2019

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Publication Information

Banner, Dominique and Overman Cavey, Laurie. (2019). "Bottle Filling Task Reasoning: A Comparison of Matching Versus Constructed Student Responses". *Proceedings of the Forty-First Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* 1461-1465.

BOTTLE FILLING TASK REASONING: A COMPARISON OF MATCHING VERSUS CONSTRUCTED STUDENT RESPONSES

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In this paper, we compare the levels of reasoning elicited during the completion of three versions of a bottle filling task: high school level matching; middle school level matching and constructed response. The goal of the tasks was to make visible secondary students covariational reasoning methods. Video of students completing the task while explaining their reasoning during one-on-one interviews were analyzed. Analysis demonstrated a wide range of reasoning when provided a matching version with a greater incidence of accuracy with students who exhibited lower levels of reasoning. Conversely, the constructed response task demonstrated higher levels of reasoning more consistently with decreased accuracy. Implications for assessment are discussed.

Keywords: Algebra and Algebraic Thinking, High School Education, Middle School Education

In this study, video from a larger study in which students completed modified versions of the well-known bottle filling task (Thompson & Carlson, 2017) while explaining their thinking were analyzed (Cavey et al., in press). The solutions and reasoning methods of four groups of students were analyzed for three separate versions of the task: a high school matching, middle school matching, and a constructed response in which the students were asked to draw their own graphs to represent the given situations. None of the versions provided any measures or scales. Across all groups, students were encouraged to verbally explain their reasoning after completing the task.

Assessing student understanding in mathematics from students' written work can be difficult. Students can be successful at completing a task without understanding the targeted learning goal or understand the targeted learning goal but be unsuccessful in the task for an unrelated reason. Although, in mathematics, there is usually one correct answer, it is also important to recognize, and acknowledge, student success in their thought processes. Relatedly, there has been a recent shift in standardized test structures from entirely multiple choice to include some constructed response items (e.g. the addition of "grid-in" questions on the SAT). This shift in testing structure, along with our interest in supporting secondary student's covariational reasoning, motivated us to examine the student reasoning elicited across different types of tasks.

Purpose

In this study, student work on three versions of a bottle filling task, two matching and one constructed response, were compared with the purpose of identifying aspects of the covariational and graphical reasoning students appeared to use and to compare the results between the groups. The questions which guided our analysis are:

- How do the aspects of covariational reasoning exhibited by students compare across the three tasks?
- Do students complete the task more accurately in one format over the other?

In this paper, we describe our methods, results, and analysis of the student work. We also discuss the implications for teaching and student assessment.

Theoretical Framework

This study is informed by research concerning covariational reasoning and assessment of students' mathematical knowledge. The intent of the bottle filling task is to elicit covariational reasoning by asking students to imagine the dynamically changing scenario of a bottle filling with a liquid and think about how the relationship between the height and volume of liquid is represented graphically.

Covariational Reasoning

Covariational reasoning involves coordinating how two quantities vary together and is essential to the understanding of functions (Thompson & Carlson, 2017). Various versions of the bottle filling task have been used in studies designed to assess covariational reasoning. For example, Carlson, Jacobs, Coe, Larsen and Hsu (2002) used a version of the bottle filling task with college undergraduates in which students were asked to create a graph of the height as a function of volume. In addition, Johnson (2012, 2013) used a version in which volume was shown as a function of height and asked high school students to sketch a picture of a bottle which would generate the given graph. The researchers in these studies also developed frameworks for analyzing student covariational reasoning. As the students in our study consist of both middle school and high school level students and we were interested in how the student understanding was translated to a graph, these frameworks informed the creation of our framework, but did not completely align with our purposes.

Assessment

Assessment is an integral component to teaching and learning. Assessment provides teachers and students information regarding a student's level of comprehension. Student work can be assessed: during whole class instruction, small groups, or one-on-one; through verbal or written interaction; and can take the form of formal or summative assessment. Information gathered from assessment can be used to inform instruction, group students, or develop interventions.

Black and Wiliam (2009) discuss the structure of formative assessment in which the teacher presents a task, the learner responds, and the teacher then acts in response to the student. To provide the most appropriate response, it is essential for the teacher to understand the student's reasoning as well as possible. However, as Black and Wiliam (2009) point out, this can be a very difficult task for teachers as it can be impossible to know entirely what a student is thinking.

One method for enhancing teachers' ability to understand a student's reasoning is for the teacher to request the student articulate their thought process, prompting the student to expand the description of their reasoning where necessary and to reflect on their own thinking. Black and Wiliam (2009) cite work by Shayer and Adey (2002) who found that articulating and reflecting on one's own thought is essential to the learning process.

Teachers must determine the types of tasks necessary to elicit student reasoning and questions to create this rich learning environment. Chaoui (2011) cites research regarding the differences in cognitive demand of multiple choice questions compared to constructed response, but also states there can be great difference in the level of cognitive demand required in the way a multiple choice question is constructed. Chaoui (2011) also cites several research studies showing similar performance on multiple choice questions compared to constructed response. This study looks at these types of comparisons specific to the bottle filling task.

Methods

Participants

This study involved the analysis of one-on-one interviews, obtained from a larger study, of secondary students completing one of three versions of a bottle filling task. Thirty-five students completed the task in the first round of interviews (15 high school and 20 middle school) and 16 in the second (9 high school and 7 middle school). It is important to note, high school students in the first round were enrolled in a Calculus class while those in the second round were enrolled in lower level mathematics courses. As we are comparing the reasoning and accuracy levels for each version of the assessment, we do not consider this an issue.

Student Task

During the first round of interviews, students were provided a matching version of a bottle filling task. All versions the bottle filling task used of were adapted from a task created by Swan (1985). The task pictured 3 bottles with different shapes: an evaporating flask, an ink bottle, and a bucket. Two versions of the matching task were developed, one for high school students, which provided 5 graph options, and one for middle school students, which provided 3 graph options. Each version depicted height as a function of volume. Students were also provided the option of creating their own graph.

In the second round of interviews, the bottle filling task was redesigned as a constructed response problem. There were no longer two separate versions, the same three shaped bottles were used, and students were asked to create a graph for each. To assist the students in creating their graph, partial planes were provided with two perpendicular rays representing the first quadrant of a graph. There were no markings on the axes or grid in the plane to indicate any type of scale. The axes were labeled to indicate volume as a function of height. The reason for this change was due to the student's ability to conflate volume with time, and thereby arriving at a correct answer without demonstrating the desired level of covariational reasoning.

Data Analysis

To compare student reasoning and accuracy, videos and transcripts of the interviews were reviewed and coded. Table 1 shows the framework developed for coding student reasoning elicited across all versions of the task:

Table 1: Student Reasoning Coding Framework

Level	Description
1	Recognizes that both quantities increase or provides no explanation related to volume or height (E.g. student graphs a straight line)
2	Makes connections between the shape of the bottle and how one quantity (either volume or height) changes (E.g. Verbally (and with gestures))
3	Coordinates the changes in one quantity with characteristics of a graphical representation (E.g. student explains how one quantity changes over time and translates their ideas to a graph)
4	Makes connections between the shape of the bottle and how volume and height change together (E.g. The student can verbally explain (or show with gestures) how the quantities change together)
5	Coordinates changes in volume and height with characteristics of a graphical representation (E.g. in addition to being able to explain, the student can also translate those ideas to a graph)

While reviewing the student interview videotapes, it was determined that students would occasionally employ differing levels of reasoning for different bottles. Therefore, student work and reasoning for each bottle were coded individually.

For the purpose of coding student accuracy, three different scoring frameworks were implemented, depending on the version of the task. A score of 0 was given for incorrect responses on all versions. For the middle school version of the matching task, students were given a score of 1 for a correct response. The high school version of the matching task was scored as follows: Evaporating Flask, 1 for a correct response; Ink Bottle, 1 for selecting option A and 2 for selecting option D; and Bucket, 1 for selecting option E and 2 for selecting option C. The task was scored in this manner due to the similarities of options A and D (continuous graphs containing 3 sections), as well as options C and E (smooth curves with differing concavity). The constructed response version of the task was scored as follows: 1 for a graph than contained a correct feature (e.g. number of changes in the graph, concavity, etc.) and 2 for a correct graph.

Subsequent to coding student reasoning elicited and the accuracy of their work, a ratio chart was completed cross-referencing the reasoning and accuracy scores. Relative frequencies of each cell were color coded by percentile group (1-9%, 10-19%, etc.) and compared, looking for trends. Trends within individual students were also analyzed.

Results

Through our analysis of the descriptor code ratio charts, it was found students completing both matching versions of the task had a greater incidence of accurately completing the task with lower level reasoning skills. In addition, there was a higher incidence of accurately completing the task for those who exhibited higher levels of reasoning. Students completing the constructed response were more apt to employ higher levels of reasoning: middle school students were much more likely, high school students only slightly more likely (possibly due to the discrepancy between the first round of high school students being enrolled in Calculus). Finally, students completing the constructed response did not show the same level of accuracy as those completing the matching.

During the evaluation of individual students for trends, it was also found, while completing the matching version of the task, students were more likely to employ different levels of reasoning for the variously shaped bottles. Whereas, those students completing the constructed response were more likely to consistently employ the same level of reasoning throughout.

Discussion and Conclusion

Results indicate matching and constructed versions of a mathematical task, such as the bottle filling task, elicit different aspects of student knowledge. Depending on the purpose of the task for the teacher, one version may be more useful over another. For example, a matching version could be more useful as a pre-test to determine prior knowledge on a topic; where constructed response could be more useful to elicit higher levels of reasoning. However, teachers must be mindful of the propensity of a student to perform higher on matching, or lower on constructed response, than their actual ability. To counteract this possibility, teachers may also wish to construct questions aimed at prompting students to articulate their reasoning.

It is important to note the limitations of this study. The groups studied were small and consisted of students who volunteered for a study with a different focus. Although all available videos were included in the study, it was not meant as a representative sample. Therefore, the research conducted here is meant only as an exploratory study.

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Acknowledgements

The larger study referenced is supported by the National Science Foundation (Award No. 1726543).

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