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Residual Self-Interference Cancellation and Data Detection in Full-Duplex Communication Systems

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***Abstract*—Residual self-interference cancellation is an important practical requirement for realizing the full potential of full-duplex (FD) communication. Traditionally, the residual self-interference is cancelled via digital processing at the baseband, which requires accurate knowledge of channel estimates of the desired and self-interference channels. In this work, we consider point-to-point FD communication and propose a superimposed signaling technique to cancel the residual self-interference and detect the data without estimating the unknown channels. We show that when the channel estimates are not available, data detection in FD communication results in ambiguity if the modulation constellation is symmetric around the origin. We demonstrate that this ambiguity can be resolved by superimposed signalling, i.e., by shifting the modulation constellation away from the origin, to create an asymmetric modulation constellation. We compare the performance of the proposed detection method to that of the conventional channel estimation-based detection method, where the unknown channels are first estimated and then the data signal is detected. Simulations show that for the same average energy over a transmission block, the bit error rate performance of the proposed detection method is better than that of the conventional method. The proposed method does not require any channel estimates and is bandwidth efficient.**

***Index Terms*—Full-duplex communication, symbol detection, self-interference cancellation, superimposed signaling.**

I. Introduction

For many years simultaneous signal transmission and reception in the same frequency band, known as full-duplex (FD) communication, was considered to be impractical due to the large self-interference signal [1]. More recently, with advanced multi-stage self-interference cancellation techniques, FD communication is becoming a reality and it is expected to be used by the next generation of wireless communication systems [2], [3].

In FD communication systems, self-interference cancellation is performed in two stages. In the first stage, which is known as *passive cancellation*, the radio frequency (RF) antennas are well-isolated to minimize the amount of interference [4]. In the second stage, which is known as *active cancellation*, the residual interference signal from the previous stage is cancelled either at RF or at digital baseband [5]–[8]. Due to channel estimation errors, the RF canceller cannot completely remove the interference. Hence, the residual interference after the RF canceller is still higher than the receiver noise floor and needs to be cancelled via digital processing at baseband [4], [5], [9]. However, effective self-interference cancellation at baseband requires accurate knowledge of the digital channels, which are the channels observed by the receiver at baseband after the passive and RF cancellation stages [10]. Consequently, for reliable FD communication first the digital channels are estimated and then the received signal is processed for data detection [5], [6], [11]. However, the digital channel estimation is not bandwidth efficient because it requires pilot transmission.

In this paper, we focus on the received signal after the passive and RF cancellation stages in a point-to-point FD communication system. Different from existing works, we propose a data detection technique based on superimposed signaling which does not require any channel estimates. We show that superimposed signaling can overcome the ambiguity inherent in the data detection problem when channel estimates are not used. The main contributions of this work are:

- We formulate a maximum a posterior (MAP) detector, based on the posterior probability distribution (PDF) function of the data, to detect the data symbols in FD communication without any requirement of channel estimation.
- We show that if the modulation constellation is symmetric around the origin, the data detection in FD communication results in ambiguity when the channel estimates are not available. We demonstrate that one simple method to resolve this detection ambiguity is to use superimposed signalling, i.e., to shift the modulation constellation away from the origin and create an asymmetric modulation constellation.
- We compare the bit error rate performance of the proposed detection method to that of the conventional channel estimation-based detection method, where the unknown channels are first estimated and then the data signal is detected, under the constraint of same average energy over a transmission block. The results show

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that the proposed method outperforms the conventional method. Since the proposed method does not require any channel estimates, it enhances bandwidth and power efficiency.

This paper is organized as follows. Section II presents the system model. Section III formulates the MAP detector for data detection in the absence of channel estimates and illustrates the ambiguity problem associated with the MAP detector. Section IV proposes a superimposing technique to resolve the detection ambiguity problem. Section V presents and discusses the simulation results. Finally, Section VI concludes the paper.

Notations: The following notation is used in this paper. Bold face lower case letters, e.g., $\mathbf{x}$, are used for vectors. $\mathrm{j} \triangleq \sqrt{-1}$, and the real and imaginary parts of a complex quantity are represented by $\Re\{\cdot\}$ and $\Im\{\cdot\}$, respectively. x^* and $|x|$ indicate scalar complex conjugate and the absolute value of complex number x, respectively. $\sum_{\sim i}$ means summation over all possible values except i. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian distribution with mean μ and variance σ^2. Finally, $f(x)$ denotes the PDF of random variable x.

II. System Model

We consider the data detection problem for the single-input single-output (SISO) FD communication system, as shown in Fig. 1. Nodes a and b each have a pair of antennas, which is used for simultaneously transmit and receive on the same frequency band. Due to the inherent symmetry of the problem, we only investigate the data detection problem for node a, as identical results are expected for node b.

The received signal at node a is given by

$$\mathbf{y}_a = h_{aa}\mathbf{x}_a + h_{ba}\mathbf{x}_b + \mathbf{w}_a, \tag{1}$$

where, $\mathbf{y}_a \triangleq [y_{a_1}, \cdots, y_{a_N}]^T$ is the $N \times 1$ vector of received symbols, $\mathbf{x}_a \triangleq [x_{a_1}, \cdots, x_{a_N}]^T$ is the $N \times 1$ vector of self-interference symbols, $\mathbf{x}_b \triangleq [x_{b_1}, \cdots, x_{b_N}]^T$ is the $N \times 1$ vector of desired communication symbols, $\mathbf{w}_a \triangleq [w_{a_1}, \cdots, w_{a_N}]^T$ is the $N \times 1$ vector of independent identically distributed (IID) Gaussian noise with zero mean and variance σ^2, i.e., $w_{ai} \sim \mathcal{CN}(0, \sigma^2)$.

We make the following assumptions in this paper:

- Since the digital channels are the channels observed after the passive and RF cancellation stages, the direct line-of-sight (LoS) components of these channels have already been canceled and the residual components are due to the scatterers [4], [5]. Consequently, similar to [12], [13], we assume h_{aa} and h_{ba} are flat-fading and Rayleigh distributed with zero mean and variance one, i.e., $h_{aa}, h_{ba} \sim \mathcal{CN}(0, 1)$.
- The transmitted symbols are modulated using the modulation set $\mathcal{A} = \{A_1, A_2, ..., A_M\}$, with size M. Modulation set $\mathcal{A}$ contains all constellation points of any given standard modulation constellation, such as M-ary phase shift keying (MPSK) modulation, and the transmitter is likely to send each constellation point with equal probability.

III. Data detection in FD communication without knowledge of channel estimates

In this section, we first derive a MAP symbol detector for the FD communication system. Then we show that this detector suffers from the detection ambiguity problem because of the symmetry of conventional modulation constellations around the origin.

A. MAP detector

The main results in this section are presented in the following propositions.

Proposition 1: The maximum MAP symbol detector for the SISO FD communication system presented in Section II is given by

$$\tilde{x}_{b_i} = \max_{x_{b_i}} f(x_{b_i}|\mathbf{y}_a). \tag{2}$$

where the marginal probability distribution $f(x_{b_i}|\mathbf{y}_a)$ is proportional to

$$f(x_{b_i}|\mathbf{y}_a) \propto \underbrace{\sum_{j_N=1}^{M} \cdots \sum_{j_1=1}^{M}}_{\sim j_i} \frac{1}{\lambda} \exp\left(\frac{|\xi|^2}{\lambda\sigma^2}\right), \tag{3}$$

where M is the size of modulation set $\mathcal{A}$, N is the length of the transmitted vector, i.e., number of transmitted symbols in a transmission block, and

$$\lambda \triangleq \sum_{n=1,n\neq i}^{N} |A_{j_n}|^2 - \frac{1}{\gamma}\left|\sum_{n=1,n\neq i}^{N} x_{a_n}^* A_{j_n} + x_{a_i}^* x_{b_i}\right|^2 + |x_{b_i}|^2 + \sigma^2 , \tag{4a}$$

$$\xi \triangleq \sum_{n=1,n\neq i}^{N} y_{a_n} A_{j_n}^* + y_{a_i} x_{b_i}^* - \frac{1}{\gamma}\sum_{n=1}^{N} y_{a_n} x_{a_n}^* \left(\sum_{n=1,n\neq i}^{N} x_{a_n}^* A_{j_n} + x_{a_i}^* x_{b_i}\right)^*, \tag{4b}$$

$$\gamma \triangleq \sum_{n=1}^{N} |x_{a_n}|^2 + \sigma^2. \tag{4c}$$

Proof: See Appendix A. ■

Note that the proportionality in (3) does not depend on the residual self-interference symbol x_{b_i} and, hence, does not affect the decision in (2).

Remark 1: The posterior PDF $f(x_{b_i}|\mathbf{y}_a)$ is independent of both the self-interference and communication channels, i.e., h_{aa} and h_{ba}. Hence, the MAP detector as proposed by Proposition 1 is independent of the channel estimates. In other words, the symbols can be detected without requiring the interference or communication channel to be estimated. The MAP detector also directly detects the symbols without requiring a separate self-interference cancellation stage.

Proposition 2: We call $\mathcal{A}$ a symmetric modulation set, if and only if for $x_k \in \mathcal{A}$, there exists $-x_k \in \mathcal{A}$, $\forall k$. The posterior

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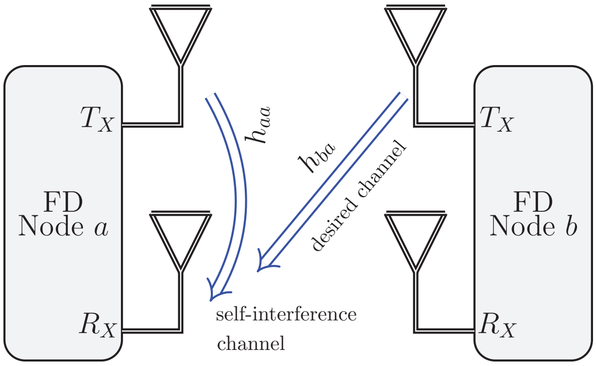

Fig. 1: Full Duplex System with single transmit and receive antenna. The single antenna at each node is shown separately for the transmission and reception for ease of illustration.

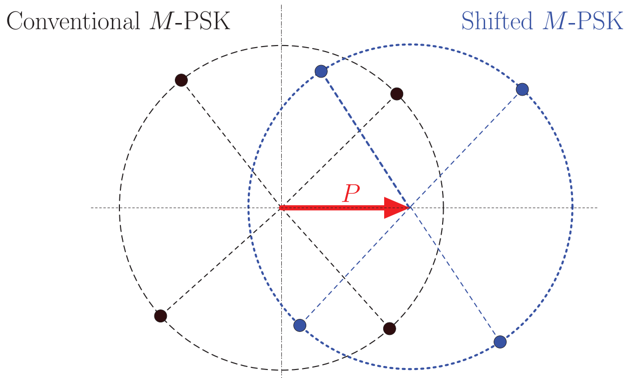

Fig. 2: Effect of superimposed signalling on the modulation constellation of $M = 4$-PSK.

PDF $f(x_{b_i}|\mathbf{y}_a)$ does not have a unique maximum if and only if x_{b_i} in (3) comes from a symmetric modulation set $\mathcal{A}$.

Proof: See Appendix B. ∎

Corollary 1: Since conventional modulation constellations are symmetric around the origin, data detection in FD communication with no channel estimation will result in ambiguity.

IV. Superimposed Signaling for Resolving the Data Detection Ambiguity in FD Communication

In this section we present a superimposed signalling technique to tackle the inherent ambiguity problem in data detection with no available channel estimates.

A. Why Superimposed Signalling?

The rationale for using superimposed signalling is as follows. From Proposition 2, the data detection ambiguity in FD communication in the absence of channel estimates, arises because of the symmetry of the modulation constellation around the origin. Consequently, an obvious approach to resolve the data detection ambiguity is to alter the symmetry of the modulation constellation around the origin and create a suitable asymmetric modulation constellation.

One simple way to achieve an asymmetric modulation constellation around the origin is to add (superimpose) a constant known signal to the transmitted signal.[2] We call this approach superimposed signalling. For illustration, Fig. 2 shows the effect of superimposed signalling with constant P on the constellation of an $M = 4$-PSK modulation set. Once the M-PSK constellation is shifted, then the new constellation is asymmetric around the origin and can be used for ambiguity-free MAP detection with no need for channel estimation.

B. Modified System Model

If both nodes a and b superimpose a common constant and known signal P to the transmitted symbols, then (1) can be written as:

$$\mathbf{y}_a = h_{aa}(\mathbf{x}_a + P) + h_{ba}(\mathbf{x}_b + P) + \mathbf{w}_a. \tag{5}$$

[2]The design of optimum asymmetric modulation constellations is outside the scope of this paper and is the subject of future work [14].

It is again clear from (5) that the effect of superimposed signalling with constant signal P is the same as shifting the modulation constellation by P along the horizontal axis.

C. Power Normalization

As illustrated above, superimposed signalling increases the average energy per symbol of the modulation constellation. Conventional (symmetric) modulations operate under an average transmit power constraint, which places limits on the average energy per symbol. A fundamental question regarding superimposed signaling is, therefore, how to choose a fair value of the extra power which is required to superimpose a known signal on the data symbols to shift the modulation constellation.

If the channels were perfectly known there would be no need to allocate power for channel estimation. However, in reality the channels are unknown and hence it is inevitable to expand extra power for channel estimation. The proposed superimposed signalling approach is similar in spirit to superimposed training in the literature, which has been extensively used as a bandwidth-efficient channel estimation technique in half-duplex (HD) communication systems [15], [16]. In superimposed training, the extra power in the superimposed pilots is used for channel estimation. In our case, we do not use the extra power for channel estimation. Rather, we use it only for achieving an asymmetric modulation constellation. Consequently, to ensure that the proposed method does not exceed the average transmit power constraint, we shift the modulation by $P \triangleq \sqrt{E_p}$, where E_p is the average energy used for channel estimation in conventional pilot based channel estimation systems.

V. Simulation Results

In this section, we present the simulation results. First we demonstrate that detection without channel estimation, using symmetric modulation constellation can result in ambiguity. Then we show that this ambiguity is resolved once the modulation set is shifted to a asymmetric modulation set, i.e., a known signal is superimposed on the data signal. We find

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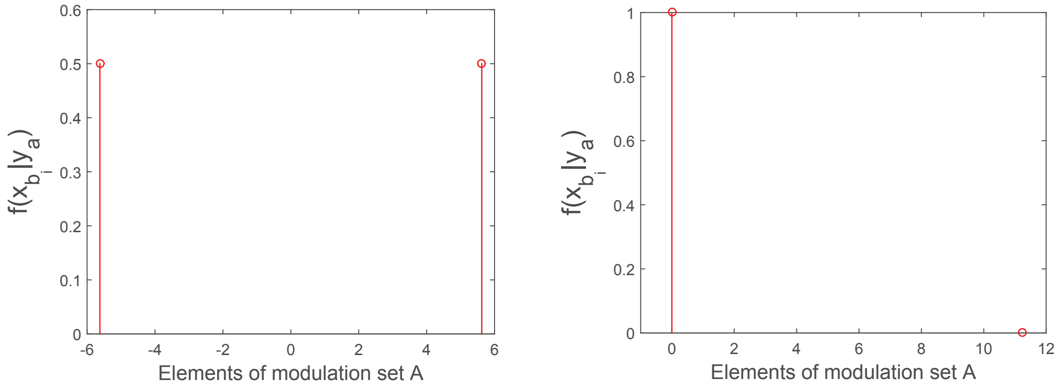
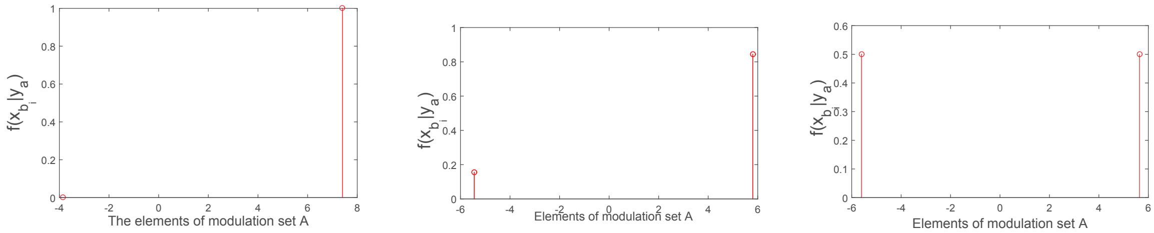


(a) Symmetric BPSK modulation.

(b) Asymmetric BPSK modulation.

Fig. 3: Posterior function $f(x_{b_i}|\mathbf{y}_a)$ at $\frac{E_b}{N_o} = 15$ dB.



(a) $\beta = 0.1$

(b) $\beta = 0.001$

(c) $\beta = 0.00001$

Fig. 4: Posterior function $f(x_{b_i}|\mathbf{y}_a)$ for different values of β

the minimum power required for superimposed signaling to resolve the ambiguity problem. Finally, we investigate the BER performance of the proposed detector. Throughout this section we make the following assumptions:

- *Channel and noise*: For each run of the simulation, the random channels h_{aa} and h_{ba} are generated according to a Rayleigh distribution and are assumed constant for blocks of N symbols, i.e. block fading. We assume independent block fading for simulation purposes which means channels are independent from block to block, i.e., quasi-static.
- *Modulation*: For the sake of simplicity, we only present the result for binary shift keying (BPSK) modulation. Consequently, the modulation set $\mathcal{A}$ has two elements.
- *Noise and shift powers*: We assume the average bit energy of the modulation is E_b and noise power is $N_0 = 1$.

A. Symmetric Modulation Set

In this section we highlight the result of Proposition 2 through simulations.

For symmetric BPSK modulation the posterior function $f(x_{b_i}|\mathbf{y}_a)$ takes two discrete values. Fig. 3(a) shows the posterior function at $\frac{E_b}{N_o} = 15$ dB when symmetric BPSK modulation is used. It is clear from Fig. 3(a) that when this modulation constellation is used the posterior function does not have a unique maximum and hence the MAP detector of (2) results in ambiguity. This ambiguity is seen as equal probability for the elements of modulation set $\mathcal{A}$ in Fig. 3(a). Fig. 3(b) shows the posterior function at $\frac{E_b}{N_o} = 15$ dB when the modulation constellation is shifted by $P \triangleq \sqrt{E_b}$. It is clear that in this case, the posterior function has only one maximum and consequently the MAP detector as proposed by Proposition 1 results in no ambiguity. This is because now the elements of modulation set $\mathcal{A}$ have different probabilities, hence, the detector can determine which element is more likely to be transmitted given the received data.

B. Minimum Required Energy for Superimposed Signalling

Although Fig. 3(b) shows that the ambiguity of the MAP detector is resolved by shifting the modulation constellation, this comes at the cost of increasing the transmit power by the shift power ($|P|^2 \triangleq E_b$). We are interested in the minimum required power for ambiguity-free MAP detector. Consequently, for $0 < \beta < 1$, we set the shift to $P \triangleq \sqrt{\beta E_b}$ and numerically investigate the minimum value for β.

Fig. 4 shows the posterior function $f(x_{b_i}|\mathbf{y}_a)$ for different values of β. Clearly, as β decreases, the difference between the maximum and minimum value of the posterior function increases, such that for $\beta = 0.00001$, the posterior function does not have a unique maximum. Fig. 4(b) shows that $\beta = 0.001$ is sufficient enough for ambiguity-free MAP detection. However, our simulation results show that for the FD system

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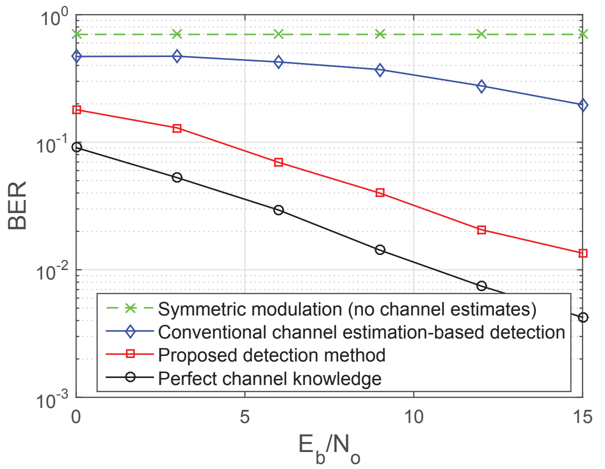

Fig. 5: BER performance of FD communication system with different availability of channel estimates.

under consideration to have a stable detection performance for different channel realizations, the minimum value for β is 0.1.

C. Bit error rate (BER) Performance

In this section we investigate the BER performance of the proposed detector. For simplicity, we only present the results for BPSK modulation in the presence of a self-interference signal which is as strong as the desired signal. We also set the shift to $P \triangleq \sqrt{0.1E_b}$. The BPSK BER with perfect channel knowledge is plotted as a reference. The performance of the proposed detector is compared with a conventional channel estimation-based detection method, assuming the channel estimation uses the same extra power as the superimposed signal for channel estimation. In the channel estimation-based detection method, the channels are first estimated using the same extra energy as the superimposed signal and then these estimates are used for data detection.

Fig. 5 shows that when the modulation constellation is symmetric around the origin and no channel estimates are available, then the detector fails to detect the symbols, i.e., all the possible outcomes are equally likely for the transmitted symbols (c.f. Fig. 3(a)). However, shifting the modulation set to an asymmetric modulation set resolves the ambiguity. In addition, the performance of the proposed detection method is better than the conventional pilot-based detection method.

VI. Conclusion

In this paper, we demonstrated that the detection of symbols in FD communication systems with no channel estimation results in ambiguity. We proposed a solution to this ambiguity problem using superimposed signaling, which involves shifted modulation constellations. We proposed a MAP detector to be used with the shifted modulation constellation in FD communication system for data detection without channel estimation. Our results showed that the proposed detection method has better BER performance, compared to conventional channel estimation-based detection method. The proposed method is bandwidth efficient and can be used in any system model where the self-interference signal is known, such as in two-way relay networks and multi-hop one way relay networks [17], [18].

Appendix A
Proof of Proposition 1

We start the proof by deriving the conditional density function $f(\mathbf{y}_a|\mathbf{x}_b)$ as follows

$$f(\mathbf{y}_a|\mathbf{x}_b) = \int_{h_{aa}}\int_{h_{ba}} f(\mathbf{y}_a|\mathbf{x}_b, h_{aa}, h_{ba}) f(h_{aa}, h_{ba}) \quad dh_{aa}dh_{ba},$$
$$= \int_{h_{aa}}\int_{h_{ba}} f(\mathbf{y}_a|\mathbf{x}_b, h_{aa}, h_{ba}) f(h_{aa}) f(h_{ba}) \quad dh_{aa}dh_{ba}. \tag{A.1}$$

where in (A.1),

$$f(\mathbf{y}_a|\mathbf{x}_b, h_{aa}, h_{ba}) =$$
$$\frac{1}{(\pi\sigma^2)^N}\prod_{i=1}^{N}\exp\left(-\frac{|y_{a_i} - h_{aa}x_{a_i} - h_{ba}x_{b_i}|^2}{\sigma^2}\right), \tag{A.2}$$
$$f(h_{aa}) = \frac{1}{\pi}\exp\left(-|h_{aa}|^2\right), \tag{A.3}$$
$$f(h_{ba}) = \frac{1}{\pi}\exp\left(-|h_{ba}|^2\right). \tag{A.4}$$

Rewriting (A.1), we arrive at

$$f(\mathbf{y}_a|\mathbf{x}_b) = \frac{1}{(\pi\sigma^2)^N\pi^2}\int_{h_{ba}}\exp\left(-|h_{ba}|^2\right)$$
$$\times\int_{h_{aa}}\exp\left(-\sum_{i=1}^{N}\frac{|y_{a_i} - h_{aa}x_{a_i} - h_{ba}x_{b_i}|^2}{\sigma^2}\right)$$
$$\times\exp\left(-|h_{aa}|^2\right) \quad dh_{aa}dh_{ba}. \tag{A.5}$$

Note that in performing the integration in (A.5), we can use the fact that the total probability of a complex Gaussian random variable is one.

Using the Bayes' rule

$$f(\mathbf{x}_b|\mathbf{y}_a) = \frac{f(\mathbf{y}_a|\mathbf{x}_b)f(\mathbf{x}_b)}{f(\mathbf{y}_a)}, \tag{A.6}$$

where $f(\mathbf{x}_b) = \left(\frac{1}{M}\right)^N$ since the transmitted symbols come from a equiprobable modulation set, i.e., $f(x_{b_i}) = \frac{1}{M}$ and $f(\mathbf{y}_a|\mathbf{x}_b)$ is given in (A.5). Substituting and simplifying, we can obtain the result in (3).

Appendix B
Proof of Proposition 2

To prove Proposition 2, we first define permutation $\Pi(\cdot)$ as a one-to-one and onto function on the index set of modulation set $\mathcal{A}$, i.e., $\mathcal{K} \triangleq \{1, 2, \cdots, M\}$. If $x_k \in \mathcal{A}$, then, $x_{\Pi(k)} \in \mathcal{A}'$, $\forall k \in \mathcal{K}$, where $\mathcal{A}'$ is one possible permutation of original modulation set $\mathcal{A}$. Without loss of generality, we further assume that both $\mathcal{A}$ and $\mathcal{A}'$ are ordered set and A_k and A'_k are the kth elements of $\mathcal{A}$ and $\mathcal{A}'$, respectively.

For simplicity of analysis, we show the proof for constant power M-PSK modulation sets in here. The extension to QAM modulation is straightforward and omitted here [14].

Lemma 1: For the ith transmitted symbol, the posterior function $f(x_{b_i}|\mathbf{y}_a)$ does not have a unique maximum if and only if for the modulation set $\mathcal{A}$ there exists a permuted set $\mathcal{A}'$ for which $\frac{x_k}{x_{\Pi(k)}} = -1$, $\forall k \in \mathcal{K}$.

Proof: For the first part of the proof we assume that the permutation $\pi(\cdot)$ that satisfies the condition of the lemma exists and then for a permuted set $\mathcal{A}'$, for which $\frac{x_k}{x_{\Pi(k)}} = -1$, $\forall k \in \mathcal{K}$, we assume that $x'_{b_i} = x_{\Pi(k)} = A'_{\Pi(k)}$ and $x_{b_i} = A_k$ maximizes the posterior density. Then $f(x_{b_i}|\mathbf{y}_a)$ can be rewritten as

$$f(x_{b_i} = A_k|\mathbf{y}_a) \propto \underbrace{\sum_{j_N=1}^{M} \cdots \sum_{j_1=1}^{M}}_{\sim j_i} \frac{1}{\sum_{n=1,n\neq i}^{N} |A_{j_n}|^2 - \vartheta + |A_k|^2 + \sigma^2} \exp\left(\frac{|\xi|^2}{\left(\sum_{n=1,n\neq i}^{N} |A_{j_n}|^2 - \vartheta + |A_k|^2 + \sigma^2\right)\sigma^2} \right), \tag{B.1}$$

where,

$$\vartheta \triangleq \frac{|A_k|^2}{\gamma} \left| \sum_{n=1,n\neq i}^{N} x^*_{a_n} \frac{A_{j_n}}{A_k} + x^*_{a_i} \right|^2. \tag{B.2}$$

It is easy to see from (B.1) that the posterior PDF of $x'_{b_i} = A'_{\Pi}(k)$ differs from the posterior PDF of x_{b_i} only in the term ϑ as the rest of the terms depend on the power of the modulation constellation, which is constant for M-PSK modulation set. We define

$$\vartheta' \triangleq \frac{|A'_{\Pi(k)}|^2}{\gamma} \left| \sum_{n=1,n\neq i}^{N} x^*_{a_n} \frac{A'_{\Pi(j_n)}}{A'_{\Pi(k)}} + x^*_{a_i} \right|^2. \tag{B.3}$$

To prove the lemma we need to show that

$$f(x_{b_i} = A_k|\mathbf{y}_a) = f(x'_{b_i} = A'_{\Pi(k)}|\mathbf{y}_a), \tag{B.4}$$

and hence no unique maximum. (B.4) holds true if and only if

$$\vartheta = \vartheta'. \tag{B.5}$$

Finally, (B.5) holds true if and only if

$$\frac{A_{j_n}}{A_k} = \frac{A'_{\Pi(j_n)}}{A'_{\Pi(k)}}. \tag{B.6}$$

Since we know that $\frac{x_k}{x'_{\Pi(k)}} = \frac{A_k}{A'_{\Pi(k)}} = -1$ $\forall k$, consequently, (B.6) is valid, which in turn means (B.4) holds true and the posterior function does not have a unique maximum. For the second part of the proof, it is clear that when no permutation exists to satisfy the condition of the lemma then ϑ can never be equal to ϑ' and consequently, the lemma holds if and only if such a permutation exists. ■

It is easy to see that the condition of Lemma 1 is met if and only if the modulation constellation is symmetric around the origin. This is because with symmetric modulations around the origin if $x_k \in \mathcal{A}$ so is $-x_k \in \mathcal{A}$. Consequently, there always exists a permutation for which the condition of Lemma 1 holds. Therefore, the posterior function does not have a unique maximum if and only if the modulation constellation is symmetric around the origin.

This concludes the proof of the proposition.

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