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Current Knowledge and Future Directions: Proportional Reasoning Interventions for Students with Learning Disabilities and Mathematics Difficulties

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Abstract

The purpose of this systematic review was to examine the effectiveness of proportional reasoning interventions for students with learning disabilities (LD) or mathematics difficulty (MD). We evaluated 5th - 9th grade interventions on proportional reasoning content, instructional features, and disability and difficulty identification. The nine studies met inclusion criteria yielded intervention effects ranging from g = – 0.10 to 1.87 and from Tau U = 0.88 to 1.00. Two of the nine studies were deemed high quality and very few studies included participants with LD. Although most studies identified the concepts addressed in the interventions, authors rarely provided in-depth descriptions of how the concept was taught. The results suggest that proportional reasoning interventions for students with LD and MD is under investigated. We posit that intervention research in proportional reasoning can and should be expanded upon and offer suggestions in terms of how researchers can continue to develop the knowledge base.

Keywords: intervention, learning disability, mathematics difficulty, proportional reasoning

Proportional reasoning is an important mathematical concept for students to learn through middle school, algebra, and beyond. It is a major component of curriculum standards in sixth and seventh grades (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010) and is predictive of later performance in higher mathematics coursework, such as algebra (National Mathematics Advisory Panel [NMAP], 2008). Algebra is a "demonstrable gateway to later achievement" (NMAP, 2008, p. 5) in higher mathematics, a major factor in high school graduation rates, and provides a foundation to concepts studied in calculus (i.e., velocity, acceleration, instantaneous rate of change, and derivatives), a stepping-stone to postsecondary scholastic and professional success.

Research has shown that students with learning disabilities (LD) underperform compared to their peers in proportional reasoning (Grobecker, 2000; Hecht et al., 2006; Mazzocco & Devlin, 2008). Students with LD experience more significant difficulties in acquiring proportional reasoning than typically progressing students (Jitendra et al., 2019; Mazzocco & Devlin, 2008). Thus, it is important to consider how proportional reasoning interventions can benefit students with LD; as well as for students with chronic low mathematics achievement in absence of a formal disability (i.e., mathematics difficulty; MD). In the following paragraphs, we define LD and MD, and expand the discussion on

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proportional reasoning in terms of its foundational nature for middle and high school mathematics. We review previous syntheses of mathematics interventions, highlighting conclusions and what is yet to be explored. Finally, we introduce the research questions for this study.

Learning Disabilities and Mathematics Difficulty

Results of prior work leave unanswered questions related to differences students with LD and MD in terms of definition as well as their difficulties with proportional reasoning. Throughout this systematic review, we refer to students who have LD and MD. Many authors define LD as students who have a learning disability using state criteria or school records, or performance at or below a cut off score (i.e., 10th percentile) on a standardized measure of mathematics achievement (Chong & Siegel, 2008). In contrast, there is less consistency with how students with MD are identified in the literature (Nelson & Powell, 2018). For example, authors use varying criteria, including but not limited to, teacher referral, performance below the 25th percentile, or lowest performing students.

Often, students with LD and MD exhibit similar behaviors in mathematics, including difficulty with counting, inefficient computation strategies, and difficulty with performing multiple steps to solve problems. Distinctive characteristics of the difficulties include poor fact retrieval and differences in working memory, processing speed, and spatial representations of numbers (Chong & Siegel, 2008; Cirino et al., 2015). In terms of proportional reasoning specifically, both similar and more nuanced characteristics of mathematics understandings emerge. Students with LD and MD both utilize additive reasoning, not proportional reasoning, to solve problems because they view relations as differences as opposed to rates of change (Grobecker, 2000; Hecht et al., 2006; Mazzocco & Devlin, 2008). For example, when examining students' rational number reasoning, Mazzocco and Devlin (2008) reported that middle school aged students with LD demonstrated statistically significant differences in identifying relationships between two fractions presented both pictorially and symbolically. In fact, due to the additive nature of their reasoning, students with LD identified a significant number of incorrect relationships.

Looking at proportional reasoning in students across grade levels, Grobecker (1997) presented students with a task-based scenario where a lion ate three grains in a bundle at a time and an elephant ate two grains in a bundle at a time. Students were given various scenarios requiring them to utilize the relationships between the animals and bundles to solve problems. She found four stages of understanding exhibited by students with LD and MD: (a) The inability to manipulate grains and bundles at the same time; (b) An additive ability to count and add grains and bundles; (c) Representing grains and bundles as groups and then adding the groups; and (d) Using proportional reasoning (i.e., using the multiplicative relationship between grains and bundles to manipulate values) to scale a relationship up and down. Students with both LD and MD experienced difficulty thinking proportionally (i.e., Stage 3 or Stage 4). Students with LD were unable to advance beyond Stage 2, using mostly additive structures, while older students with MD demonstrated higher stages of reasoning.

Framework: Proportional Reasoning

The difficulties students with LD and MD demonstrate underscores a major difference between additive and proportional reasoning and a need to support students to transition to more sophisticated reasoning. Proportional reasoning entails utilizing a multiplicative relationship between two or more quantities. When two quantities are related proportionally, the ratio of one quantity to the other is invariant as the numerical values of both quantities change by the same factor. This can be represented as a:b = c:d or $\frac{a}{b} = \frac{c}{d}$. For example, consider how many cups of sugar it would take to make 20 gallons of lemonade if three cups of sugar make four gallons of lemonade. Students who reason proportionally may notice that the number of gallons of lemonade was quintupled, so the number of cups of sugar would also be quintupled. In this respect, reasoning proportionally does not require that students use, manipulate, or are even aware of the notation initially.

Arguably, the difficulties students with LD and MD experience with additive reasoning can be traced back to experiences, or the lack thereof, that students have to reason proportionally (NMAP, 2008). Research in learning trajectories (e.g., Battista & van Auken Borrow, 1994; Confrey et al., 2014) suggest potential pathways students may travel to build up conceptions of ratio and proportional reasoning. This research states that *linking two composite units* together is foundational to understanding ratios and rates conceptually. Students often connect such work to whole number multiplicative reasoning. For example, to conceptualize multiplication, such as 3×4, a student should be able to iterate three four times either in action or mentally. The result is a composite unit that consists of a unit of 12, which consists of a unit of three iterated four times.

Iterating linked composite units requires the enacted or mental iteration of two composite units at once - a multiplicative relation (Battista & van Auken Borrow, 1994). In other words, understanding that the ratio 3:4 represents the same proportional relation as 15:20 requires a concurrent iteration of the composite units three and four, five times. In the previous scenario involving sugar and lemonade, students may (1) add three five times and add four five times; (2) multiply 3×5, then 4×5 sequentially to obtain $\frac{15}{20}$; or (3) add 16 to 3 to get 19 because they added 16 to 4 to get 20. The first and second strategies are an iteration of the linked composite using variations of multiplicative thought. The third is an additive view of the relation.

Students who reason proportionally realize that the composite values covary between equivalent classes yet the within relation remains invariant. For example, 4 is $\frac{4}{3}$ times as large as 3, as is 20 $\frac{4}{3}$ times as large as 15. Put simply, all denominators of ratios written in the form $\frac{a}{b}$ are n times as big as the numerator. Knowing that invariant relation ($1\frac{1}{3}$) is also the unit rate ($1{:}\ 1\frac{1}{3}$), or the constant of proportionality, is important. Effective instruction in ratios, then, might encourage the trajectory of thought structures students build to understand ratios to promote advances from additive to multiplicative understandings (Daro et al, 2011; Siegler et al., 2010).

Previous Reviews and Rationale

Previous syntheses have examined the effectiveness of interventions in mathematics for students with LD and MD (e.g., Cook et al., 2019; Jitendra et al., 2018; Stevens et al., 2018). For example, Jitendra et al. (2018) examined middle and high school mathematics interventions for students with LD or MD and found a mean effect size of g = .37; authors further reported that the eight studies that used visual models combined with other strategies produced the largest effect size (g = .52). Stevens et al. (2018) examined mathematics intervention for students with MD in grades 4 through 12; the authors reported extreme variability in the effectiveness of interventions ranging from d = .66 to 4.65. Cook et al. (2019) reviewed the effectiveness of schema-based instruction (SBI) on solving word problems for students with LD, and reported that SBI is a potentially evidence-based practice, but that more research needs to be done to strengthen the evidence base. Although SBI is a common instructional approach to teach proportional reasoning, Cook et al. did not focus specifically on proportional reasoning studies.

Although previous reviews examining the effectiveness of mathematics interventions may have included studies focused on proportional reasoning, research syntheses have not exclusively focused on the effectiveness of proportional reasoning interventions for students with LD or MD. This makes it difficult to generalize results of previous reviews specifically for students in the middle grades with LD or MD in the area of proportional reasoning because syntheses generally report summary effects for a pool of studies (e.g., such as by the same grade). In order to improve student outcomes, practitioners need targeted information about interventions, including what population of students the interventions were found to be effective. Many syntheses have targeted wide grade bands, making it difficult to generalize results of reviews that examine SBI or word problem solving to students who are typically taught proportional reasoning skills. Given that proportional reasoning skills are critical for success with more advanced mathematics and are also needed for success with daily living skills, such as cooking and financial management, it is essential that researchers and practitioners understand the research base supporting interventions in this area for students with LD and MD. Conducting a systematic review of the literature on proportional reasoning interventions for students with LD and MD will provide researchers and practitioners with information about what is currently available for students with LD or MD in the area of proportional reasoning interventions.

Purpose and Research Questions

The purpose of this study is to investigate proportional reasoning interventions for students with LD and MD in 5th – 9th grades. Specifically, we explored intervention effects as they related to how researchers defined LD and MD and determined how researchers measured intervention outcomes. We aimed to answer the following research questions: (1) What are the characteristics of studies that investigated the effectiveness of proportional reasoning interventions for students with LD or MD (e.g., proportional reasoning content, study quality)?; (2) How do authors define LD and

MD in studies that investigate the effectiveness of proportional reasoning interventions?; (3) How effective are interventions focused on ratios and proportional reasoning?; (4)What measures, other than mathematics achievement, do authors use to measure students' growth in understanding proportional reasoning?

Method

Literature Review

We were primarily interested in examining literature that reflected more current practices; therefore, we reviewed intervention studies published in the last 20 years, from January 1998 to December 2018. We searched Academic Search Premier, Education Research Complete, ERIC, ProQuest Dissertations & Theses Global, PsycINFO, and Web of Science databases with combinations of the following search terms: "*composite unit*," *covariation*, *disabilit**, *dyscalcul**, *intervention*, *iterating*, "*linked composite*," *"math* difficult*," "multiplicative reasoning," partitioning*, *proportion**, "*proportional reasoning," rate*, *ratio**, and *"unit* coordination."* We considered peer-reviewed journals, dissertations, conference proposals, book chapters, and research reports published in English for inclusion.

Inclusion Criteria

We used the following criteria to determine study inclusion.

1. The study examined the effects of a proportional reasoning intervention. We defined proportional reasoning as mathematical reasoning that involves co-variation and multiple comparisons, with the ultimate goal to understand that proportion binds two variables (i.e., a change in one requires a change in another to maintain an equivalent relation). We defined intervention as instruction that supplemented or supplanted regular mathematics instruction that reflected a change to typical instruction, such as increased intensity (Wanzek & Vaughn, 2007). Studies that examined curriculum or intervention packages that included units on ratios and proportional reasoning were excluded unless the studies provided separate results for instruction in ratios and proportional reasoning.

2. Participants were in middle grades (i.e., 5th – 9th grade). Studies focused on other grades were excluded (e.g., Hunt, 2015), unless authors provided disaggregated results by grade level. We focused on 5th – 9th grades for a few reasons. First, according to the Common Core State Standards (CCSS), proportional reasoning is heavily emphasized beginning with 6th grade and extending to 7th grade (NGA & CCSSO, 2010); so, it makes sense to include these grade levels. Second, in terms of Multi-Tiered Systems of Support, the school structure would justify the use of proportional reasoning interventions beyond 7th grade with students who struggle; it is reasonable to extend the search to include 8th and 9th grades. Finally, early transitions to proportional reasoning can occur as early as 5th grade (Christou & Philippou, 2001; Ham & Gunderson, 2019; Lamon, 2007); in addition, we considered studies with 5th grade participants in an attempt to (a) gather studies conducted prior to the broad adoption of the CCSS; (b) gather studies with mixed grade level groupings of students without disaggregated data (e.g., 5th and 6th grade); and (c) gather studies conducted in non-US countries who may introduce proportional reasoning knowledge and skills earlier than the U.S.

3. The study identified a subset of participants as having a LD or MD. Although a common approach to identifying MD is with performance at or below the 25th percentile (Nelson & Powell, 2018); we were less restrictive in our inclusion criteria due to the small number of intervention studies that have examined proportional reasoning for students with either LD or MD. We included any study where authors specified that students had MD. When studies included students with and without LD or MD, studies that did not provide disaggregated data were excluded unless students with LD or MD made up more than 50% of participants (e.g., Xin et al., 2005). Studies that focused on disabilities other than LD were excluded (e.g., Deaf or hard of hearing; Nunes & Moreno, 2002).

4. The study employed a group design or single case design (SCD) and included a measure of effectiveness (e.g., means, *SD*s, percentage of non-overlapping data).

5. The study included at least one researcher-developed or norm-referenced mathematics measure of proportional reasoning or provided disaggregated scores for items that measured proportional reasoning. Studies that did not include a proportional reasoning measure or disaggregated results were excluded (e.g., Sawrey, 2018).

The comprehensive literature search resulted in 831 studies, which we reviewed in two phases (see PRISMA in supplementary materials S1). First, we reviewed titles, keywords, and abstracts of all 831 studies to determine which studies were outside the scope of this systematic review; 88% of articles were excluded at this point for the following reasons: irrelevant focus (63.7%), participants were not in 5th through 9th grades (8.5%), the study was an intervention but did not focus on proportional reasoning (5.8%), the topic was proportional reasoning but the study was not an intervention (4.5%), and other (5.1%; e.g., commentaries, meta-analyses, duplicates of peer-reviewed articles and conference proposals). We identified 104 studies for full-text review; we reviewed the method and results sections of all articles to determine if the study met all of the inclusion criteria for our systematic review. Many studies were excluded for more than one reason, including: the topic of the study was proportional reasoning but was not an intervention (n = 12); the study was a mathematics intervention not focused on proportional reasoning (n = 43); participants were not in 5th through 9th grades (n = 21); a subset of participants did not have LD or MD (n = 47); participants were not administered a measure of proportional reasoning (n = 20); and the study was irrelevant (n = 15). Nine studies met inclusion criteria for this systematic review, including three SCD studies and six group design studies.

<u>Coding Procedures</u>

We coded the studies that met inclusion criteria for (a) study and intervention information; (b) measures; (c) participants; (d) instructional content; and, (e) study quality.

Study and Intervention Information

We coded studies for location, publication year, design, instructional arrangement, intervention agent, and intensity (duration, number of sessions). Regarding intervention features (Gersten et al., 2009; Siegler et al., 2010), we coded studies for the occurrence of scripted lessons, teacher modeling, meaningful practice opportunities, scaffolded instruction, explicit and systematic instruction, and pictorial and concrete representations. We also coded if interventions emphasized development progressions, student verbalizations and discourse, giving students "think time," metacognition, and teaching underlying problem structures.

Measures

We coded all dependent mathematics measures. We identified each dependent measure as aligned (i.e., proximal) or not-aligned (i.e., distal) to the intervention content and if the measure encompassed many different skills (i.e., broad) or focused on a single skill (i.e., narrow). We also recorded if studies provided reliability and validity information.

Participants

We coded participants' grade/age, ethnicity, gender, free/reduced lunch (FRL) status, and English learner (EL) status. Because the focus of this systematic review was on students with LD or MD, we also coded how authors identified participants as at-risk. We coded if studies identified students as having a LD and the specific criteria (e.g., Individualized Education Program [IEP] goals in mathematics) as well as MD and the specific criteria used for identification (i.e., percentile on a mathematics screening measure).

Instructional Content

We coded if interventions addressed the following content: ratio, unit rate, covariance, invariance, composite units, proportional reasoning, missing value, additive comparison (National Research Council [NRC], 2001). For each, we coded for the level of specificity including: (1) study identified or listed the content that was addressed; (2) study provided a definition of the content in relation to the intervention; and (3) study illustrated or described in depth how the concept was taught, and student actions during intervention.

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Study Quality

We coded studies for quality according to guidelines recommended for group-design studies (Gersten et al., 2005) and SCD studies (Horner et al., 2005). For group-design studies, we coded essential quality indicators (QIs) on a 0-2 scale (0 = did not meet; 1 = partially met; 2 = fully met) and when studies met all but one essential QI (as outlined by Gersten et al.), we coded studies for desirable indicators on a 0-1 scale (0 = did not meet or partially met; 1 = fully met). For SCD studies, we coded all QIs on a 0-2 scale (0 = did not meet; 1 = partially met; 2 = fully met) as Horner et al. outlined one set QIs and did not specify essential versus desirable QIs.

Interrater Agreement

All five authors shared coding responsibilities for all studies included in this systematic review. Each study (100%) was coded once by two authors to determine interrater agreement. Interrater agreement was calculated as [agreements ÷ disagreements × 100]. The mean interrater agreement across categories was 91% (range = 82% to 96%). The authors resolved discrepancies and determined the final code prior to data analysis.

Data Analysis

We calculated effect sizes for group design studies as unbiased Hedges' *g* to account for small sample sizes (WWC, 2017). If studies with a pre-post design did not report adjusted post-test means, effect sizes were calculated as the "difference between the mean outcome for the intervention group and the mean outcome for the comparison group, divided by the pooled within-group standard deviation of the outcome measure" (WWC, 2017, p.13). When studies reported adjusted post-test scores or only reported post-test scores, effect sizes were calculated as the mean standardized difference between the experimental and control condition divided by the pooled *SD* at posttest. Group design effect sizes were interpreted as 0.20 = small, 0.50 = moderate, and 0.80 or greater = large (Cohen, 1988).

For SCD studies, we calculated effect sizes as Tau U, which measures the amount of overlap between phases and controls for the trend in baseline data (Parker et al., 2011). First, we uploaded all graphs in WebPlotDigitzer Version 4.2 (Rohatgi, 2019) to extract raw data from the baseline and intervention phases from each study. Second, we inputted the raw data into a web-based calculator for SCD analysis (Vannest et al., 2016) to calculate Tau U and the 95% CI. Generally, Tau U effect sizes are interpreted as no effect (less than .50); small (.50 - .60); moderate (.60 - .90); and large (greater than .90). Due to the small number of studies included, we do not provide summary effects; we provide only individual study effect sizes.

Results

Descriptive Results

Nine studies met inclusion criteria (see Tables 1, 2). All studies were conducted in the U.S. and were published between 2009 and 2018 (*M* = 2013). There were 2,804 total participants who were: 50% female, 57% Caucasian, 28% Hispanic, 21% African American, 6% Asian American, and 2% Other (percentages do not add to 100% as some studies reported participants as White Hispanic). Fifty-two percent of participants received FRL and 15% were ELs.

Intervention Characteristics

The average duration was 4.7 weeks (*SD* = 1.7 weeks) and the majority of studies (67%) used the intervention as a replacement curriculum; one study (Hunt & Vasquez III, 2014) used the intervention in addition to the core mathematics curriculum, and two studies did not report whether the intervention was in addition to a replacement of core mathematics instruction (Xin, 2008; Xin et al., 2005). All but one study reported using features of explicit and systematic instruction and emphasized teaching the underlying problem structure (Hunt & Vasquez III, 2014). Authors used pictorial representations in all studies except one (Brawand, 2013) to represent and model solution strategies. The most common pictorial representation reported was schematic diagrams. All but two studies (Brawand, 2013; Xin et al., 2005) reported using meta-cognitive strategies such as teaching the 'think aloud' process for problem solving. In contrast, only a few studies reported incorporating the following intervention features: emphasizing developmental progressions (Hunt & Vasquez III, 2014) and emphasizing student discourse and verbalizations (Hunt & Vasquez III, 2014; Jitendra et al., 2018).

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Intervention Content

All studies included at least one skill in the area of proportional reasoning, and the most common skills addressed in interventions were: unit rate (*n* = 8); ratios (*n* = 7); and proportional reasoning (*n* = 7). Other skills were only addressed by a few interventions including: missing values (*n* = 3); invariance (*n* = 2); covariance (*n* = 1); and numerical comparisons (*n* = 1). Only one intervention identified focusing on composite units (Hunt & Vasquez III, 2014) and no interventions focused on qualitative comparisons. Although most studies *identified* concepts that interventions focused on, very few studies provided a *definition* of the content in relation to the intervention and only one study *illustrated* how the concept was presented to students and supported through instruction (Hunt & Vasquez III, 2014). Across all studies, there were 29 instances of identified concepts we coded for; however, authors provided a definition of the concepts in only 15 instances (52%) and authors provided an in-depth illustration or description of how the concept was taught in only 5 instances (17%).

Study Quality

Across the six group-design studies, the average quality rating was 1.74 (*SD* = 0.16; *max* = 2.0). We identified two high- or acceptable-quality studies (Jitendra et al., 2017; Jitendra et al., 2009). The remaining four studies had more than one essential QI with a score below 2 and were not deemed high or acceptable quality. The area of essential QIs that studies scored the lowest (*M* = 1.0; *SD* = 0.58) was "sufficient information to confirm whether the participants had the disabilities or difficulties presented" followed by "control condition described with replicability" (*M* = 1.33; *SD* = 0.82) and "outcome measures administered at appropriate times" (*M* = 1.5; *SD* = 0.84). Regarding the two studies (eligible for high or acceptable), both met four or more desirable indicators, meaning they were each identified as high-quality. For SCD studies, the average quality rating was 1.58 (*SD* = .23; *max* = 2.0). Horner et al. (2005) did not propose a recommended number of indicators needed for a study to be deemed high- or acceptable-quality. Therefore, we identified a high- or acceptable quality study as any study that met all but one QI (similar to Gersten et al.); however, none of the SCD studies met this criterion. We observed the lowest average scores for "social validity" (*M* = 1.00; *SD* = 0.58), "experimental control" (*M* = 1.0; *SD* = 1.0) and "three demonstrations of experimental effect" (*M* = 1.33; *SD* = 0.58).

How Authors Defined LD and MD

With this research question, we examined how authors identified participants with LD and MD within intervention studies (see Tables 1 and 2). Studies specifically identified participants as having a LD in four studies totaling 28 participants. Across the four studies, authors identified participants as having a LD according to school or district criteria and in two studies, authors reported that students also had mathematics IEP goals. In all studies with participants with LD, authors also ensured that students demonstrated difficulty with the targeted intervention content (e.g., through additional intervention study screening measures).

In seven studies, authors identified many more students (*n* = 1,491) as having MD, using varying criteria. Only one study identified students as MD according to performance at or below the 25th percentile on a mathematics achievement test; this was surprising given that the 25th percentile is one of the most common cutoffs used by researchers to identify MD (Nelson & Powell, 2018). Other studies used less restrictive cutoffs (e.g., 35th percentile) or criteria that did not specify a percentile cutoff, such as performance on a state test. Finally, only two studies with participants with MD identified students using measures that specifically assessed an aspect of students' understanding of proportional reasoning (Jitendra et al., 2018; Xin et al., 2005), meaning that studies more often used students' performance in mathematics broadly to identify participants as MD for the proportional reasoning intervention study.

The Effectiveness of Proportional Reasoning Interventions

With the third research question, we examined the effectiveness of proportional reasoning interventions. Because we did not identify enough studies to statistically investigate moderators in intervention effectiveness, we were not able to explore patterns in effectiveness across participants characteristics, such as criteria for LD or MD. We first present the results for the three SCD studies followed by the results for the six group design studies.

The three SCD studies each reported that participants made gains from average scores during the baseline to average scores during the intervention phases. The authors reported overall gains between 52.3% and 60.4% (Brawand, 2013), 71% (Hunt & Vasquez III, 2014), and 80.5% (Xin, 2008). The average Tau U for all studies ranged from moderate to large (0.88 to 1.00); Table 1 presents the Tau U effect sizes for each study.

(Insert Table 1 about here)

The remaining six studies used group design methods and reported intervention outcomes at posttest and delayed posttest on a variety of measures. Effect sizes for proximal measures of proportional reasoning ranged from g = – 0.01 to 1.87 at immediate post-test and from g = 0.16 to 2.69 at delayed post-test. At immediate post-test, three treatment groups yielded negligible or small effects, four treatment groups yielded moderate effects, and only one treatment group yielded large effects. Effect sizes for distal measures of generalized mathematics performance ranged from g = – .22 to 1.33.

How Do Studies Report Improvement?

With our final research question, we were interested in exploring how studies defined improvement of student understanding. In other words, did studies only use typical mathematics measures to report intervention effectiveness, or did studies also use other methods (e.g., social validity, anecdotal information) to report intervention effectiveness? All studies reported improvement on students' ratio and proportion problem-solving skills, with fewer studies reporting improvement of general mathematics skills. However, one study also reported students' results on measures of social validity and self-efficacy as related to the intervention. Regarding social validity, Brawand (2013) reported that the majority of participants enjoyed learning how to solve word problems using SBI format and that their word problem skills had improved as a result of using the SBI process. Most students also reported that they planned to continue to use the process learned in intervention. Brawand (2013) also measured students' self-efficacy and the author reported that one more student at posttest compared to pretest reported that they felt they could *not* manage a solution strategy even after the intervention; however, three more students at posttest reported they were surer of themselves solving word problems. The same number of students at pre and posttest reported difficulty with solving word problems.

One study also examined students' metacognitive strategy knowledge as a measure of intervention effectiveness (Jitendra et al., 2009). The results of the metacognitive assessment indicated that students in the low achieving classrooms were not completely sure how to organize information from a word problem using a diagram after the intervention, compared to students in the average and high achieving classrooms; though the difference between groups was not statistically significant. The authors also reported that high achieving students perceived all three solution strategies for solving ratio and proportion word problems (cross multiplication, equivalent fractions, unit rate) to be equally easy to apply. In contrast, both average and low achieving students identified cross multiplication as an easier strategy to use compared to equivalent fractions or unit rate strategies.

Hunt and Vasquez III (2013) examined strategy use through the lens of students' own mathematical thinking and reported that all three students increased their reasoning as defined by increasingly sophisticated strategy usage evidenced across the intervention. Students' use of build-up, emergent unit, and unit strategies evidenced increased multiplicative understanding of ratios, suggesting conceptual development. The authors also reported that all three students also increased their performance on ratio problems across the intervention, showing an iterative development of both procedural skill and multiplicative reasoning (NRC, 2001). In contrast, none of the other reported measures defined student proficiency in terms of student development.

Discussion

The purpose of this systematic literature review was to investigate proportional reasoning interventions for middle grade students with LD or MD. We explored the type of proportional reasoning content that researchers included in interventions and examined the level of study quality. Moreover, we investigated how researchers defined participants as having LD or MD and the effectiveness of the proportional reasoning interventions. Results of this systematic review uncover interesting information pertaining to the state of the knowledge base for proportional reasoning interventions and students with MD and LD. Taken together, findings support opportunities for the field to expand intervention work in ratio and proportional reasoning in the middle grades for students with more clearly defined MD or specific LD. Moreover, opportunities exist for studies that utilize more carefully articulated or illustrated

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foundational concepts and specially designed pedagogies. We discuss the state of the knowledge base and opportunities for growth in the area of proportional reasoning interventions for students with LD or MD as related to the results of this systematic review.

Characteristics of Included Studies

First, we examined the specific proportional reasoning content and study quality of the included studies. The dearth of research in specific, foundational areas of ratio and proportion content and specially designed pedagogy is also notable. Within the small number of studies we reviewed on ratio and proportion overall, very few addressed students' prior or informal reasoning, focused on informal concepts of proportional reasoning (i.e., invariance, covariance, linked composites, qualitative comparison), or a careful sequencing of problems that could work to support students to advance their reasoning (Daro et al., 2011; Hunt & Vasquez III, 2013). These topics have been researched more extensively with populations of students without disabilities or difficulties (e.g., Lamon, 2007; Streefland, 1997); thus, researchers in special education may continue to develop and investigate the efficacy of interventions in proportional reasoning, paying specific attention to less investigated concepts and skills (e.g., invariance). Additionally, authors rarely defined or illustrated the concepts addressed by the interventions, reflecting a further lack of specificity. Future researchers may consider providing detailed supplementary materials such as lesson scripts or sequences that illustrate how a specific concept was presented. It may be difficult for researches to expand on the interventions conducted, to date, without explicit information about how researchers modeled and demonstrated content.

In terms of pedagogy, most interventions in this study used some form of explicit instruction, such as explicit teacher modeling, scripting of teacher language, and explicit modeling of proportional structure. This finding was not surprising given the evidence-base that supports explicit and systematic instruction, especially for students with LD and MD (Gersten et al., 2009). The interventions and instructional strategies addressed in this systematic review provide a good foundation for future exploration and investigation of other potentially effective instructional practices for students with LD and MD. The need for more diverse research related to effective instructional strategies for students with LD or MD, however, cannot be ignored. Many of the design features across the studies were similar (likely due to the same research team conducting the majority of studies), reflecting a lack of variability in the interventions. One core component used heavily in the reviewed studies is SBI, which works to address students' understanding of problem structures (Fuchs et al., 2004; Fuchs et al., 2008). In SBI, teachers train students to explicitly identify a problem structure and diagram key information into a schematic drawing (Gersten et al., 2009). SBI is showcased in IES practice guides for effective instructional practices in fractions and ratios (Siegler et al., 2010) and is potentially an evidence-based practice to teach students with MD (Cook et al., 2019; Star et al., 2015).

Yet, the field also explicates that the predominance of SBI should not preclude the design, development, and testing of alternate intervention approaches (Gersten et al., 2009; Siegler et al., 2010). One major contribution of this systematic review rests with the potential contributions new approaches may yield for the field. For example, only one study utilized approaches that began with the student solving problems (Hunt & Vasquez III, 2014) and utilized knowledge of development to inform task design, and only two studies utilized discourse and verbalizations of students' thought processes to support student achievement (Hunt & Vasquez III; Jitendra et al., 2018). Despite their generally positive effects, we have very little evidence about how such features support students with particular LD or MD in learning concepts of ratio and proportion in ways not addressed by the interventions in this systematic review. More research is needed to better understand how relatively novel intervention topics (invariance, covariance, qualitative comparison; problems used to nudge or challenge students' prior reasoning), or pedagogical design features (e.g., task design, student verbalizations, knowledge of concept development) might support students to advance their reasoning.

We also examined study quality, and we reported that only two studies met QI to be deemed high quality; however, on a scale of 0 to 2, the average scores across group and SCD were relatively high, indicating that studies met most indicators fully or partially. Regarding the indicators with the lowest quality scores, future research may consider stricter or even different inclusion criteria for participants around disability and difficulty. Group design studies scored the lowest for providing "sufficient information provided to determine whether the participants demonstrated the disabilities or difficulties presented." To score a 2, studies were required to a) document that participants had LD or b) document that participants had MD *according to performance ≤25th percentile*. We were firm in our criteria for studies to score a 2 because more restrictive criteria provided greater confidence that students in the study actually represented a population of students with LD or MD, allowing for more specific recommendations according to our

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systematic review. These results are similar to the results reported by Cook et al. (2019); the authors applied the Council for Exceptional Children (CEC) quality indicators to group and SCD studies and reported that there were not enough participants across studies identified as having LD to classify SBI as an evidence-based practice.

Studies might also ensure that post-test measures are administered immediately (Gersten et al., 2005) in order to have confidence in the reported intervention effectiveness; other QI frameworks also note the importance of appropriate timing of assessments (CEC, 2014). Authors may also consider providing more detailed information regarding the instruction provided in the control condition, as most group design studies in this systematic review only provided information about the textbooks and materials used in business as usual settings; these results are similar to other reviews of interventions regarding the lack of information about the control condition (Nelson & McMaster, 2019). Without comprehensive information about what instruction looks like in the control condition, it is difficult to have confidence in the efficacy of the intervention, even when authors report large effect sizes.

For SCD studies, one study (Xin. 2008) did not exhibit strong levels of experimental control or effect due to not including at least 3 data points in each phase for each participant (i.e., there were intervention phases with 1 or 2 data points); future research that utilizes SCD method may consider following recent advances in single case design to demonstrate the effectiveness of the intervention (Kratochwill et al., 2013). Moreover, although Brawand (2013) utilized measures of social validity, none of the SCD design studies employed typical intervention agents or discussed the cost or feasibility of the study. Authors in future research may consider providing this information, along with information about how feasible the intervention is to implement with little training in the content (e.g., paraprofessionals). It is important to note that the rubric we designed to code for the quality of studies only represents the information that authors actually reported in the published manuscripts and dissertations; quality as it is presented here, may not be reflective of the actual research that occurred. For that reason, we encourage authors of future intervention studies to consider reporting full intervention study information in online supplementary materials in order to allow for replication and expansion of the studies.

Characteristics of Participants

Across the nine studies that addressed proportional reasoning interventions in the middle grades, approximately 2-3% of the students had LD as defined by school criteria and/or IEP goals specific to mathematics, a figure that is below the percentage of the population of middle grade students who possess LD reported nationally (McFarland et al., 2019). In Xin (2008), researchers included some participants without a formal LD diagnosis. Three of four participants were referred for difficulty in word problem solving; however, the authors did not indicate students' initial level of mathematics difficulty as measured by an achievement test.

Furthermore, the majority of students served by the reported interventions had MD, a population often defined in the included studies by performance on a state test or by cutoff scores that fell above the commonly accepted criterion of the 25th percentile. Holistically, our results suggest ambiguities in terms of the nature or specificity of the LD and the severity of the MD these interventions addressed. Cognitive characteristics of both MD and LD vary as a function of the cut off scores and diagnostic criteria utilized to define the population (Murphy et al., 2007). Thus, there is a need for additional studies that address the unique needs of students with LD or students who exhibit mathematics difficulties as defined by more selective sampling criteria.

Effectiveness of Proportional Reasoning Interventions and Outcome Measures

With our third research question, we examined the effectiveness of proportional reasoning interventions; however, we were unable to report a summary effect due to the small number of studies in the literature base focused on students identified with LD or MD. For group design studies, studies yielded variable effects, ranging from negligible to large. What is interesting is the variation in intervention effects given the fact that most of the group design studies were conducted by the same research team implementing a very similar SBI program with 7th grade students. The variable results suggest that more research with students with LD and MD is needed in order to identify practices that produce consistent results. For SCD studies, Tau U ranged from 0.88 to 1.00, which suggests a moderate to large effect for students in these studies. However, the lower bound of the 95% CI for two studies bordered the range of "no effect" or "small effect." This also indicates a need for more research in proportional reasoning interventions to identify what works, for what group of students, and under what conditions.

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Finally, another area for expanding the intervention literature base in ratio and proportion lies in the ways that researchers measure student improvement across the studies. All studies reported some type of performance as the dominant method of improvement for students with LD and MD in proportional reasoning. Many of the studies utilized researcher-developed measures, which reflected improvement in intervention content as opposed to development, advancement in reasoning, or applicability of the knowledge to the construct of ratio and proportion generally. Moreover, because improvement on the distal measure was not always significant, it is unclear to what extent the interventions improved students' global knowledge of ratio and proportion. Finally, only two studies examined other factors, such as strategy formulation, multiplicative reasoning, self-efficacy, or metacognition, as a gauge of improvement, despite the relationships between such factors and student achievement (NRC, 2001). In contrast, researchers have measured or investigated the role of self-efficacy with students with LD or MD in word problem solving (Owen & Fuchs, 2002) and fraction word problem solving (Fuchs et al., 2016), for example. Researchers have also investigated how different problem types may affect problem difficulty differently for students depending on the type of their learning difficulty (Powell et al., 2009). Taken together, these findings reveal a need to broaden the ways in which students' improvement of knowledge in ratio and proportion is measured beyond the use of pre-post testing or performance-driven gauges of improvement.

Implications for Practice

The primary purpose of this study was to provide researchers with detailed information about the state of the knowledge base of proportional reasoning interventions for students with LD and MD; however, there are some practical implications to consider. Many studies in this systematic review reported moderate students gains on proximal outcomes measures, suggesting that interventions in ratios and proportional reasoning are effective, on average, for students with varying degrees of academic risk. These results suggest that when practitioners intervene in this area of mathematics, they can see improvement in understanding of ratios and proportional reasoning concepts. However, practitioners may need to consider a few aspects of these results. First, treatment effects were extremely variable, ranging from negligible to large. The interpretation of the results of this systematic review are constrained by a small research base and practitioners should consider generalizing treatment outcomes with caution. Second, practitioners may also want to consider using additional measures of student understanding in addition to traditional mathematics assessments, such as self-efficacy and strategy use as these concepts are also critical for students' success in all areas of mathematics. To date, these outcomes have largely been uninvestigated for students with LD or MD in the area of proportional reasoning; however, teachers may gain valuable information regarding making instructional decisions and designing supplementary learning activities based on student responses on assessments related to self-efficacy and strategy use.

In addition, practitioners may consider the fact that specific aspects of proportional reasoning have been investigated more often (e.g., unit rate, ratios) compared to other aspects (e.g., missing values, invariance, covariance). Practitioners may consider the fact recommendations in these areas of proportional reasoning will grow and change as the research base grows. Practitioners may carefully examine interventions for how aspects of proportional reasoning are defined by researchers and how it is taught to students in these studies. Very few studies, in their published form, provided lesson content in a manner that was replicable. Practitioners may consider requesting access to lesson plans and interventions to determine how the concepts were taught in order to make the best decisions for their own instruction.

Limitations

There are limitations with this systematic literature review. First, only nine studies met our inclusion criteria. This small number of studies limits the ability to perform meta-analytic techniques to find a summary effect for proportional reasoning interventions. Due to the small number of studies, we also were not able to investigate how variables moderated treatment effects. Second, the majority of studies focused on 7th grade and were conducted by similar research teams. This limits the generalizability of the results to students in different grade levels. Because similar research teams conducted the majority of the studies, the results of our study may also be limited due to researchers pulling intervention samples from similar geographic areas and testing similar interventions. The purpose of this study was to focus on middle grade students who have LD or MD, but future research may broaden the scope of the systematic review by expanding to studies with participants with other types of disability, or including studies the focus on typically achieving students. Finally, we only identified 28 participants as having a documented LD, and many other studies used less restrictive criteria for MD. This limits our ability to determine the effectiveness of interventions for students with LD and MD, in particular, students with the most persistent and chronic learning needs. Future research should attempt to include more students with varying ability levels (with disaggregated results), specifically those students who may need more support in the area of proportional reasoning.

Conclusion

The results of this systematic review add to the evidence base on proportional reasoning interventions for students with LD and MD and expose the need for more research in this area. Future researchers may use the results of this review as a foundation for exploring what other instructional strategies are effective for students with the most persistent learning needs. Our intent with this systematic review was to summarize the current knowledge base so that other researchers may make progress in aspects of proportional reasoning that have not yet been investigated. Given that proportional reasoning is the foundation of mathematics that all students learn in middle and high school, it is essential that more research in this area is conducted.

References

Battista, M. T., & Van Auken Borrow, C. (1995, October). *A proposed constructive itinerary from iterating composite units to ratio and proportion concepts*. Paper presented at the annual meeting of the North American Chapter of the International Group for Psychology of Mathematics Education, Columbus, OH.

*Brawand, A. E. (2013). Proportional reasoning word problem performance for middle school students with high-incidence disabilities. *Unpublished doctoral dissertation.*

Chong, S.L., & Siegel, L.S. (2008). Stability of computational deficits in math learning disability from second through fifth grades. *Developmental Neuropsychology, 33*, 300-317. https://doi.org/10.1080/87565640801982387

Christou, C., & Philippou, G. (2001). Mapping and development of intuitive proportional thinking. *The Journal of Mathematical Behavior*, *20*, 321-336. https://doi.org/10.1016/S0732-3123(02)00077-9

Cirino, P. T., Fuchs, L. S., Elias, J. T, Powell, S. R., & Schumacher, R. F. (2015). Cognitive and mathematical profiles for different forms of learning difficulties. *Journal of Learning Disabilities, 48*, 156–175. https://doi.org/10.1177/0022219413494239

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Hillsdale, NJ: Erlbaum.

Confrey, J., Maloney, A. P., Nguyen, K. H., & Rupp, A. A. (2014). Equipartitioning, a foundation for rational number reasoning: Elucidation of a learning trajectory. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. 61–96). Charlotte, NC: Information Age.

Cook, S. C., Collins, L. W., Morin, L. L., & Riccomini, P. J. (2019). Schema-based instruction for mathematical word problem solving: An evidence-based review for students with learning disabilities. *Learning Disability Quarterly*, *43*(2), 75-87. https://doi.org/10.1177/0731948718823080

Council for Exceptional Children. (2014). *Council for exceptional children standards for evidence-based practices in special education.* Arlington, VA: Author.

Daro, P., F. Mosher, T., & Corcoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction*. Consortium for Policy Research in Education.

Fuchs, L. S., Fuchs, D., Prentice, K., Hamlett, C. L., Finelli, R., & Courey, S. J. (2004). Enhancing mathematical problem solving among third-grade students with schema-based instruction. *Journal of Educational Psychology*, *96*(4), 635-647. https://doi.org/10.1037/0022-0663.96.4.635

Fuchs, L. S., Malone, A. S., Schumacher, R. F., Namkung, J., Hamlett, C. L., Jordan, N. C., ... & Changas, P. (2016). Supported self-explaining during fraction intervention. *Journal of Educational Psychology*, *108*(4), 493-508. https://doi.org/10.1037/edu0000073

Fuchs, L. S., Seethaler, P. M., Powell, S. R., Fuchs, D., Hamlett, C. L., & Fletcher, J. M. (2008). Effects of preventative tutoring on the mathematical problem solving of third-grade students with math and reading difficulties. *Exceptional Children*, *74*(2), 155-173. https://doi.org/10.1177/001440290807400202

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). Assisting students struggling with mathematics: Response to Intervention for elementary and middle schools. Washington, DC: Institute of Education Sciences, U.S. Department of Education.

Gersten, R., Fuchs, L. S., Compton, D., Coyne, M., Greenwood, C., & Innocenti, M. S. (2005). Quality indicators for group experimental and quasi-experimental research in special education. *Exceptional Children*, *71*(2), 149-164. https://doi.org/10.1177/001440290507100202

Grobecker, B. A. (1997). Partitioning and unitizing in children with learning differences. *Learning Disability Quarterly*, *20*(4), 317-335. https://doi.org/10.2307/1511228

Grobecker, B. (2000). Imagery and fractions in students classified as learning disabled. *Learning Disability Quarterly*, *23*(2), 157–168. https://doi.org/10.2307/1511143

Ham, L., & Gunderson, E. A. (2019). Utilizing Analogical Reasoning to Aid Children's Proportional Reasoning Understanding. *Journal of Numerical Cognition*, *5*, 140-157. https://doi.org/10.5964/jnc.v5i2.193

Hecht, S., Vagi, K. J., & Torgesen, J. K. (2006). Fraction skills and proportional reasoning. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children?* (pp. 121–132). Baltimore, MD: Brookes.

Horner, R. H., Carr, E. G., Halle, J., McGee, G., Odom, S., & Wolery, M. (2005). The use of single-subject research to identify evidence-based practice in special education. *Exceptional Children*, *71*(2), 165-179. https://doi.org/10.1177/001440290507100203

Hunt, J. H. (2015). Notions of equivalence through ratios: Students with and without learning disabilities. *The Journal of Mathematical Behavior*, *37*, 94-105. https://doi.org/10.1016/j.jmathb.2014.12.002

*Hunt, J. H., & Vasquez III, E. (2014). Effects of ratio strategies intervention on knowledge of ratio equivalence for students with a learning disability. *The Journal of Special Education*, *48*(3), 180-190. https://doi.org/10.1177/0022466912474102

*Jitendra, A. K., Dupuis, D. N., Star, J. R., & Rodriguez, M. C. (2016). The effects of schema-based instruction on the proportional thinking of students with mathematics difficulties with and without reading difficulties. *Journal of Learning Disabilities*, *49*(4), 354-367. https://doi.org/10.1177/0022219414554228

*Jitendra, A. K., Harwell, M. R., Dupuis, D. N., & Karl, S. R. (2017). A randomized trial of the effects of schema-based instruction on proportional problem-solving for students with mathematics problem-solving difficulties. *Journal of Learning Disabilities*, *50*(3), 322-336. https://doi.org/10.1177/0022219416629646

*Jitendra, A. K., Harwell, M. R., Im, S. H., Karl, S. R., & Slater, S. C. (2018). Using regression discontinuity to estimate the effects of a tier-1 research-based mathematics program in seventh grade. *Exceptional Children*, *85*(1), 46-65. https://doi.org/10.1177/0014402918784541

*Jitendra, A. K., Harwell, M. R., Karl, S. R., Simonson, G. R., & Slater, S. C. (2017). Investigating a Tier 1 intervention focused on proportional reasoning: A follow-up study. *Exceptional Children*, *83*(4), 340-358. https://doi.org/10.1177/0014402917691017

Jitendra, A. K., Lein, A. E., Im, S., Alghamdi, A. A., Hefte, S. B., & Mouanoutoua, J. (2018). Mathematical interventions for secondary students with learning disabilities and mathematics difficulties: A meta-analysis. *Exceptional Children*, *84*(2), 177–196. https://doi.org/10.1177/0014402917737467

*Jitendra, A. K., Star, J. R., Starosta, K., Leh, J. M., Sood, S., Caskie, G., ... & Mack, T. R. (2009). Improving seventh grade students' learning of ratio and proportion: The role of schema-based instruction. *Contemporary Educational Psychology*, *34*(3), 250-264. https://doi.org/10.1016/j.cedpsych.2009.06.001

Kratochwill, T. R., Hitchcock, J. H., Horner, R. H., Levin, J. R., Odom, S. L., Rindskopf, D. M., & Shadish, W. R. (2013). Single-case intervention research design standards. *Remedial and Special Education*, *34*(1), 26-38. https://doi.org/10.1177/0741932512452794

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 629–667). Reston, VA: NCTM.

McFarland, J., Hussar, B., Zhang, J., Wang, X., Wang, K., … Barmer, A. (2019). *The Condition of Education 2019* (NCES 2019-144). U.S. Department of Education. Washington, DC: National Center for Education Statistics.

Mazzocco, M., & Devlin, K. (2008). Parts and "holes": Gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Science, 11*(5), 681–691. https://doi.org/10.1111/j.1467-7687.2008.00717.x

Murphy, M. M., Mazzocco, M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities*, *40*(5), 458-478. https://doi.org/10.1177/00222194070400050901

National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common Core State Standards mathematics.* Washington, DC: Authors.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. US Department of Education.

National Research Council (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Nelson, G., & McMaster, K. L. (2019). The effects of early numeracy interventions for students in preschool and early elementary: A meta-analysis. *Journal of Educational Psychology,* 111, 1001–1022. doi:10.1037/edu0000334

Nunes, T., & Moreno, C. (2002). An intervention program for promoting deaf pupils' achievement in mathematics. *Journal of Deaf Studies and Deaf Education*, *7*(2), 120-133. https://doi.org/10.1093/deafed/7.2.120

Owen, R. L., & Fuchs, L. S. (2002). Mathematical problem-solving strategy instruction for third-grade students with learning disabilities. *Remedial and Special Education*, *23*(5), 268-278. https://doi.org/10.1177/07419325020230050201

Powell, S. R., Fuchs, L. S., Fuchs, D., Cirino, P. T., & Fletcher, J. M. (2009). Do word-problem features differentially affect problem difficulty as a function of students' mathematics difficulty with and without reading difficulty? *Journal of Learning Disabilities*, *42*(2), 99-110. https://doi.org/10.1177/0022219408326211

Parker, R. I., Vannest, K. J., Davis, J. L., & Sauber, S. B. (2011). Combining nonoverlap and trend for single-case research: Tau-U. *Behavior Therapy*, *42*(2), 284-299. https://doi.org/10.1016/j.beth.2010.08.006

Rohatgi, A. (2019). WebPlotDigitzer (Version 4.2). https://automeris.io/WebPlotDigitizer/

Sawrey, K. B. (2018). Upper elementary students' connecting and interpreting function representations. *Unpublished doctoral dissertation.*

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Wray, J. (2010). Developing effective fractions instruction for kindergarten through 8th grade: A practice guide. Washington, DC: Institute of Education Sciences, U.S. Department of Education.

Streefland, L. (1997). Charming fractions or fraction being charmed? In T. Nunes & P. Bryant (Eds.), Learning and Teaching Mathematics: An International Perspective. (pp. 347- 370). East Sussex, U.K.: Psychology Press.

Stevens, E. A., Rodgers, M. A., & Powell, S. R. (2018). Mathematics interventions for upper elementary and secondary students: A meta-analysis of research. *Remedial and Special Education*, *39*(6), 327-340. https://doi.org/10.1177/0741932517731887

Vannest, K.J., Parker, R.I., Gonen, O., & Adiguzel, T. (2016). Single Case Research: web based calculators for SCR analysis. (Version 2.0) [Web-based application]. College Station, TX: Texas A&M University. http://www.singlecaseresearch.org/calculators

Wanzek, J., & Vaughn, S. (2007). Research-based Implications from extensive early reading interventions. *School Psychology Review*, *36*(4), 541–561.

What Works Clearinghouse. (2017). *WWC Procedures Handbook Version 4.0.* Washington, DC: Author, Institute of Education Sciences, US Department of Education.

*Xin, Y. P. (2008). The effect of schema-based instruction in solving mathematics word problems: An emphasis on prealgebraic conceptualization of multiplicative relations. *Journal for Research in Mathematics Education*, *39*(5), 526-551.

*Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word Problem—Solving instruction on middle school students with learning problems. *The Journal of Special Education*, *39*(3), 181-192. https://doi.org/10.1177/00224669050390030501

Table 1

Summary of Single Case Design Studies and Effects

Study	*N*	Grade	Risk (*n*)	Criteria for LD or MD	Hours of Intervention[a]	Agent	Study Quality	Measure	Tau *U* (95% CI)
Brawand (2013)	9	7, 8	LD (*n* = 6)	IEP indicating LD and IEP goal in math; teacher referral for WPS difficulty	8.75 hrs	Researcher	1.55	Proportional WPS	0.88 (0.55; >1.00)
Hunt & Vasquez III (2014)	3	6, 7	LD (*n* = 3)	State criteria for disability and students had an IEP goal in math; teacher referral for difficulty with ratio strategies	6.25 hrs	Researcher	1.81	Missing equivalent ratio	1.00 (.74; >1.00)
Xin (2008)	4	5	LD (*n* = 1)	School criteria for disability; teacher referral for difficulty with WPS	6 hrs	Researcher	1.36	WPS	1.00 (0.53; > 1.00)
			MD (*n* = 3)	Teacher referral for difficulty with WPS					

Note. IEP = individualized education program; LD = learning disability; MD = math difficulty; WPS = word problem solving.

[a]Hours of Intervention = approximate hours of intervention based on length of session and number of sessions

Table 2

Summary of Group Design Studies and Effects

Study	*N*	Grade	Risk (*n*)	Criteria for LD or MD	Hours[b]	Agent	Quality Rating[c]	Measure	Hedges' *g* Immediate	Hedges' *g* Delayed
Jitendra et al. (2016)	260	7	MD (*n* = 108)	Scores ≤ 25th percentile on math subtest of the state test	24.2 hrs	Teacher	1.83	WPS	.44	.29
								Transfer	.15	
			MDRD (*n* = 152)	Scores ≤ 25th percentile on math and reading subtests of the state test				WPS	0.37	0.57
								Transfer	0.01	
Jitendra, Harwell, Dupuis, et al. (2017)	806	7	MD (*n* = 806)	Scores < 35th percentile on a general measure of math problem solving	22.5 hrs	Teacher	1.5	PPS	0.26	0.17
Jitendra et al. (2018)	1,120	7	MD[a] (*n* = 129)	Scores ≤ 35th on a general measure of mathematical problem solving (GMADE)	17.5 hrs	Teacher	1.75	PPS	0.53	0.40
								GMADE	0.36	
			MD[a] (*n* = 138)	Scores ≤ 35th on a researcher-developed measure of PPS				PPS	0.63	0.43
								GMADE	0.19	

Jitendra, Harwell, Karl, et al. (2017)	373	7	MD (*n* = 253)	Received a score of 1 or 2 (out of 5) on the 6th grade end of year state math test	22.5 hrs	Teacher	1.92	PPS GMADE	0.54 0.32	0.48
Jitendra et al. (2009)	148	7	MD (*n* = 28)	Participants in low achieving class according to previous years' test scores	6.7 hrs	Teacher	1.83	PST PSSA	-0.01 -.022	0.16
Xin et al. (2005)	22	6, 7, 8	LD (*n* = 18)	School disability criteria; teacher referral of students having problems with WPS; ≤ 70% on WPS pretest	12.0 hrs	Mix	1.58	WPS Generalized	1.87 1.33	2.69
			MD (*n* = 3)	Teacher referral of students having problems with WPS; ≤ 70% on WPS pretest						

Note. GMADE = Group Mathematics Assessment and Diagnostic Evaluation; IEP = individualized education program; LD = learning disability; MD = math difficulty; MDRD = mathematics and reading difficulty; Mix = mix of researcher and teacher; PPS = Proportional problem solving; PSSA = Pennsylvania System of School Achievement; PST = problem solving test; WPS = word problem solving.

[a]For the Jitendra et al. (2018) article, two groups of students with MD are represented. These groups are not mutually exclusive.

[a]Hours of Intervention = approximate hours of intervention based on length of session and number of sessions

Figure S1. PRISMA diagram outlining the literature search process.