in this manner, so these answers were simply classified as "agree" or "disagree".

Code	Description from GAIMME modeling process [15]	Examples
ID	Identify and specify the problem to be solved	"Identify a problem", "Collect data", "theory"
DEV	Make assumptions	"create hypothesis", "analyze data" (if mentioned before "create model"/solve)
VAR	Define essential variables	specific mention of variables
SOL	Do the math: get a solution	"solve the problem", "come up with a unique solution", "attempt to build models"
IMP	Implement the model	"try the model", "experiment with it"
REP	Report the results	"present", "share"
IT	Iterate as needed to refine and extend the model	"test", "repeat", "solve more"
AA	Analyze and assess the model and the solutions	"analyzing numbers/data", "ensure math is correct"
Code	**Researcher-generated description**	**Examples**
VIS	Visual representations of data	"draw some diagrams",
SIM	Simulation mentioned	"simulation", "computer program"
IRL	Reference to real world problems	"applying data to real life scenarios"

Table 2. Code descriptions and student examples for each category.

Results

Student focus on iteration and analysis

After completing the simulation, students focused less on identifying the problem and more on iteration and analysis of the problem, as shown in Table 3. Student A stated that researchers “explain diagrams/solve more math” as a step in the presurvey, but expanded to “test to ensure math is correct” in the postsurvey. Other students did not include iteration initially, but they mentioned it in the postsurvey. Student B used the phrase “they experiment” in the presurvey, and wrote “test equations, make adjustments, repeat” in the postsurvey. Student C included iteration as a change to the diagram, as shown in Figure 2.

Pre-Survey Counts

ID	DEV	VAR	SOL	IMP	REP	IT	AA	VIS	SIM	IRL
13	5	1	12	5	8	7	6	5	0	2

Post-Survey Counts

ID	DEV	VAR	SOL	IMP	REP	IT	AA	VIS	SIM	IRL
9	5	0	12	7	4	10	11	1	3	2

Table 3. Counts for pre- and post-survey results.

Describing modeling as a simulation

After completing the simulation, students were more likely to mention simulation or computer programs as part of the modeling process. There were no students who mentioned the word “simulation” in the description or diagram of the modeling process in the presurvey, but three students mentioned it in the post survey. For example, student D defined mathematical modeling as “modeling with mathematical equations” initially, then changed in the post survey to “modeling a mathematical concept using computer programs.” Student E stated that mathematical modeling is “using models such as graphs, pictures, etc… to explain mathematical topics” in the presurvey, and changed to “using pictures, charts, and graphs and [sic] simulations to learn mathematical topics.” It is also interesting that this particular student changed the word “explain” to “learn.” This might indicate the tendency for students to narrow their definition of modeling when they are exposed to one particular scenario rather than a variety of scenarios, as discussed in the individual anomalies section below.

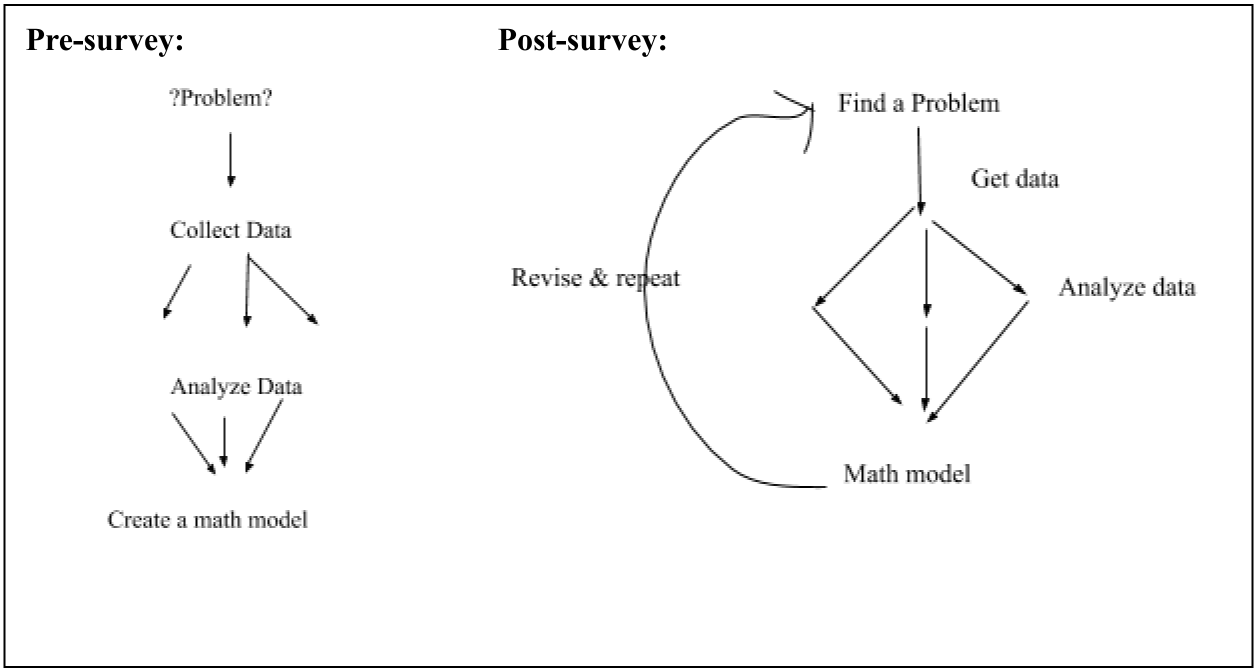


Figure 2. Sample of student work that included iteration explicitly in the postsurvey.

Researcher vs. student comparison to the formal definition

The simulation activity does not fully engage students in all steps of the modeling process. Specifically, students did not fully “identify and specify the problem to be solved,” “iterate as needed to refine the model,” or “implement the model and report the results.”, as the problem was given to the students. Although students might “iterate” as they test their equations, they do not refine the model, because they did not modify the simulation itself. Furthermore, this problem has an exact solution, so students who correctly differentiated and assigned variables were able to “solve” the problem immediately.

All students agreed that their definition matched the formal definition of modeling, although five students mentioned that their definitions were not as specific. All but four students responded that the simulation included all steps of the formal modeling process. These results are summarized in Table 4. All four students excluded iteration, mostly citing that they changed values, but the model itself did not change.

Student	Steps Excluded	Comments
A	IMP, IT, AA	“we did not change the model itself, or apply the equations to other situations” “we had no part in the analysis of the model”
F	IT, AA	“equation model not redefined after initial composition, only altered measure of degrees” “no comprehensive analysis or summary of model results.”
G	IMP, IT	“didn’t change the model”
K	IT	“we made a few adjustments to our equations due to miscalculations but we never really ‘fixed’ the model itself”

Table 4. Steps of the formal modeling process that were excluded by four students.

Individual anomalies

There were particularly interesting variations for students whose pre-surveys indicated they were outliers in their understanding of mathematical modeling. Students who did not have a basic understanding of modeling in the pre-survey showed little growth after completing the simulation. Student H focused on visual representations both times, and while this individual did mention “scientific procedures” in the pre-survey and “guessing and checking” and “they experiment” in the post-survey, the diagrams were both simply representations of data. Student I used the words “I don’t know” along with each answer in the pre-survey, defining mathematical modeling as “using mathematic [*sic*] principles/simulations to simulate a scenario” and listing processes as “possibly checking for unintended variable influences”. The post-survey data focused on the specific activity that the student completed, including “using angles to create equations” and “create equations based on little info”.

On the other end of the spectrum, student J started with a well-defined initial understanding of modeling that narrowed after completing the simulation. In the initial description, this student included five of the steps of the modeling process, while in the final description only included three. Student J also narrowed his emphasis to focus on an “external scenario based upon [a] specific mathematical topic” instead of “a tool… utilized for further understanding of a certain problem and/or… another scenario in which this model would fit”.

Discussion

Student descriptions of mathematical modeling included more iteration and assessment and analysis after the simulation task. The simulation was created for students to assess and analyze the mathematical equation that they developed, so this increase is consistent with the design of the simulation. The iteration piece, however, may illustrate the discrepancy between “educational modeling” [4] and the way scientists and engineers engage in modeling and design processes. Students used repeated trials to change their equation and to identify possible reasoning for problems with the model. They included this repetition as an additional step in their diagram of the model. However, they did not refine the model to make it more accurate, or change the code to account for potential singularities (as engineers would need to do for a physical robot arm).

There was also a wide discrepancy in student responses. This simulation is designed as part of a larger BC Calculus unit about parametric and polar equations, so these discrepancies may be due to where students fall in the Zone of Proximal Development [21]. Students who struggle to understand basic concepts after initial instruction, or those who have little background in mathematical modeling, might not be ready to integrate the knowledge gained from the simulation into a more complex modeling framework. This could be mitigated with additional scaffolding or whole class review of modeling before and after the simulation.

Students were asked to explain whether their definition of modeling matched the formal model. However, many students only vaguely described that their model was not as specific as the formal definition. Some of the discrepancy here might have occurred due to the format of the survey. In a classroom setting, a written survey is akin to a worksheet or assignment, and students may feel uncomfortable challenging a formal or “professional” model. Asking students to explicitly indicate whether they included each section of the model or conducting face-to-face interviews may result in a more precise understanding of which areas students considered and which areas they left out when they developed their own definition of mathematical modeling.

Student tendency to list “simulation” as part of the modeling process after completing this activity indicates the importance of varying student experiences with modeling. Students complete many activities where they model equations visually, and this was reflected in their pre-survey results. After experiencing the simulation, many students indicated this as an explicit part of the modeling process, even though it is not necessarily required. Exposure to a wider variety of modeling tasks that include simulation may broaden student definitions.

Future Work

One purpose of this simulation was for students to engage in the mathematical modeling process by using the simulation to test the velocity equations that they derived. However, some groups looked at the structure of the pre-programmed equations instead of using the simulation to find their errors. The structure of the horizontal and vertical motion equations (horizontal contained cosine and vertical contained sine) led students to find and correct their errors prior to entering values in the simulation. The simulation also assumed that students algebraically distributed all values prior to solving for $\frac{d\theta_2}{dt}$, but many students solved without distributing. By modifying the simulation to display four equation choices without designating them as "horizontal" or "vertical", students would be able to implement their equations and use the simulation itself to check for accuracy.

In this study, researchers created a simulation of the two-joint robot simulation for students in BC Calculus to explore complex math topics using an engineering task and engaging in the mathematical modeling process. Students completed surveys before and after completing the simulation to assess any changes in their perceptions of mathematical modeling after participating in the engineering task. After modifications to the simulation, it will be available for other instructors or researchers to use, along with the accompanying student handout and teacher guide, available at https://sites.google.com/view/twojointrobot. Students seemed to increase their understanding of mathematical modeling as an iterative process, although some students narrowed their description to focus on the role of simulation. This study was quite small with varied results, and further exploration with additional classes may show different results. Introducing multiple engineering tasks using simulation and other methods may help students refine their definition of mathematical modeling to include broader understanding. It would also be interesting to measure the efficacy of this simulation on student math performance.

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References

[1] C. R. Hirsch and A. R. McDuffie, *Annual Perspectives in Mathematics Education 2016: Mathematical Modeling and Modeling Mathematics*. National Council of Teachers of Mathematics, 2016.

[2] J. A. Gutiérrez and R. R. Gallegos, "Theoretical and Methodological Proposal on the Development of Critical Thinking through Mathematical Modeling in the Training of Engineers," in *Proceedings of the Seventh International Conference on Technological Ecosystems for Enhancing Multiculturality*, León, Spain, Oct. 2019, pp. 941–948, doi: 10.1145/3362789.3362828.

[3] A. McKenna and A. Carberry, "Characterizing the role of modeling in innovation," *Int. J.*

Eng. Educ., vol. 28, no. 2, pp. 263–269, 2012.
[4] M. Develaki, “Using Computer Simulations for Promoting Model-based Reasoning,” *Sci. Educ.*, vol. 26, no. 7, pp. 1001–1027, Nov. 2017, doi: 10.1007/s11191-017-9944-9.
[5] A. J. Magana and T. de Jong, “Modeling and simulation practices in engineering education,” *Comput. Appl. Eng. Educ.*, vol. 26, no. 4, pp. 731–738, 2018, doi: 10.1002/cae.21980.
[6] B. Bowen and V. W. DeLuca, “Comparing traditional versus alternative sequencing of instruction when using simulation modeling,” *J. STEM Educ. Innov. Res.*, vol. 16, no. 1, Jun. 2015, Accessed: Feb. 01, 2020. [Online]. Available: https://www.jstem.org/jstem/index.php/JSTEM/article/view/1896.
[7] S. J. Dickerson and R. M. Clark, “A classroom-based simulation-centric approach to microelectronics education,” *Comput. Appl. Eng. Educ.*, vol. 26, no. 4, pp. 768–781, 2018, doi: 10.1002/cae.21918.
[8] A. Sokolowski, “The Effect of Math Modeling on Student’s Emerging Understanding,” *IAFOR J. Educ.*, vol. 3, no. 2, pp. 142–156, 2015.
[9] P. Corke, “Velocity kinematics in 2D | Masterclass,” *Robot Academy*, Feb. 21, 2017. https://robotacademy.net.au/masterclass/velocity-kinematics-in-2d/ (accessed Feb. 01, 2020).
[10] J. Ichnowski, “Configuration Space Visualization of 2-D Robotic Manipulator,” 2010. https://www.cs.unc.edu/~jeffi/c-space/robot.xhtml (accessed Feb. 01, 2020).
[11] Y. Zhang, “Using MATLAB and Simulink for Robotics - Video,” Mar. 25, 2015. https://www.mathworks.com/videos/using-matlab-and-simulink-for-robotics-100488.html (accessed Feb. 01, 2020).
[12] A. Sultan, “Robotics 101,” *Math. Teach.*, vol. 104, no. 8, pp. 614–619, Apr. 2011.
[13] E. Berkove and R. Marchand, “The Long Arm of Calculus,” *Coll. Math. J.*, vol. 29, no. 5, p. 376, Nov. 1998, doi: 10.2307/2687252.
[14] “AP Calculus AB and BC Course and Exam Description, Effective Fall 2019,” p. 248.
[15] “Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME).” Consortium of Mathematics and Its Applications (COMAP), Bedford, MA, and Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA., 2016, [Online]. Available: http://www.siam.org/reports/gaimme.php.
[16] H. M. Doerr and L. D. English, “A Modeling Perspective on Students’ Mathematical Reasoning about Data,” *J. Res. Math. Educ.*, vol. 34, no. 2, pp. 110–136, 2003, doi: 10.2307/30034902.
[17] R. P. Leland, J. A. Richardson, T.-Y. Lee, and J. Dantzler, “Mathematical Problem Solving For Engineering Students,” 2005.
[18] *Unity*. Unity Technologies, 2019.
[19] “Harris Corporation T7™ Multi-Mission Robot on an EOD Mission.” https://www.youtube.com/watch?v=5zX2LwiT0xA (accessed Feb. 01, 2020).
[20] “Three Approaches to Qualitative Content Analysis - Hsiu-Fang Hsieh, Sarah E. Shannon, 2005.” https://journals.sagepub.com/doi/abs/10.1177/1049732305276687 (accessed Feb. 01, 2020).
[21] L. S. Vygotsky, *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press, 1980.

Appendix A. Two-Joint Robot Arm Simulation Student Handout

PART 1: POSITION

In this portion of the activity, you will determine whether your expressions for the x- and y-coordinates are correct.

1. Write your expressions for x and y as functions of θ_1 and θ_2.
2. For this simulation, the length of arm #1 is 1 unit, and the length of arm #2 is 2 units. Select three pairs of values for θ_1 and θ_2. Use your equation to predict the x- and y-values for the end effector. Write your angles and predictions below:
   NOTE: Please use **degree mode** on your calculator!
   a. $\theta_1 =$ $\theta_2 =$ x = y =
   b. $\theta_1 =$ $\theta_2 =$ x = y =
   c. $\theta_1 =$ $\theta_2 =$ x = y =
3. Enter your values for θ_1 and θ_2 in the simulation. You will notice that each time you enter an angle, the arm will move to the appropriate location.
   Record the x and y values from the simulation below :
   (NOTE: There is a bug in the y-value; please subtract 1 from the y-output of the simulation)
   a. x = y =
   b. x = y =
   c. x = y =
4. Do the x- and y-values from the simulation match your predicted values? If so, move on to question 6. If not, complete question 5.
5. Go back to your original equation, and see if you can find your error.
   a. Explain your error and write your new equation here.
   b. Calculate your predicted x and y values and record them below.
      i. x = y =
      ii. x = y =
      iii. x = y =
   c. Repeat this step until your predicted values match the simulation values, then move on to question 6.
6. How did you select the angles that you tested? Are there any additional angles that you think you should test? Why or why not? If so, please list them here and verify your x- and y-values.

PART 2: DEVELOP MOTION EQUATIONS

This simulation assumes that the $\frac{d\theta_1}{dt}$ is constant, and the $\frac{d\theta_2}{dt}$ is updated for either horizontal or vertical motion. The length of each arm is also constant.

1. Given the above constraints, develop an equation for $\frac{d\theta_2}{dt}$ if the end effector needs to move horizontally. Record your equation below.
2. Explain how you determined the equation above. Be sure to explain the assumptions that you are making.

3. Using the same constraints, develop an equation for $\frac{d\theta_2}{dt}$ if the end effector needs to move vertically. Record your equation below.
4. Explain how you determined the equation above. Be sure to explain the assumptions that you are making.
5. Move on to part 3.

PART 3: VERIFY MOTION EQUATIONS

Equation 1 is HORIZONTAL MOTION.

The length of arm #1 is 1 unit, and the length of arm #2 is 2 units.

1. Select initial values for θ_1, θ_2, and $\frac{d\theta_1}{dt}$. Record your values and write your equation below. DO NOT convert trigonometric values to decimals. For example, leave sin(45) as is, not as 0.707.
2. The simulation is designed for a certain equation format. Enter your values for θ_1, θ_2, and $\frac{d\theta_1}{dt}$. Verify that your starting position is correct.
3. Enter values for A-F corresponding to the following equation. It's okay to enter negative numbers if necessary!

$$\frac{d\theta_2}{dt} = \frac{A * cos(B) + C * cos(D)}{E * cos(F)}$$

4. Click the button for the horizontal equation, then click start. Observe the motion of the robot arm.
5. Record your observations. Did the robot arm move as you expected?
6. Does your equation work for other angles? Develop a plan and test your equation to see if it works in a variety of cases. Explain why you chose the values that you did.

Equation 2 is VERTICAL MOTION.

7. Repeat your experiments with vertical motion, but enter values for A-F corresponding to the following equation:

$$\frac{d\theta_2}{dt} = \frac{A * sin(B) + C * sin(D)}{E * sin(F)}$$

8. Record your process and observations.

PART 4: CONSTRAINTS

You might have noticed that even when you successfully achieved the motion you wanted, there were still some strange things going on. In this part of the simulation, you will investigate why these things happen.

There is a value called "Max Theta 1" that allows you to stop the motion of the simulation for a value of θ_1 between 0 and 359 degrees.

Design a plan to determine appropriate constraints for different starting values. Use the simulation to test your plan, and use your equations to back up your reasoning. In the space below, record your processes and your findings.

Appendix B. Survey Questions

Pre-Activity Survey Questions

1. What is mathematical modeling?
2. What processes do mathematicians/scientists/engineers go through to create a mathematical model?
3. Draw a diagram or flow chart of the procedure you described in question #2.

Post-Activity Survey Questions (Part 1)

1. What is mathematical modeling?
2. What processes do mathematicians/scientists/engineers go through to create a mathematical model?
3. Draw a diagram or flow chart of the procedure you described in question #2.

Post-Activity Survey Questions (Part 2)

(Students were given a copy of the GAIMME formal modeling process [15])

1. Put a check mark next to each step of the modeling process that you engaged in during the simulation activity. Give an example of how you used each step that you checked.
2. Place a check next to any of the steps that you think were missing from the simulation activity. Explain your reasoning. (students were given two copies of the table below, one for each question).
3. Does the modeling process illustrated in the diagram match your idea of the modeling process? Briefly explain.

Step of Modeling Process	Check	Example/Explanation
Identify and specify the problem to be solved		
Make assumptions and define essential variables		
Do the math: Get a solution		
Implement the model and report the results		
Iterate as needed to refine and extend the model		
Analyze and assess the model and the solutions		