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Leslie Atkins Elliott
Boise State University, leslieatkins@boisestate.edu

André Bolliou
Boise State University

Hanna Irving
Boise State University

Douglas Jackson
Boise State University

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Modeling Potential Energy of the Gaussian Gun

Leslie Atkins Elliott, André Bolliou, Hanna Irving, and Douglas Jackson, Boise State University, Boise, ID

The Gaussian gun is an arrangement of magnets and ball bearings (pictured in Fig. 1) such that—when the leftmost ball is released—the rightmost ball is ejected at high speeds. The device has been described in several articles on energy education.[1-5] The sudden appearance of kinetic energy offers a productive context for considering a range of challenging ideas: the often-counterintuitive relationship between force and potential energy, the escape velocity for attractive forces, why energy is required to break bonds, and why energy is released when bonds form.[3] Beyond these ideas, it is also useful for motivating the representation of a potential well and bound states for both quantum mechanics and chemistry.

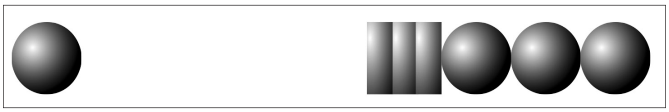

Fig. 1. The Gaussian gun: four ferromagnetic ball bearings and three strong neodymium disk magnets. When the leftmost ball is released, it strikes the magnet and the rightmost ball is ejected with great speed.

The goal of the activity described in this article is the construction of a gravitational analog of the Gaussian gun (GG). That is, to create a curve such that the pull of gravity mimics the magnetic attraction, and thus the dynamics, of the Gaussian gun. Such a model supports students in understanding the ideas described above: force vs. energy, escape velocity, breaking bonds, potential wells, and bound states.

The techniques to construct this slope were developed by preservice secondary teachers in a course on the nature of scientific research and its role in science teaching. Our semester began by observing the Gaussian gun and working to develop models of energy that could account for the sudden appearance of kinetic energy. As the students debated ideas regarding the origin of the energy in this phenomenon, one student rolled a ball down a three-ring binder to represent the energy of the incoming ball, losing potential energy as it increases in potential—a counterargument to a group locating potential energy in the magnet. This led to a conversation about whether or not this shape—a linear slope—adequately represented the ball's energy as it rolled toward the magnet. With this question posed, the lab group sought to characterize and build a slope that recreated the motion of the ball in the Gaussian gun.

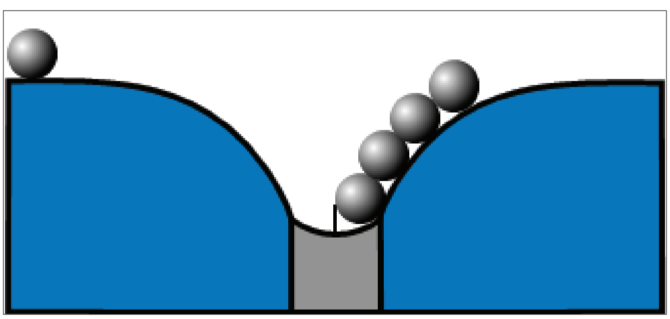

Fig. 2. A schematic of a gravitational analog of the Gaussian gun. A ball released from the left will cause the rightmost ball to be ejected.

A schematic of the final product, which reproduces key features of the balls' dynamics, is pictured in Fig. 2; the central region represents the area of the magnet. A video of the 3D printed object in use, showing the incoming ball from the left- and the rightmost ball being ejected, can be viewed at *TPT Online.*[7]

This article does not describe the design process, rich with prototypes and failures, that characterized the efforts involved in creating this. While these were a central element of the course (which was focused on research methods of scientific inquiry and their relationship to the secondary classroom), we focus below on the outcome of that process, with information on how to replicate the production of this curve in more traditional physics classrooms, and the affordances of the representation for understanding the Gaussian gun.

How to create the gravitational analog: Overview

In this section, we describe a lab activity for students that reproduces the technique developed in our class. In the activity, students record the force that the ball bearing experiences due to the magnet as a function of distance. From this data, students can approximate the work done by the magnet as the ball is pulled away from the magnet and, consequently, the potential energy of the ball/magnet system as a function of their separation. In doing so, students construct a potential well, and this representation has affordances for understanding features of the Gaussian gun in particular, and for a range of physics ideas in general. Note that our goal for this activity is that students construct a shape that is steep close to the magnet, with the slope decreasing towards zero as you move away from the magnet; greater precision than this is welcomed but not a part of the steps below. Finally, we discuss how to translate these measurements into producing a gravitational analog of the potential well.

Measuring the force on the ball due to the magnet

To begin, set up the magnets and ball as shown in Fig. 3 with the magnets affixed to the table (we used masking tape). Using string, fashion a tight lasso around the ball or glue the string and ball together. This string is then attached to a force probe, which is read out to a computer. When the ball is gently but firmly pulled from the magnet, the force will increase until the ball begins to move

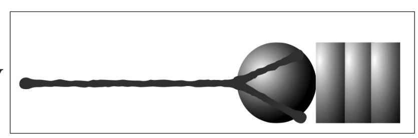

Fig. 3. String (leading to force probe) and the Gaussian gun.

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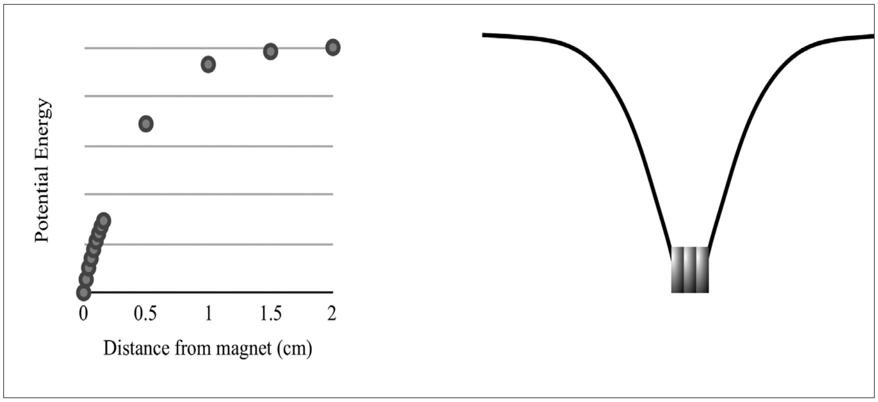

Fig. 4. Potential energy as a function of distance from the magnet (left), and a symmetric potential well based on this data (right).

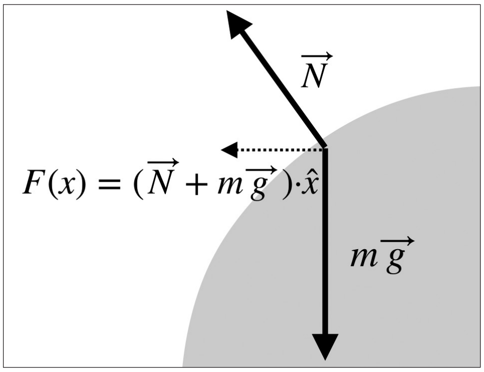

Fig. 5. The shape of a curve to replicate the speed of the Gaussian gun.

away; the highest recorded force is equal to the force of the magnet on the ball.

Place index cards between the ball and the magnet to gather data for distances close to, but not touching, the magnet and repeat the force measurement. Continue adding index cards until the force is no longer appreciably changing. At this point, move the ball 0.5 cm to collect force data. Stop once the force is imperceptible. By knowing the width of an index card (e.g., measure the height of a stack of 50 cards and divide), students can generate a table of distance and force data.

From here, creating a plot of work done as the ball is pulled away from the magnet—and hence potential energy in the system—is straightforward. With a potential energy set to zero when the ball is touching the magnet, the work done by the string is $\boldsymbol{F} \cdot \boldsymbol{d}$ for each increment of measurement; since very little energy is lost to heat, we approximate the potential energy gained as equal to the work done. Figure 4 (left) shows this plot for Ball A from our data. And with this data, students can sketch a symmetric profile of the potential due to the interaction of the magnet with the ball (Fig. 4, right). (The symmetry is an approximation, which we discuss below.)

Making sense of the representation

This representation, a potential well, supports a range of discussions that facilitate an understanding of the forces and energy of the Gaussian gun; these questions emphasize understanding the representation and the forces and energy involved. In the next section, we outline questions that support an understanding of the dynamics of the GG:

- How does the strength of the force compare to the potential energy? (In this scenario, there is less height, and therefore less PE, in regions of more slope, and therefore more force.)
- Which of the balls has the most energy before the GG is launched? (The single ball farthest from the magnets. This usually generates a long discussion, often prompting the idea of a negative energy by the magnet and zero energy far from the magnet.)
- Does the rightmost ball have much energy before the GG is launched? (Yes. Despite the fact that it experiences very little force, it has a great deal of energy relative to the other balls on that side.)
- If this was a universe of only the magnet and ball, would the PE ever reach a maximum? (Theoretically, no, though our data suggest otherwise.)
- Does that imply that the PE increases without limit? (Surprisingly to many students, no.)

In addition, the following questions support an understanding of potential wells more broadly:

- If the well was a parabolic shape instead, what would that tell us about the force? (It gets stronger as you get farther away; there is more PE in regions with stronger force.) What kinds of interactions might have that kind of shape? (Springs, quarks.)
- If the well was a linear shape (like a V), what would that tell us about the force? (It is constant, independent of distance.) What kinds of interactions might have that kind of shape? (Gravity on Earth's surface. All forces over short distances. Infinite planes of charge.)
- Could you imagine a scenario with an "upside down" well? (Repulsive interactions.)

The gravitational analog: Building a working model

Our goal was to create a slope along which balls could be placed and roll to reproduce the dynamics of the Gaussian gun. In particular, we hoped to produce a curve such that an incoming ball would gain speed gradually at first and then rapidly as it approaches the "magnet" at the center; that it would strike a chain of balls and eject one; and that the ejected ball would have enough energy to escape the well entirely. Clearly gravity cannot produce the accelerations that the GG produces for the ball bearing: close to the magnet, the force of attraction is many times stronger than the gravitational attraction. However, we can scale the force so that the dynamics are similar, but slower.

The simplest construction is for the curve to be a scaled version of the potential well itself. In this case, the height of the well (where the ball has energy *mgh*) represents the amount of potential energy; the distance along the base represents the distance from the magnet; the total kinetic energy at a given distance, then, is found by the height lost.

If, instead, the goal is to develop a curve such that, when viewed from above, the rolling ball reproduces the motion of the ball rolling towards the magnet (the original question that drove our inquiry), the shape is slightly different. To determine this shape, the *x*-component of the sum of the forces (due to gravity and the normal force) should be proportional to the force of the magnet, as shown in Fig. 5. The measured value of the force, then, determines the slope at each point

Fig. 6. A 3D printed analog of the Gaussian gun.

along the curve.

In both cases the resulting wells are similar: steep towards the middle where the magnet lies, and flattening out at the edges where it is nearly flat. To design the bottom of the well, we needed to limit the amount of energy transferred to the well itself, leading the students to design a gradual curve with a flattened base (see Fig. 2). To hold the balls in place and transfer energy from the incoming to outgoing ball, several mechanisms were designed (including see-saws). Many of these absorbed too much energy, and ultimately the first mechanism designed, a small metal stopper, was used to hold the balls on one side of the well.

The well was printed to be just over one ball-width wide so the ball doesn't rub against the walls, sandwiched between pieces of clear acrylic, and with a drilled hole to thread in the metal stopper (see Fig. 6). With balls placed on top of the curve, one ball entering from the left could easily "eject" the ball on the right. It is at once obvious why this should be so, demystifying the dynamics of the Gaussian gun and analogous systems. Among the questions we addressed using this representation were:

- How does this shape represent the fact that the force gets weaker and weaker the farther you are from the magnet? (The slope decreases the farther you are from the magnet, so it gets increasingly easy to pull the ball away from the magnet.)
- Where is the energy in the system and in what form is that energy at the beginning? At the end? (Initially, all the energy is gravitational potential energy and so the highest ball has the most GPE. At the end, the rightmost ball has gained all of the first ball's energy; most of the energy it gained is now KE.)
- Could you push a ball in this well away from the magnet hard enough so that it would never turn around and roll back? (Yes; even though it would continuously decrease in speed, it would never stop. The minimum speed necessary is the escape velocity.)
- How is this analogous to the Gaussian gun? (The ranking is the same, but now PE is due to the interaction with the magnet instead of with Earth.)
- Why does the final ball leave with so much energy? (It had a lot of energy to begin with—it was already most of the way "out" of the well; the additional energy transferred from the incoming ball is more than enough to escape the well with a great deal of KE remaining.)

In addition, this representation supports connections to chemistry[6] and advanced physics, in which the "attached" magnet and balls represent bound states (either electrons in a potential well or two atoms that are bound):

- Is energy required or released when a bond forms? (When the leftmost ball rolls in and forms a "bond," there is excess KE released to the system.)
- Is energy required or released when a bond is broken? (Energy is required to cause the rightmost ball to be ejected.)

Limitations

While the analog does not perfectly replicate the dynamics of the magnet/ball system, it characterizes critical aspects and supports students in visualizing multiple aspects of the Gaussian gun: a move away from the magnet as adding energy; the decreasing force is represented by a decrease in slope rather than a decrease in height; rapidly changing curvature matches the kinesthetic experience of the ball being hard to pull off of the magnet but easy to move once it's no longer close; and, finally, a representation of where the energy is in the system.

However, there are limitations of this representation. First, and perhaps most significantly, this representation assumes the well is symmetric; the presence of ferromagnetic balls means that the well should not be symmetric. However, this simplification does not affect the broader understanding of energy in the system. In addition, collecting data from each side and developing an asymmetric well is a possible extension of this activity. As a second limitation, we avoid a discussion of uncertainty in the measurements. Again, this simplification does not affect the broader understanding of energy. However, if your instruction has emphasized the critical importance of reporting uncertainty in measurements, you may wish to adjust the lab accordingly. And finally, as noted above, the final printed product is scaled: the ball is not nearly as fast as the magnetic system, and thus it's significantly less dramatic.

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