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Stop-Motion Animation to Model the Analemma

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The Sun does not return to the same position in the sky every 24 hours. At local noon, for example, the Sun will appear higher in the sky as we move from winter to summer solstice. In addition, and perhaps more surprisingly, solar days (the roughly 24 hours between subsequent local noons) vary in length, causing the Sun to be east or west of its location 24 hours prior. Over a year, this variation traces out a figure 8, known as an analemma, as shown in Fig.1.[1] It can also be seen in the sundial in Fig. 2, where the gnomon incorporates the analemma to produce an accurate reading of local time.

Techniques for observing the analemma have been detailed in a prior article[2] in this journal. In this article, we describe a simple method for modeling the role of tilt in producing the analemma: stop-motion animation. While other publications detail the mathematical formalism that describes the analemma[3] and how to use this information to determine variations between the shortest day of the year and the latest sunrise of the year,[4] we find the technique presented here to be productive for non-majors for whom mathematical formalism is challenging.

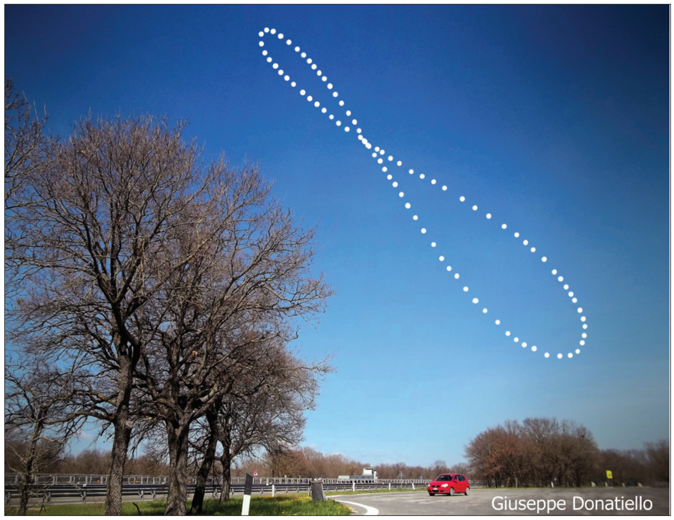

Fig. 1. The analemma as photographed in Italy.

Fig. 2. An analemmatic sundial in Boise, ID.

Stop-motion animation of the analemma

Stop-motion animation has been useful in modeling scientific phenomena in pedagogical settings, as it focuses students' attention on moment-to-moment changes that often underlie key mechanisms.[5] We begin approaching the analemma not via explanation, but by developing a stop-motion animation with a globe, using the camera itself to represent the Sun. Students are tasked with replicating the rotation and revolution of Earth, and capturing the position of Earth as viewed by the Sun every 24 hours.

In one day, as Earth moves ~1° around the Sun, Earth spins ~361° on its axis. The stop-motion animator then would rotate the globe by 1° about its axis, and then revolve the globe 1° around the Sun as the "Sun" (camera) turns 1° to "view" the globe. This is represented in Fig. 3. Thus, this animation requires three precise movements—a rotation, revolution, and rotation—iterated multiple times for a complete revolution.

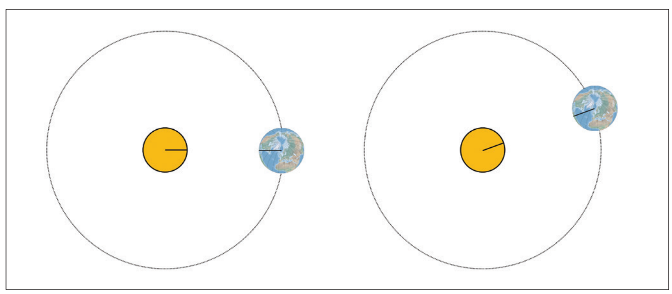

Fig. 3. Schematic of two frames for a stop-motion animation. At left, the initial position as viewed from the above the ecliptic; at right, a 20° rotation and revolution.

The above steps are sufficient for reproducing the analemma. Equivalently, but with fewer opportunities for error, the animator could center the globe on a turntable and spin the globe 1° counterclockwise about its axis and 1° clockwise on the turntable. When tracking one point on the globe over the "year," this activity reproduces a figure 8, as can be seen in the online videos at

https://drive.google.com/file/d/13vWHRyDXV5GP16FdcRAZh26DxNGjPSwE/view

and

https://drive.google.com/file/d/1qlHehAzg3FdeihwkwxYYCpp9YaNUYxhq/view.

Additional details on the production of the video are in the next section.

Creating a stop-motion animation

To produce the videos, we used a kitchen turntable, a globe centered on the turntable (the globe has a set of time zones on top that functions as a protractor), and a smartphone. A printed protractor was affixed to the turntable. The turntable protractor is marked in 15° increments clockwise;

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Fig. 4. A globe mounted on a turntable. The globe rotates 30° counterclockwise and the turntable 30° clockwise from its starting position (left) to produce the right frame of a stop-motion animation.

the protractor atop the globe is marked in 15° increments counterclockwise and affixed to the globe's axis so it will not spin with the globe. The camera is set up to be level with the center of the globe; we used boxes and the leveling app to align the camera. There are free stop-motion apps available for capturing photos and producing an animation from those; alternatively, you can take photos, markup these to show the progression of a point on the equator, and stitch these together using software, such as iMovie, to produce a stop-motion animation. Tracing a point on or near the equator will yield a figure 8, as seen in Fig. 5.[6]

Fig. 5. When selecting one point on the globe in the tropics and following it over the year, a figure 8 pattern emerges.

An explanation of the analemma

The height of the analemma is generally easy for students to explain: as we tilt toward the Sun, the Sun appears higher in the sky. An explanation for the "lobes" of the 8, indicating that some solar days are longer than 24 hours, then shorter, over each 6-month period, is not as straightforward. To explain this, we consider the steps involved in creating the animation. In particular, producing this animation involves two rotations: the rotation about the axis of Earth by $360^{\circ} + x$, and a rotation about an ecliptic axis (perpendicular to the plane of orbit) by $-x$. This indicates that a year is composed of two kinds of "day": 366.25 rotations about our axis (the sidereal day), and the day produced by the revolution about the Sun.

Because these axes of the two rotations are not parallel, the rotations do not cancel one another out. Explaining the role of tilt in the analemma, then, is equivalent to characterizing how these two different rotations affect the motion of a point on the globe over a year. The rotation about the axis of the globe must be undercompensated by the rotation of the turntable for two periods during the year, and overcompensated for an additional two periods of the year. In viewing the rotations from above the ecliptic (that is, above the turntable), the two rotations appear as shown in Fig. 6; Earth is tilted 23.5° and undergoes a $360^{\circ}+x$ counterclockwise rotation; the outer circle is not tilted and undergoes a clockwise rotation of x.

From this view, the symmetries are apparent: for two three-month periods centered on the solstices, the net rotation is west and solar days are longer than 24 hours; for two three-month periods centered on the equinoxes, the net rotation is east and solar days are shorter than 24 hours; and the two rotations cancel one another at four points (in between the solstice and equinox), the four times when a solar day is (for the circular orbit) exactly 24 hours long.[7]

Acknowledgments

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References

1. "Analemma A14 2016" by gjdonatiello is licensed under CC0 1.0.
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6. The trace of a point above/below the tropics will appear circular; it is moving in a figure 8 relative to the longitude of local noon but this is less obvious visually.
7. When considering only axial tilt, the longest solar day is only minimally longer than the average day (24 hr 20 s). However, the accumulation of these longer days causes local noon to occur 10 minutes later than would otherwise be anticipated. This is captured in the equation of time, which describes the discrepancy between mean time, in which noon occurs every 24 hours, and apparent solar time, in which noon occurs when the Sun is at its highest point that day. Further details may be seen at the Desmos link https://www.desmos.com/calculator/gd3y9fn9wg.

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