Developing a Non-Cooperative Optimization Model for Water and Crop Area Allocation Based on Leader-Follower Game

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Publication Information
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Abstract

In this paper, a mathematical model for conflict resolution among a diverse set of agricultural water users in Golestan province, Iran, is developed. Given the bi-level nature of the distribution of power in the current problem, a combination of Leader–Follower game and Nash–Harsanyi bargaining solution method is employed to find optimal water and crop area allocations. The Golestan Regional Water Authority is the leader in this setting, controlling the total water allocations; and the agricultural sectors are the followers, competing over the allocated water. Two objectives for the leader are (i) maximizing profits, and (ii) maximizing share of green water in total agricultural production through selecting more efficient crop patterns. The followers’ objective is merely maximizing obtained benefits for the selected crop patterns. Virtual water concept is also factored into the related objective functions, and the water allocation problem is solved considering spatio-temporal crop pattern along with a dynamic water pricing system. This involves using a hybrid optimization structure as a new approach to solving two level optimization problems. The results show that the leader’s income is independent of total water allocation and is only affected by crop pattern and crop area, two factors which drive water price too. The followers’ benefit also depends on crop pattern and crop area, as they influence the crop yield, cost and water price. Finally, green water plays a key role in selecting the optimal crop pattern and crop area.

Keywords: Green water; Leader-followers Game; Agricultural water allocation; Agricultural benefit; Nash bargaining model; NSGA-II multi-objective optimization model
1. Introduction

Water shortage is a global problem, which is more pronounced in arid and semi-arid areas (Sadegh et al. 2010). While prolonged droughts, change in ratio of snow to rain, global warming, and increased number of dry days all played some role in this issue, population growth and economic development and subsequent rise in water demand aggravate the problem (AghaKouchak, 2015). Agricultural sector as the largest consumer of water around the globe endures highest socio-economical loss from water scarcity, manifested in the reduction of crop yield (Khanjari Sadati et al., 2014). Su et al. (2014) introduced “Virtual Water Trade” as an effective strategy to improve sustainable use of water resources, which can also be employed as a strong tool to effectively allocate water resources at regional scales. The concept of virtual water was originally introduced by Allan (1998) to account for the water consumed in food production, and recently any product in general, which is in turn traded in regional and global markets. The concept of virtual water and other related fields such as virtual water trade, virtual water flow and water footprint have been extensively studied in the literature (Yang and Zehnder, 2007; Liu and Savenije, 2008; Verma et al., 2009; Faramarzi et al., 2010; Velázquez et al., 2011; Konar et al., 2013; Chen and Chen, 2013; Zhang et al., 2014; Su et al., 2014; Zhang et al., 2016; Ababaei and Etedali, 2017; Srinivasan et al., 2017; Wang et al, 2017). Different methods have also been applied to optimize the water-food nexus (e.g. Faramarzi et al., 2010; Su et al., 2014; Zhang et al., 2016).

Increasing water consumption and withdrawal due to population and economic growth, as well as increasing awareness for environmental protection have led to intense competition over the already stressed water resources (Sadegh and Kerachian, 2011; Taher Kahil et al. 2015). This highlights the significant role of governmental entities and watershed managers as decision-makers on how to allocate water (C. Johansson et al., 2002; O. Orubu, 2006; Hanak and Lund, 2012; Farhadi et al. 2016; Hu et al. 2016). The relationships between different water users can be effectively defined within the framework of Non-Cooperative game theory models. In this type of games, the interactions between players (stakeholders) are based on their strategic goals (Carraro et al., 2007). When players make decisions in different levels (power layers), a specific non-cooperative game, namely “leader-follower” or “Stackelberg” game can be applied (Tharakunnel et al. 2009, Jorgensen et.al 2010, Safari et al. 2013, Kicsiny et al. 2014, Taher Kahil et al. 2015, Hu et al. 2016). The application of leader-follower game in the field of optimal water allocation was first considered by Barbier and Bhaduri (2003, 2008). Ever since, the leader-follower game has been used in water resources management literature. For example, Bhaduri and Liebe (2012) evaluated the scope and sustainability of cooperation between two countries with the common basin using a Stackelberg static model, and Safari et al. (2013) developed a model for optimal water allocation to various users.
based on a leader-follower game. In the latter study, Iran Water Resources Company was considered as the leader and three water consumers as followers. Being of a single-objective nature, they used genetic algorithm to solve the water allocation model. Another example is the conjunctive allocation of surface- and ground-water resources by Parsapour-Moghaddam et al. 2015, using a single-player game with non-cooperative behavior by consumers of surface- and ground-water resources.

Hu et al. (2016), by presenting a two-level optimization model, introduced the basin executives as upper-level and farmers as lower-level decision makers. They converted the multiple objectives of their study into a weighted single objective model and solved it by the weighted-sum method. Zhang et al. (2016) presented an optimization model based on the concept of virtual water to increase the productivity of agricultural water consumption for different scenarios. The objective function used in their research is to minimize the blue water consumption. The impact of green water, as well as the possibility of intervention by the government and watershed authorities in the water allocation process in the region are, however, not considered. In another study, Galan-Martin et al. (2016) developed a multi-objective optimization model (objectives including sustainable food production and environmental protection) and solved it by applying the epsilon constraint method, without any regard for the role of the state and the watershed administrators. Furthermore, researchers such as Chen et al. (2017) used leader-follower models in the field of water pollution. They presented a bi-level interaction model in which the environmental sector and water users are defined as the upper- and lower-level decision makers, respectively. By comparing this model with one-level models, they noted the significant performance of two-level models.

While significant strides have been made in this field, the impact of virtual water trade has not yet been considered, to the best of authors’ knowledge, in a leader-follower game framework to allocate water to consumers and resolve potential conflicts. Also most models in the literature are either single-objective or are converted into single objective form (weighted average of multiple objectives). In the model presented in this study, the leader has two objective functions, namely maximizing profits and maximizing share of green water in agricultural production, and the proposed method forms a Pareto front between the two objective functions. Strategic planning for employing virtual water to reduce crop water demands in joint groundwater–reservoir irrigation systems is also not fully explored, which we will address in this paper. Moreover, we define water price as a dynamic variable, which is vital to preventing a surge in cultivation area and water demand, and achieve self-sufficiency in crop production. Regional self-sufficiency is defined as a specific level of crop production that can supply the annual consumption for that crop in the region. The concept depends on population, crop yield and crop demand per capita.
In the current research, a two-level optimization model is developed with the presence of executive managers in top-level and the agricultural sectors in low-level as leader and followers, respectively. The leader's objective functions are (i) maximizing the profit gained from selling water to the followers, and (ii) maximizing ratio of “green water to total water” consumption through strategic planning and crop selection. The followers’ objective functions are to maximize their benefits through adopting different crop patterns. To prevent intensive increase in cultivation area for some crops and to ensure self-sufficiency in crop production, the model is designed to adapt water price dynamically in different sectors for each crop. For solving the proposed optimization model, a multi-objective genetic algorithm (NSGA-II) approach is combined with an internal GA optimization model that maximizes the benefit of the followers. In the following sections, the proposed methodology, results, discussions and conclusions are discussed.

2. Methodology

The purpose of this study is to optimize agricultural water allocation, while satisfying the goals of basin’s executive manager with superior power over the agricultural sectors in the decision making process. Two defined objectives for the basin manager are (i) maximizing the profit gained from selling water to agricultural sectors, and (ii) maximizing the ratio of “green water to total water” consumption through selecting more efficient crop patterns in different sectors (three sectors in this study). Both objectives are affected by the crop pattern and the crop cultivation area. In our model, 10 crops that maximize the leader’s objective functions out of 16 crops are chosen to be planted, which may vary in different sectors. Selection of these 10 crops must guarantee maximum benefit for each of the three sectors (followers). Furthermore, by maximizing the ratio of “green water to total water” consumption, crop per drop productivity would improve. It can lead to increasing cultivation area and consequently yielding higher profit.

Another parameter that has a key role in determining the objective values of the leader and followers is the price of water. In the proposed methodology, the price of water is considered as function of the cultivation area for each crop, such that minimum water price is associated with the cultivation area and pattern that assure self-sufficiency in each sector. The so-called ideal cultivation area in this study depends on population, crop yield and crop demand per capita in each sector. Farmers that choose to diverge from the ideal cultivation area, for any reason like crops price, yield, etc., are penalized by the leader through higher water price. Therefore, the water price for each crop is a function of cultivation area of that specific crop and can vary dynamically in different sectors for each crop.
Considering the multi-level nature of the problem at hand, with the basin administrator (leader) in a higher level than agricultural sectors (followers), it is logical to apply a non-cooperative leader-follower game to model the system. To resolve the related optimization problem, a combined genetic algorithm (GA) structure is applied. While an internal GA optimizes the objective of followers, a multi-objective genetic algorithm (MOGA) as an outer loop, optimizes the leader’s objectives.

In non-cooperative single leader-multi followers game, the followers decide simultaneously for their movement without any knowledge about each other’s strategies, and based solely on the leader’s total water allocation to the system and their perception of other players’ behaviour. The objective function of the followers is defined based on non-symmetric Nash-Harsanyi function (Harsanyi and selten, 1972), in which each follower’s function takes a power proportional to its influence in the bargaining process. In this study, it is assumed that the more population the sectors (followers) have, the more powerful they are in bargaining. Therefore, to calculate each sector’s power factor, its population is divided by the total population of all sectors.

The allocated water to each of the three sectors are the leader’s decision variables, while the follower’s decision variables include cultivation area coefficient for each crop, as well as crop patterns in the three sectors. There are 4 and 6 types of dominant crops for winter and summer, respectively, and hence the number of possible crop patterns among 16 suitable crops for each sector is equal to 2,940.

In each iteration, the internal GA structure (for the followers) randomly chooses three crop patterns among the 2,940 alternatives including 10 crops for each sector. Then it randomly selects 30 cultivation area coefficients as the followers’ decision variables. Therefore, the total number of followers’ decision variables is 33. It is also assumed that the cultivation area for a given crop is calculated by the multiplication of the total allocated water to each sector (the leader’s decision variables) by the crop’s cultivation area coefficient (Safari et al., 2014). By solving the model, a Pareto front curve with various solution points is formed. Fig. 1 shows the different steps for modelling the proposed methodology. The multi-objective optimization model is formulated as:

\[
\begin{align*}
    f(1) &= \text{Maximize } \sum_{i=1}^{3} \sum_{c=1}^{10} \left( X_i AC_{ic} r_{ic} \right) \\
    f(2) &= \text{Maximize } \frac{\sum_{i=1}^{3} \sum_{c=1}^{10} \left( X_i AC_{ic} y_{ic} VWC_{green-ic} \right)}{\sum_{i=1}^{3} \sum_{c=1}^{10} \left( X_i AC_{ic} y_{ic} VWC_{green-ic} \right) + \sum_{i=1}^{3} \sum_{c=1}^{10} \left( X_i AC_{ic} y_{ic} VWC_{blue-ic} \right)}
\end{align*}
\]
Where $X$ is allocated water to each sector (MCM), $AC$ is allocation coefficient and $r$ is water price for each crop at each sector ($), respectively. Also $y$ is crop yield (ton per hectare), $VWC_{\text{blue}}$ is blue virtual water content (m$^3$/kg) and $VWC_{\text{green}}$ is green virtual water content (m$^3$/kg), respectively. $i$ and $c$ are indices for sectors and crops, respectively. In equations (1) and (2), $f(1)$ is the benefit function ($) and $f(2)$ is the green water rate function. The objective functions are subjected to the following constraints:

\begin{align*}
    \text{Area}_i & \leq \text{Maximum Area}_i \quad (3) \\
    \sum_{i=1}^{3} X_i & \leq T.A.W \quad (4) \\
    f_{\text{followers}} & = \text{Maximize} \prod_{i=1}^{3} B_{ij}^{\omega_j} \quad (5) \\
    \text{Subject to:} \\
    \text{Area}_i & \leq \text{Maximum Area}_i \quad (6) \\
    \sum_{i=1}^{3} \omega_j & = 1 \quad (7)
\end{align*}

Where $\text{Area}$ is cultivation area (hectares), $T.A.W$ is total available water (MCM), $B_j$ is followers' benefit (million dollars) and $\omega$ denotes followers' power coefficient.

2.1. The NSGA-II Multiobjective Optimization Model

Non-Dominant Sorting Genetic Algorithm (NSGA-II) is a powerful optimization algorithm, proposed by Deb et al. (2002). This algorithm, which solves multi-objective optimization problems, has been widely used in the literature (Nikoo et al., 2011; Nikoo et al., 2012; Nikoo et al., 2014; Monghasemi et al., 2015; Alizadeh et al., 2017). Fig. S1 (Supplementary Information) explains in details the NSGA-II multi-objective optimization procedure.

2.2. The Leader – Follower Game

This method was introduced by Von-Stackelberg, as a non-cooperative game in 1934. The hierarchical nature of decision making in this game necessitates an equilibrium solution concept. In this game, the optimal move of the leader is subject to existing Nash equilibrium among followers. In other words, the leader is completely aware
about the followers’ payoff functions before making a decision and can determine the equilibrium in followers’
game. Similarly, for every leader’s decision, the followers are able to calculate their equilibrium reaction in order
to maximize their individual payoff function (Tharakunnel et al., 2009). The best move by the followers is
associated with the strategy that maximizes their payoff. How to make this best move depends on the leader’s
decision that is clear to all followers, and also the solution method that is assigned by the leader. The followers
can interact with one another based on non-cooperative, or hierarchical, behaviour (Safari et al., 2013). Since the
followers compete over limited resources, they all should make simultaneous decisions without any knowledge
about the others’ moves. In this condition, it is rational that the followers consider one of the bargaining solution
methods like the Nash-bargaining solution method. In this paper, a non-cooperative interaction is proposed among
the followers (agricultural sectors) with different powers in the bargaining process. Therefore, a non-symmetric
Nash-bargaining method can be used to define the followers’ objective functions. Interested readers are referred
to examples S1 and S2 (Supplementary Information) for more details.

Fig.1 Flowchart of the proposed non-cooperative optimization model for water and crop area allocation
based on leader-followers game

2.3. Non-symmetric Nash-bargaining Solution Method

The symmetry assumption as one of the Nash axioms was criticized by some researchers because different
players may not possess similar negotiating power (Matsumoto and Szidarovszky, 2016). This idea is the
foundation of the non-symmetric Nash-bargaining method (Harsanyi and selten, 1972). The mathematical
representation of this method describes the solution method by introducing a positive power vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, where $\omega_1 + \omega_2 + \ldots + \omega_n = 1$, as well as a unique solution function $\varphi(H, d)$, which is the unique solution for
the following optimization problem:

$$\text{Maximize } Z = (f_1 - d_1)^{\omega_1} \cdot (f_2 - d_2)^{\omega_2} \cdot \ldots \cdot (f_n - d_n)^{\omega_n}$$

(8)

Subject to:

$$f_i \geq d_i \quad (i = 1, 2, 3, \ldots, n)$$

(9)

$$f_1, f_2, \ldots, f_n \in H$$

(10)
in which $Z$ is the objective function for the Nash-based problem, $f$ is the payoff function for each follower, and $d$ is the disagreement payoff vector.

2.4. Water price function

As mentioned before, in this study water price is a function of crop’s cultivated area. It is assumed that the minimum water price for each crop is associated with the ideal cultivation area, in which self-sufficiency is met for each crop. This area is readily computed based on crop yield, crop per capita demand and population in each region. A second degree polynomial equation can model this function:

$$P_w = a \cdot Area^2 + b \cdot Area + c$$  (11)

Where $P_w$ is the water price, $Area$ is defined as cultivation area and $a$, $b$, $c$ are equation coefficients.

3. Case study

Performance of the proposed methodology is examined for a specific part of Golestan province in Iran, which includes an irrigation network named Narmab, supplying water to Minoo Dasht, Azad Shahr and Gonbad Kavoos cities with maximum cultivable area of 2,000, 7,000 and 10,000 hectares, respectively. The network water demand is supplied by Narmab reservoir (Fig. 2) and groundwater resources. The dam reservoir with a capacity of 115 MCM was constructed on Narmab River. Groundwater resources, consisting of 1,535 wells, 11 qanats, and 218 springs, also supply water to the agricultural sectors. The related aquifer characteristics such as average aquifer thickness and storage coefficient are 95 meters and 5%, respectively. The aquifer transmissivity varies between 20 to 2,000 $m^2$ per day, and average annual precipitation is about 500 mm delivered mostly from January to April, while average potential evapotranspiration is roughly 1000 mm (Golestan Regional Water Authority, 2010). The current prolonged drought condition has led to significant stress on the water resources in the region, with maximum extraction from surface and subsurface water resources being about 70 and 50 MCM, respectively. This research considers two planting seasons (i.e. winter; from November to April and summer; from June to October) for cultivation, as it is common in the study area with 16 possible crops (summer Rice, summer Cotton, Cucumber, Soybean, Potato, Tomato, Mung bean, Water Melon, Corn, Pea, Wheat, Onion, Barley, Spinach, Canola and Kidney bean).

Fig. 2 Location of the study area in Iran
Fig. 3 schematically presents the relationship between different players in the study area. Agricultural sectors as Economical Actor push Water Company Authority to access more water through Parliamentary representatives and other political powers in their region. However, supplying the environmental water demand is a very important issue for environmental sectors such as Department of Environment and related NGOs. Therefore, they try to force Water Company Authority not to allocate excess water to the agricultural sectors. Forming some negotiations between the political and environmental actors is both possible and pragmatic.

### 4. Results and discussion

The proposed methodology, as schematically presented in Fig. 1, starts with gathering data and determining parameters such as virtual water content (Green and Blue) of different crops in the region and the crop price functions. The ideal cultivation area, that satisfies self-sufficiency for each crop, is also calculated to then be used for water pricing. Subsequently, the developed NSGA-II multi-objective optimization model (NSGA-II MO) with an internal GA optimization structure is executed. The NSGA-II MO model optimizes the leader’s objective functions, while the internal GA optimizes the followers’ objective functions. NSGA-II MO has three decision variables namely agricultural water allocation to each of the three agricultural sectors. The population size and maximum number of generations in NSGA-II are set to 60 and 150, respectively. For internal GA, there exist 33 decision variables including 30 cultivation area coefficients (10 for each of the sectors) and 3 crop patterns (one for each sector). The GA model assigns 330 chromosomes to the population size and 300 to the maximum number of generations, with a two point crossover function with fraction value 0.8. Stopping criteria is defined based on TolFun parameter of 1e-6 for StallGenLimit parameter value 50. For the NSGA-II model, since the number of decision variables is small (3), it is expected that the number of solution points is small too. So the largest probable value (1) is assigned to the Pareto fraction parameter to get the maximum number of solution points. Using an Intel® Core™ i7 and CPU @ 2.4GHz processing system, the model’s run took about 72 hours. By running the model, a Pareto front that consists of 5 solution points is obtained (Fig. 4). Different solutions on the tradeoff curve include the optimal values for agricultural water allocation to each sector, crop cultivation area and optimal crop pattern in each sector. Also the agricultural benefit as a function of the cultivation area and the water price are calculated. It should be noted that the water price appears both in the leader’s objective function (maximization of the leader’s income) and the followers’ (maximization of the followers’ benefit).
4.1. Optimal agricultural water allocation

Leader’s decision variable is optimized through maximizing the leader’s income and share of green water in total agricultural water consumption, as the two leader’s objectives. Fig. 5 shows the values of agricultural water allocation to the different agricultural sectors for the 5 obtained solution points on the Pareto front. Alternative 3 is associated with the highest total agricultural water allocation, and the highest water allocation to sectors 1, 2 and 3 are related to alternatives 3, 5 and 4, respectively. This is due to the selected crop pattern for each sector.

Note that higher agricultural water allocations do not guarantee more profit for the leader (Table 1 and Fig. 5). For example, for alternatives 4 and 5 with agricultural water allocations of 100.11 and 102.72 MCM, respectively, the obtained incomes are 12.3 million dollars (alternative 4) versus 8.8 million dollars (alternative 5). This is because of the selected crop patterns and the calculated water price, which are assigned to each crop in different sectors based on its price function.

Table 1 Values of leader’s objective functions and total water allocation for different alternatives

Table 2 presents total crop water consumption (green + blue water) calculated for different alternatives associated with the values of maximum “green to total water” consumption ratio, agricultural water allocation and irrigation efficiency (0.5). Comparing the alternatives in terms of “green to total” water consumption, alternative 5 with greater volume of green water is ranked more favourably in comparison with alternative 3, although alternative 5 consuming more blue water. This stems from the crop patterns with different ability to extract soil water content (green water) for alternative 5 compared to alternative 3.
4.2. Optimal crop area

As mentioned earlier, one of the decision variables that is optimized during the optimization process is crop pattern, which can vary from one agricultural sector to another. Table 3 details the optimal crop patterns for alternatives 4 and 5 as the least and most water consuming alternatives, respectively. Each crop pattern consists of 6 summer and 4 winter crops (10 total), which are selected among 9 summer crops and 7 winter crops. In this table, Spinach as a winter crop is not considered in any of the crop patterns calculated for different sectors.

Table 3 Optimal crop pattern in different agricultural sectors for alternatives 4 and 5

In addition to crop pattern, the cultivation area for each crop also plays a significant role in water consumption in each crop pattern. Figs. 6 and 7 separately compare the total cultivation area for the two previously mentioned alternatives (4 and 5) for summer and winter crops, respectively.

Fig. 6 Cultivation area comparison for summer crops for two alternatives 4 and 5

Fig. 7 Cultivation area comparison for winter crops for two alternatives 4 and 5

As depicted in Fig. 6, summer crops, excluding watermelon, have greater or similar cultivation area in alternative 5 as compared to alternative 4. Also among winter crops (Fig. 7), wheat has a greater area for alternative 4 than alternative 5, but canola with the same monthly water consumption and longer cultivation period increases water demand (total water consumption, Table 2) for alternative 5. Note that crop area for canola in alternative 4 and kidney bean in alternative 5 are 56 and 0 hectares, respectively.

Also total cultivation area for alternatives 4 and 5 are 24,018 hectares and 26,442 hectares, respectively (Table 4). Hence, alternative 5 is expected to have a greater amount of water consumption in comparison with alternative 4. As mentioned earlier, maximum cultivable area in sectors 1, 2 and 3 are 2,000, 7,000 and 10,000 hectares, respectively.
4.3. Agricultural benefit

Maximizing agricultural benefit is the objective of each agricultural sector (followers). Since all followers make their decisions simultaneously, the Nash-Harsanyi bargaining method has been used to formulate their objective functions. Fig. 8 shows the agricultural benefit for different sectors and alternatives, with alternatives 3 and 4 yielding the most and the least benefits, respectively. Total crop area for alternatives 3 and 4 are 30,245 hectares and 24,018 hectares, respectively (Table 4). In addition to several factors such as crop yield, crop price, cultivation cost and water price, the total crop area is the main factor in the followers’ benefit.

Fig. 8 Agricultural benefit (10^6 $) for different sectors and alternatives

Fig. 8 also shows that comparing each sector, the benefit of alternative 3 is greater than that of alternative 4. For alternatives 1 and 5 with almost the same total cultivation area (Table 4), the difference in benefit is about 12 million dollars, which is attributed to the crop patterns for alternative 1 that have lower water consumption (Table 2), lower costs (cultivation area and water price), and finally greater cultivation area for the more expensive crops with greater yield (Table 5).

Table 5 Average agricultural benefit ($/ha) for alternatives 1 and 5

In this table, the parameters “Cost”, “Y.P” and “Water-price” are calculated as weighted average of the cultivation area for each crop. Also the parameter “Water allocation” (m³/ha) is calculated through dividing the agricultural water allocation (Table 2) by the total cultivation area (Table 4) for each alternative.

4.4. Agricultural water price

The agricultural water price for each crop can vary in different sectors for a specific alternative. The water price depends on the divergence from ideal cultivation area, “the area which satisfies the demand for that crop”, and the initial prices, which are set to $0.05, $0.0625 and $0.075 for sectors 1, 2 and 3, respectively. The parameters “ideal area” and “initial price” are used to determine the water price function for each crop. The
objective function also calculates the lowest water price associated with the ideal cultivation area for each crop.

As an example, the values of water price in different sectors for alternative 1 are compared to each other in Figs. 9 and 10, for the summer and winter crops, separately.

![Fig. 9 Water price for summer crops in different agricultural sectors based on the results of alternative 1](image)

![Fig. 10 Water price for winter crops in different agricultural sectors based on the results of alternative 1](image)

Among summer crops (Fig. 9), corn in sector 3 has the highest water price, because of higher divergence from the ideal cultivation area (greater ratio of cultivation area to the ideal cultivation area, as in Table 6). Fig. 10 depicts the water price for winter crops. Spinach in sector 3 and pea in sector 2 have the highest water prices among other crops. As illustrated in Table 7, the main reason for this behaviour is divergence from the ideal cultivation area for these crops. It is worth mentioning that the higher water prices lead to less benefit for the agricultural sectors, although this can lead to maximizing the leader’s profit as one of its objectives. Hence, it is rational that the model calculates some cultivation areas with a higher water price.

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Ratio of cultivation area to the ideal cultivation area (hectares) for summer crops based on the results of alternative 1</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Ratio of cultivation area to the ideal cultivation area (hectares) for winter crops based on the results of alternative 1</th>
</tr>
</thead>
</table>

These results are only comparable to that of Safari et al. (2014) to some extent, given the difference of adopted methodology. Safari et al. (2014) optimized water price for different users (domestic, industrial and agriculture) and crops using historical cultivation area for different crops. They, however, did not consider any water price function for the users and crops. In addition, they did not optimize the cultivation area. In our study, both water price function and crop area optimization have been considered. A key strategy to manage cultivation area to serve regional needs for each crop is to set water price as a function of the ratio of cultivation area to ideal cultivation area.
5. Conclusion

In this study, a non-cooperative game theory model was developed to optimize agricultural water allocation and crop cultivation area. Considering the bi-level decision making nature of the problem, a Leader – Follower game was applied with Iran Water Resources Management Company as the leader and agricultural sectors as the followers. Two defined objective functions for the leader are (i) maximizing profit gained from selling water to the agricultural sectors, and (ii) maximizing the ratio of “green water to total water” consumption through selecting the most efficient crop patterns. Since the followers’ behaviour is non-cooperative and they make their decisions simultaneously, their objective functions, i.e. maximizing benefits for the selected crop patterns, are formulated based on the Nash-Harsanyi bargaining solution method. The developed optimization model is solved by the multi-objective genetic algorithm (NSGA-II) approach linked with an internal GA optimization model that maximizes the benefit of the followers. The proposed methodology is applied to the Narmab irrigation network in Golestan province in Iran to examine the model’s performance. The results show that the leader’s profit is affected by crop pattern and crop area as two factors that also influence the agricultural water price. The alternative with the highest total water consumption is not identical to the one for which the highest blue water was allocated (Table 2), because the ratio of “green water to total water” consumption (i.e. leader’s second objective) affects the total water consumption. This ratio plays a significant role in selecting the optimal crop pattern and crop area, through which it also affects the followers’ benefits. Note that optimal crop pattern and crop area are functions of crop yield, cost and agricultural water price. Future studies can develop a stochastic model to consider uncertain parameters such as water availability and green water content (dependant on precipitation). Furthermore, an agent-based model could be developed to account for the role of interactions among agents in determining crop pattern and area allocation.


Golestan Regional Water Authority, 2010. The final report of updating the Golestan basin and water resource balance study.


Table 1: Values of leader’s objective functions and total water allocation for different alternatives

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Leader’s Income ($10^6$)</th>
<th>Green-water/total water consumption</th>
<th>Total water allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.9</td>
<td>0.361</td>
<td>97.67</td>
</tr>
<tr>
<td>2</td>
<td>11.7</td>
<td>0.363</td>
<td>98.05</td>
</tr>
<tr>
<td>3</td>
<td>10.2</td>
<td>0.371</td>
<td>107.12</td>
</tr>
<tr>
<td>4</td>
<td>12.3</td>
<td>0.340</td>
<td>100.11</td>
</tr>
<tr>
<td>5</td>
<td>8.8</td>
<td>0.411</td>
<td>102.72</td>
</tr>
</tbody>
</table>

Table 2: Total water consumption for different alternatives

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Obj. 2</th>
<th>Agricultural water allocation (MCM)</th>
<th>Net irrigation (blue) water (MCM)</th>
<th>Green water (MCM)</th>
<th>Total water consumption (MCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.361</td>
<td>97.67</td>
<td>48.83</td>
<td>27.59</td>
<td>76.42</td>
</tr>
<tr>
<td>2</td>
<td>0.363</td>
<td>98.05</td>
<td>49.02</td>
<td>27.94</td>
<td>76.96</td>
</tr>
<tr>
<td>3</td>
<td>0.371</td>
<td>107.12</td>
<td>53.56</td>
<td>31.59</td>
<td>85.15</td>
</tr>
<tr>
<td>4</td>
<td>0.340</td>
<td>100.11</td>
<td>50.06</td>
<td>25.79</td>
<td>75.84</td>
</tr>
<tr>
<td>5</td>
<td>0.411</td>
<td>102.72</td>
<td>51.36</td>
<td>35.84</td>
<td>87.20</td>
</tr>
</tbody>
</table>

*Obj. 2 is the maximum green water to total water consumption ratio

Table 3: Optimal crop pattern in different agricultural sectors for alternatives 4 and 5

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Crop</th>
<th>Summer Crops</th>
<th>Winter Crops</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>C₁</td>
<td>C₂</td>
</tr>
<tr>
<td>4</td>
<td>1,2,3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2,3</td>
<td>1,2</td>
<td>3</td>
</tr>
</tbody>
</table>


1: Minoo Dasht agricultural sector, 2: Azad Shahr agricultural sector, 3: Gonbad Kavoos agricultural sector
**Table 4** Cultivation area (hectares) for different agricultural sectors and alternatives

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Agricultural Sector 1</th>
<th>Agricultural Sector 2</th>
<th>Agricultural Sector 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>summer crops</td>
<td>winter crops</td>
<td>summer crops</td>
</tr>
<tr>
<td>1</td>
<td>1999</td>
<td>1696</td>
<td>4150</td>
</tr>
<tr>
<td>2</td>
<td>1990</td>
<td>1762</td>
<td>3449</td>
</tr>
<tr>
<td>3</td>
<td>1992</td>
<td>2000</td>
<td>4188</td>
</tr>
<tr>
<td>4</td>
<td>1610</td>
<td>1966</td>
<td>3338</td>
</tr>
<tr>
<td>5</td>
<td>1929</td>
<td>1864</td>
<td>3801</td>
</tr>
</tbody>
</table>

**Table 5** Average agricultural benefit ($/ha) for alternatives 1 and 5

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Cost*</th>
<th>Y.P*</th>
<th>Water-price</th>
<th>Water allocation (m$^3$)</th>
<th>Water-Cost</th>
<th>Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>273.72</td>
<td>3961.62</td>
<td>0.11</td>
<td>3735.39</td>
<td>419.30</td>
<td>3268.60</td>
</tr>
<tr>
<td>5</td>
<td>312.50</td>
<td>3387.08</td>
<td>0.08</td>
<td>3884.70</td>
<td>327.29</td>
<td>2747.29</td>
</tr>
</tbody>
</table>

*Y, P and Cost are yield (kg/ha), crop price ($/kg) and cultivation cost ($/ha), respectively

**Table 6** Ratio of cultivation area to the ideal cultivation area (hectares) for summer crops based on the results of alternative 1

<table>
<thead>
<tr>
<th>Agricultural Sector</th>
<th>Crop</th>
<th>R</th>
<th>C$_1$</th>
<th>C$_2$</th>
<th>S$_1$</th>
<th>P$_1$</th>
<th>T</th>
<th>M</th>
<th>W$_1$</th>
<th>C$_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.43</td>
<td>1.93</td>
<td>0</td>
<td>1.89</td>
<td>1.35</td>
<td>5.48</td>
<td>0</td>
<td>0</td>
<td>9.07</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.29</td>
<td>2.94</td>
<td>2.80</td>
<td>0</td>
<td>0.91</td>
<td>0</td>
<td>0</td>
<td>0.31</td>
<td>5.10</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.20</td>
<td>4.38</td>
<td>1.36</td>
<td>0</td>
<td>0</td>
<td>3.21</td>
<td>3.65</td>
<td>0</td>
<td>11.10</td>
</tr>
</tbody>
</table>

R: Rice, C$_1$: Cotton, C$_2$: Cucumber, S$_1$: Soybean, P$_1$: Potato, T: Tomato, M: Mung bean, W$_1$: Water Melon, C$_3$: Corn
### Table 7 Ratio of cultivation area to the ideal cultivation area (hectares) for winter crops based on the results of alternative 1

<table>
<thead>
<tr>
<th>Agricultural Sector</th>
<th>Crop</th>
<th>P₂</th>
<th>W₂</th>
<th>O</th>
<th>B</th>
<th>S₂</th>
<th>C₄</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>6.93</td>
<td>0.85</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>3.83</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>11.86</td>
<td>0.77</td>
<td>0</td>
<td>1.61</td>
<td>0</td>
<td>0.63</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0</td>
<td>0.67</td>
<td>0</td>
<td>0</td>
<td>6.30</td>
<td>0.16</td>
<td>1.95</td>
</tr>
</tbody>
</table>

Fig. 1 Flowchart of the proposed non-cooperative optimization model for water and crop area allocation based on leader-followers game.
Fig. 2 Location of the study area in Iran

Fig. 3 Interactions between different stakeholders (players) in the study area
Fig. 4 Pareto front solution points as a result of running multiobjective genetic algorithm

Fig. 5 Agricultural water allocation to the different agricultural sectors (MCM)
Fig. 6 Cultivation area comparison for summer crops for two alternatives 4 and 5

Fig. 7 Cultivation area comparison for winter crops for two alternatives 4 and 5
**Fig. 8** Agricultural benefit ($10^6$ $\text{\$} $) for different sectors and alternatives

**Fig. 9** Water price for summer crops in different agricultural sectors based on the results of alternative 1
Fig. 10 Water price for winter crops in different agricultural sectors based on the results of alternative 1