Optimal and Objective Placement of Sensors in Water Distribution Systems Using Information Theory

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Publication Information  

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Using Information Theory

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Optimal and Objective Placement of Sensors in Water Distribution Systems Using Information Theory

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Abstract

Optimization-based deployment of contamination warning system in water distribution systems has been widely used in the literature, due to their superior performance compared to rule- and opinion-based approaches. However, optimization techniques impose an excessive computational burden, which in turn is compensated for by shrinking the problem’s decision space and/or using faster optimization algorithms with less accuracy. This imposes subjectivity in interpretation of the system and associated risks, and undermines model’s accuracy by not exploring the entire feasible space. We propose a framework that uses information theoretic techniques, including value of information and transinformation entropy, for optimal sensor placement. This can be used either as

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pre-selection, i.e. pinpointing best potential locations of sensors to be in turn used in optimization framework, or ultimate selection, i.e. single-handedly selecting sensor locations from the feasible space. The proposed framework is then applied to Lamerd water distribution system, in Fars province, Iran, and the results are compared to the suggested potential locations of sensors in previous studies and results of TEVA-SPOT model. The proposed information theoretic scheme enhances the decision space, provides more accurate results, significantly reduces the computational burden, and warrants objective selection of sensor placement.

**Keywords**

Sensor placement; value of information; transinformation entropy; water distribution system; contamination warning system

**Highlights**

- Optimal placement of sensors in distribution systems ensures safety of drinking water
- Value of Information (VOI) can be used for accurate and robust placement of sensors
- VOI enhances the decision space and enables exploring the entire feasible space
- Transinformation Entropy (TE) minimizes redundant information from multiple sensors
- TE maximizes probability of detecting contamination events

1. **Introduction**
Delivering safe drinking water through water distribution systems (WDSs), although seemingly straightforward, is a challenging task. A WDS is a distributed and easy-to-access infrastructure, which could deliver contaminants along with potable water to a large population (Shafiee and Zechman, 2013). Water quality in WDSs can be easily compromised by accidental or intentional incidents (Berry et al. 2008; Hu et al. 2018; Janke et al. 2017), and severely damage public health. Such incidents can claim thousands of civilian lives, create horror in a community, harm public confidence in the water supply system, and potentially impair the economy of an entire nation due to post-incident expenses and long-term impacts. Literature corroborates the WDSs’ vulnerability to contaminant intrusion, and its catastrophic impacts (Gavriel et al., 1998; Yokoyama, 2007; Forest et al., 2013). For example in May 2000, contaminated drinking water in Walkerton, Ontario, Canada, severely affected 2,300 people and claimed 7 lives (Hrudey et al. 2003).

In the face of preventive actions being near impossible to protect an entire WDS, deploying online and real-time monitoring system of water quality has been regarded as the best alternative to mitigate risks of delivering contaminated water to consumers (Hart and Murray 2010; Rathi and Gupta 2014). This is also referred to as Contamination Warning System (CWS), and is a network of multiple quality sensors placed at different locations of a WDS with a centralized monitoring system that detects the time and location that safe drinking water is compromised. Due to expensive costs of purchase, installation and maintenance, placement of sensors at every node in WDS is not economically justified (Zeng et al., 2016). For example, PSA 10.255 analyzer is an online water quality sensor for measuring various elements.
such as Mercury, Selenium and Arsenic in potable water with 1 micro-grams/Liter accuracy, which costs between 3000$ to 5000$ (P. S. Analytical Co., 2018). Therefore, optimal deployment of CWS has been the focus of several studies (e.g. Berry et al. 2005a; Krause et al. 2006; Shastri and Diwekar, 2006; Berry et al. 2008; Ma et al. 2010; Afshar and Marino, 2012; Berry et al. 2012; Zhao et al., 2016; Mukherjee et al., 2017; Janke et al. 2017); in which, an optimization model is developed to find a layout of sensors with objective functions such as minimizing affected population, time to detection, and volume of contaminated water, and maximizing probability of detection.

Although optimization approach is proven to be superior to rule- and opinion-based approaches, one of the obstacles in its application for real-world large utility networks is computational burden. Due to expensive computational nature of optimization algorithms, usually a fixed set of nodes or a fixed number of sensors is selected to reduce the number of decision variables (e.g. Weickgenannt et al., 2010; Bazargan-Lari, 2014; Afshar and Khombi, 2015; Tinelli et al., 2017; Janke et al. 2017). Another approach is to use more computationally efficient optimization algorithms that search for a near optimal solution, at the expense of lower accuracy, objectivity and robustness (Berry et al. 2008; Berry et al. 2012; Janke et al. 2017). While most of the literature have focused on developing different optimization techniques to solve for subjectively selected decision variables, Diao and Rauch (2013) argued a more precise and objective method to pre-determine the potential locations of sensors in WDS should be adopted. In an effort to tackle this issue, they proposed an approach inspired by the concept of control theory for complex system analysis (Liu et al.,
2011). The goal of the proposed controllability analysis is to reduce the number of decision variables of the CWS deployment optimization problem by eliminating nodes with “redundant signals” from the set of potential locations, i.e. decision variables. Therefore, the retained nodes would provide unique signals. The advantage of this method lies in being merely dependent on the hydraulic of the WDS. However, the focus of their study was only to determine a minimum number of nodes by virtue of which any contamination event could be detected (full coverage). Fast detection of contamination or minimum health impact cannot be guaranteed by this method (Diao and Rauch, 2013). To account for these factors, possible contamination events should be simulated and a complementary optimization model should be used along with this method.

Due to the stochastic nature of how, when and where a contamination could be injected in WDS, simulation of possible contamination events seems an inevitable part of optimization approach to deploying CWS (Hart and Murray 2010). Therefore, this will lead to obtaining a series of probability distribution functions (pdfs) of time to detection, and affected population, among others, for any potential node for placement of sensors. Previous studies have focused on optimization of one or a few signatures of these pdfs as objective function. These signatures were usually the pdfs’ mean (e.g. Berry et al. 2003, 2005a, 2008 and 2012; Preis and Ostfeld, 2008; Bazargan-Lari, 2014; Janke et al. 2017), or the pdfs’ tale (e.g. Berry et al. 2008 and 2012; Janke et al. 2017; Naserizade et al. 2018). However, aggregating the pdf into a few signatures, which are in turn used in the optimization process, loses a lot of valuable information and might not be good representative of the entire system.
functionality. These pdfs can deliver paramount inferable information, and different nodes of a WDS provide diverse information regarding exposed population to contaminated water, and time interval between injection and detection of contaminant, among others. This information describes the behavior of the WDS and the consequences of a contaminant intrusion, and details a corresponding action by decision maker. Obviously, equipping every node of a WDS with sensors would provide maximum information about the WDS. However, the installation and maintenance costs of such comprehensive CWS is prohibitive. Moreover, information provided by different nodes may not be unique. Even similar information obtained from a given pair of nodes may have different value to the decision maker, which can be defined by a utility as a function of objective pdfs. Such perception would lead to definition of a WDS as a finite set of probabilistic curves, and hence, provide the ability to compare the locations for placement of sensors in a more objective way. Consequently, choosing an optimal subset from the original space not only warrants objectivity but also would hugely reduce the runtime and memory requirements of the optimization model. This allows for searching for global optimum in a complete and comprehensive decision space, as opposed to the traditional sensor placement studies that either had shrunk the decision space or employed more-efficient but less-accurate optimization algorithms to tackle the computational burden (Berry et al. 2008; Berry et al. 2012; Janke et al. 2017).

We adopt an information theory-oriented approach to determine potential and/or optimal locations for CWS in WDS. In doing so, we use the concept of Value of Information (VOI) to determine a set of nodes which can provide maximum
information values, and can be used as closest representatives of other nodes. Grayson (1960) developed the VOI technique to assess the value of received information for decision-making. Ever since, the VOI technique has found a wide range of applications in water science, including groundwater quality assessment (Wagner et al., 1992; Shaqadan, 2008), designing groundwater quality monitoring systems (Reichard and Evans, 1989; Ammar et al., 2009; Khader et al., 2012; Hosseini and Kerachian, 2017), designing CWS in agricultural systems (Roberts et al., 2009), and flood monitoring and impact assessment (Alfonso, 2010; Verkade and Werner, 2011; Alfonso and Price, 2012; and Alfonso et al., 2016). The literature has used VOI for time-series analysis, while in this study we use the technique for event-based analysis.

Different sets of selected nodes with similar count (but with different locations) may yield a comparable VOI, but with different level of information redundancy. Following a practical assumption in the literature, this study is based on using sensors which are 100% reliable above a certain concentration (e.g. Janke et al. 2017; Naserizade et al. 2018). This assumption warrants avoiding information redundancy. Therefore, Transinformation Entropy (TE) is used to minimize the mutual information of selected nodes. Also, the level of TE for any given pair of nodes depends on the spatial distance of the nodes. Hence, minimizing TE would also result in obtaining more spatially distributed CWS which yields a higher probability of detecting contamination across the WDS. Shannon (1948) introduced the Entropy theory to quantify the information content of a data set (Harmancioglu., 1981). TE is a special form of entropy that quantifies mutual information of two data sets, and has
been applied in many monitoring network design problems, including reservoir quality monitoring stations (Lee et al., 2014; Nikoo et al., 2017), groundwater quality monitoring stations (Caselton and Husain, 1980; Mogheir and Singh, 2002; Mogheir et al., 2004a, 2004b, 2005 and 2009; Masoumi and Kerachian, 2008 and 2010; and Mondal and Singh 2012), and river quality monitoring stations (Harmancioglu and Yevjevich, 1987; Ozkul et al., 2000; Karamouz et al., 2006 and 2009; Salark and Sorman, 2006; Mahjouri and Kerachian, 2011; and Memarzadeh et al., 2013).

In this study, we use the VOI and TE techniques to quantify the value and uniqueness of information of different nodes in WDS. Then, a multi-objective optimization model, namely NSGA-II (Deb et al., 2000 and 2002), is formulated with a potential to be used as a pre-selection or ultimate selection method for design of a CWS in WDS. The objectives of the proposed model are, 1) maximizing VOI, 2) minimizing TE, and 3) minimizing number of selected sensors. This method is applied for design of CWS in Lamerd City’s WDS which is a large scale WDS and previously used in other studies (Bazargan-Lari, 2014; Naserizade et al., 2018). Finally, results are compared to the potential locations for placement of sensors in previous studies and optimal CWS designs from TEVA-SPOT model (Janke et al. 2017) to assess the proposed method’s capabilities.

2. Methodology

2.1. Value of Information (VOI)

Grayson (1960) first introduced the concept of VOI, and Hirshleifer and Riley (1979) presented it for monitoring network design (Alfonso and Price, 2012). A decision maker can update their perception about the state of a system, which can be quantified
in discrete form as the vector of prior probability, $P(s)$, of having a particular state $s$. When new information comes to light, based on Bayes’ theorem, the updated belief can be represented as,

$$P(s|m) = \frac{P(m|s) P(s)}{P(m)},$$  \hspace{1cm} (1)$$

where, $P(s|m)$ is the updated belief following the receipt of message $m$; $P(m|s)$ is the conditional probability of receiving message $m$ when the state of the system is $s$; and $P(m)$ is the probability of receiving message $m$.

If one assumes that the time interval between injection and contaminant detection at a given node is the state, $s$, of that node (which we call “detection state” hereafter), vector $P(s)$ is the belief of the WDS’s utility about the possibility of contamination being detectable at any given node in each detection state, $s$. This belief may be determined roughly by utility’s experience or more precisely by simulation of random scenarios. The message $m$ can be inferred as the received contamination warning after passage of $m$ time units from injection of contamination from a sensor placed at any node. Once $P(s)$ is determined, before calculation of updated belief, $P(s|m)$, one has to calculate evidence probabilities, $P(m)$, and evidence conditional probabilities, $P(m|s)$, by gathering new information independent from prior belief.

To calculate $P(m)$ for any given node, we assume a sensor is placed at that node and probabilities of receiving contamination warning in $m$ time unit from injection is calculated by the new information. Then, the selected node should be paired with another node and conditional probabilities of receiving contamination warning from the selected node and detection states of the other node should be calculated to obtain
\(P(m|s)\). In this study, the results of simulation of random scenarios are used for calculation of both prior and evidence probabilities. Since the scenarios are generated randomly, they are independent from Bayesian viewpoint (Alfonso and Price, 2012 and Alfonso et al. 2016).

Presume a utility manager uses a sensor placed at node \(i\) to warn consumers of node \(j\) about water contamination. Timely warnings would preclude major consequences, while early or late warnings would result in panic or exposure of consumers, respectively. Hence, an action (warning), \(a\), with various lag times and its associated consequences, \(C(a,s)\), when the detection state of node \(j\) is \(s\), can be defined. The utility of those actions based on received message, \(u_m\), and without the received message, \(u_s\), can be calculated as,

\[
u_m = \sum_s C(a,s) P(s|m), \quad (2.a)
\]
\[
u_s = \sum_s C(a,s) P(s), \quad (2.b)
\]

where, \(S\), is the total number of detection states, \(s\). Therefore, the value of a chosen action \(a_m\) based on a received message, \(m\), would be \(u_m - u_s\). A rational decision maker will choose the action with maximum utility, therefore, the VOI of node \(i\) for determining the detection state of node \(j\), \((VOI_i(j))\) can be calculated by,

\[
VOI_i(j) = \sum_M P(m) \left[ \max_a \left( \sum_s C(a,s) P(s|m) \right) - \max_a \left( \sum_s C(a,s) P(s) \right) \right], \quad (3)
\]

in which, \(M\) is number of messages. For a given node \(i\), VOI can be calculated for all \(j\) nodes, which will result in a \(VOI_i\) curve. The area below the \(VOI_i\) curve is equal to
the VOI of node $i$ for detecting the states of the entire system. Obviously, the VOI of
node $i$ is always maximum for determining the detection state of node $i$. When more
than one node is selected to determine the detection state of node $j$, $VOI_{ij}(j)$ would
be maximum of $VOI_{i1}(j)$ and $VOI_{i2}(j)$; i.e. the VOI of set of nodes which is selected
for placement of sensors would be union of VOI curves of the selected nodes. Hence,
the VOI of all nodes of a WDS is maximum, and can be used as a benchmark to
assess the behavior of any set of selected nodes for placement of sensors. A numerical
example is provided in Supplementary Material which explains the steps involved in
calculation of $VOI_i(j)$.

2.2. Transinformation Entropy (TE)

In discrete form, the mutual information which also called transinformation, $TE(i, j)$,
of two selected nodes ($i$ and $j$) for placement of sensors can be calculated as (Mogheir
et al., 2004a, 2004b),

$$TE(i, j) = -\sum_s \sum_{ss} P(i_s, j_{ss}) \ln \left[ \frac{P(i_s, j_{ss})}{P(i_s)P(j_{ss})} \right],$$  (4)

where, $P(i_s, j_{ss})$ is the joint probability of having detection state $s$ at node $i$, while
the detection state of node $j$ is $ss$; and $P(i_s)$ and $P(j_{ss})$ are probabilities of having
detection states $s$ and $ss$ at nodes $i$ and $j$, respectively. Similar to VOI, TE of a set of
nodes is union of their TE curves. Also, the area below the TE curve of a set of nodes
is dependent on the spatial distribution of those nodes. Hence, minimizing TE of the
selected nodes set for placement of sensors would warrant uniqueness of information.
of nodes, and consequently selection of nodes with the highest spatial distribution in WDS. Higher spatial distribution of CWS warrants greater possibility of detecting contamination.

2.3. **Optimization model**

Both VOI and TE can be calculated for all pair of nodes in a WDS. This will result in two square matrices, where the elements in $i^{\text{th}}$ row and $j^{\text{th}}$ column are equal to $VOI_i(j)$ and $TE(i,j)$, respectively. Therefore, a multi-objective optimization model with a vector of binary decision variables ($\bar{b}$) can be formulated (as in eqs.5) to find the minimum number of nodes ($Z_3$, eq. 5.c) with maximum VOI ($Z_1$) and minimum TE ($Z_2$). The VOI and TE matrices are normalized by their maximum values, respectively (eqs. 5.a and 5.b). By including the third objective, it is implicitly assumed that the costs associated with placement of sensors have a monotonic relationship with number of placed sensors in WDS.

$$\text{maximize } Z_1 = \frac{1}{\max([VOI])} \sum_{ij} \max_i \{b_i \times VOI_i(j)\}, \quad (5.a)$$

$$\text{minimize } Z_2 = \frac{1}{\max([TE])} \sum_{ij \neq i} \max_i \{b_i \times b_j \times TE(i,j)\}, \quad (5.b)$$

$$\text{minimize } Z_3 = \sum_{i} b_i. \quad (5.c)$$

3. **Case Study**
The proposed methodology is applied for designing CWS in WDS of Lamerd City (Fig.1), Fars province, Iran. This WDS consists of 185 links (pipes), 122 junctions, 23 hydrants, 2 reservoirs and a tank, and supplies water demands of about 81,000 people. The base demand of Lamerd’s population is approximately 260 LCD (liters per capita per day), and the daily demand pattern is depicted in Fig.2.

Fig. 1. Water Distribution System (WDS) of Lamerd City. The locations of potential contamination injection: hydrants, reservoirs and tank are marked with symbols as defined in legend.
Arsenic is used as the contaminant of choice in this study as commonly used in the literature (e.g. Shafiee and Zechman, 2013; Bazargan-Lari, 2014; Naserizade et al. 2018). Arsenic is a cheap, accessible and well-known poisonous substance, which could be deadly at very low dosages. The critical dose, $C_d$ (milligrams), of this substance depends on the weight, $W_p$ (kg), of the exposed person and can be calculated as follows (Shafiee and Zechman, 2013),

$$C_d = 5.0 \times 10^{-8} \times W_p.$$  \hspace{1cm} (6)

A person with an average 70 kg weight could be critically affected by ingesting 3.5 mg of Arsenic. Throughout this study, the term “affected population” refers to the population who ingested equal to or greater than 3.5 mg Arsenic due to consuming contaminated water. Furthermore, it is assumed that ingesting contaminant could only occur through drinking, and each person drinks 0.93 L/day (Shafiee and Zechman, 2013).
A contamination event is associated with various uncertainties. Monte Carlo Simulation (MCS) is used to incorporate the uncertainties emanating from contaminant intrusion. For this purpose, mass and duration of injection are considered as stochastic variables; and time of injecting, location of injection and number of simultaneous injections are considered as scenario-based inputs in MCS. Then, all combinations of these parameters (Table 1) are generated and used as contamination injection scenarios. It is worth mentioning that 14 hydrants, of the available 23, together with the tank and two reservoirs are selected as the potential locations for contaminant intrusion. Due to hydraulic of the WDS, contamination injection from the other 9 hydrants has very low impact on the affected population. Therefore, these hydrants are not considered in contamination scenarios (Naserizade et al., 2018). The injection scenarios are simulated using EPANET (Rossman, 2000) model of Lamerd WDS, which was previously calibrated by Bazargan-Lari (2014). The simulation period is 48 hours and both hydraulic and quality time-steps are 60 seconds. Since Arsenic cannot react with materials on the pipe wall in short time, only bulk flow reaction is considered in the quality simulation (−0.05 day⁻¹). The results of the simulated scenarios are then used to calculate the time to detection at each node, the ingested mass of Arsenic, and consequently the affected population. It is assumed that the detection limit of available sensors is 0.01 (mg/L) (Naserizade et al. 2018). Assumption is that sensors are 100% reliable in detecting contamination with concentration above the detection limit, and will fail for concentrations below this threshold. These results are used to derive the VOI matrix for all pairs of nodes.
**Table 1.** Characteristics of scenarios considered in MCS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of injection</td>
<td>0100AM, 0700AM, 0900AM, 1100AM, 1200PM, 0100PM, 0200PM, 0300PM, 0400PM, 0500PM, 0600PM, 0700PM, and 0900PM</td>
</tr>
<tr>
<td>Mass of injection</td>
<td>200 mg/sec to 700 mg/sec</td>
</tr>
<tr>
<td>Duration of injection</td>
<td>40 min to 80 min</td>
</tr>
<tr>
<td>Locations of injection</td>
<td>17 points: 14 hydrants, 2 reservoirs and a tank (Fig.1)</td>
</tr>
<tr>
<td>Number of injections</td>
<td>Simultaneously from 1, 2 and 3 points</td>
</tr>
<tr>
<td>Total number of scenarios</td>
<td>216436 scenarios</td>
</tr>
</tbody>
</table>

In order to show the method’s capability, the obtained solutions are compared to the sets of potential nodes for placement of sensors from Bazargan-Lari (2014) and Naserizade et al. (2018) studies, as well as the results of TEVA-SPOT model (Janke et al. 2017). Similar to this study, the researchers use offline simulation of contamination scenarios. However, deriving the objective functions from a large set of simulation results requires huge amount of logical operations, and consume a large volume of memory, especially in an optimization process. Therefore, Bazargan-Lari (2014) and Naserizade et al. (2018) have determined a set of potential locations for placement of sensors, through sensitivity analysis, to reduce the number of decision variables. On the other hand, TEVA-SPOT model requires the user to specify the number of sensors to be placed in WDS. Also, its Graphical User Interface version, TEVA-SPOT GUI, uses a Greedy Randomized Adaptive Search Procedure (GRASP) optimization algorithm to provide near-optimal solutions with lower memory requirements and in quickest possible way. The offline calculation of VOI and TE, however, summarizes all scenarios in two matrices; hence, the decision space could be significantly enhanced. Moreover, VOI and TE approaches hugely reduce the memory requirements and provide faster runtimes, and hence, more accurate
optimization algorithm (NSGA-II — a multi-objective version of GA) is used in our framework. Ultimately, we compare our solutions with the same number of nodes to that of the decision space (potential sensors’ locations) of Bazargan-Lari (2014) and Naserizade et al. (2018) and the results of the TEVA-SPOT model.

4. Results and Discussion

A total of seven detection states are considered to derive VOI for each node (Table 2). These states are considered based on the criterion of fast detection of contamination, given a comprehensive pre-analysis of simulated scenarios (Fig.3). When an intrusion occurs, contaminated water could not necessarily reach all nodes of the WDS due to its hydraulic characteristics. From the set of nodes which contaminated water can reach, almost none is affected by contaminated water in the first 5 minutes (average affected population $\approx 0.5\%$). In the next 10 minutes, the affected population increases smoothly ($\approx 2\%$). During the first 60 minutes from injection, the contamination reaches majority of the possible nodes, however, with relatively low concentration which affect only about 6% of the population. In the next 240 minutes the affected population rapidly increases ($\approx 96\%$) due to accumulation of contamination in the consumers body and relatively high concentration of contamination.
Table 2. Detection states of each node in contaminant intrusion events.

<table>
<thead>
<tr>
<th>Detection state</th>
<th>Time interval between injection and detection of contaminant (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 to 5</td>
</tr>
<tr>
<td>2</td>
<td>5 to 15</td>
</tr>
<tr>
<td>3</td>
<td>15 to 30</td>
</tr>
<tr>
<td>4</td>
<td>30 to 60</td>
</tr>
<tr>
<td>5</td>
<td>60 to 120</td>
</tr>
<tr>
<td>6</td>
<td>120 to 300</td>
</tr>
<tr>
<td>7</td>
<td>More than 300 min</td>
</tr>
</tbody>
</table>

Fig. 3. Temporal change of affected population from the time of injection at each node in all simulated scenarios. The nodes that remain immune from contaminated water in each event are removed from the calculation.

Calculation of VOI requires determining the cost matrix, $C(a, s)$, whose elements are the cost of releasing “no consuming” warning after receipt of message $m$ from $i$ node, when the detection state at node $j$ is $s$. For example, assume in an intrusion event, contaminated water reaches nodes $i$ and $j$ in the 4th and 2nd detection state,
respectively, and a sensor is placed at node $i$ while there is no sensor at node $j$. A utility manager warns the consumers at node $j$ following the receipt of contamination warning from the sensor $i$, which is late, because consumers were already drinking contaminated water for about 45 minutes. The average affected population of node $j$ in the 45 minutes interval is the cost of action $a_4$ when the state is $s_2$, i.e. $C(a_4, s_2)$. On the other hand, early warning would result in a widespread panic among the population of node $j$. Although a more specific approach for cost assessment of panic among consumers could be considered, the costs of early warnings are considered equal to those of late warnings in this study. Using these assumptions, Fig.3 is used to determine the mean affected population of nodes in each detection state. The difference between the average fraction (%) of affected population multiplied by mean population of nodes ($\approx 663$ persons) for each pair of time intervals (the average of affected population at the beginning and at the end of the interval) with negative sign (Table 3) is set as $C(a, s)$.

**Table 3.** The cost of actions, $C(a, s)$, in each detection state for the calculation of VOI.

<table>
<thead>
<tr>
<th>Cost of actions, $C(a, s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>$s_1$</td>
</tr>
<tr>
<td>$s_2$</td>
</tr>
<tr>
<td>$s_3$</td>
</tr>
<tr>
<td>$s_4$</td>
</tr>
<tr>
<td>$s_5$</td>
</tr>
<tr>
<td>$s_6$</td>
</tr>
<tr>
<td>$s_7$</td>
</tr>
</tbody>
</table>
From over 216,000 simulated contamination scenarios, about 80% are used for calculation of prior probabilities, and the remaining 20% are used for calculation of evidence probabilities. The VOI and TE are then calculated for every pair of nodes. The normalized VOI and TE of all 122 nodes are equal to 74.24 and 68.82, respectively. For example, the normalized VOI and TE for nodes 1 and 45 are shown in Fig. 4. It is vivid in this figure that both VOI and TE are reliant on spatial distance. Although, TE quantifies the mutual information of a pair of nodes, their information content may have different value to a decision maker (the neighbors of node 1 in Figs. 4a and c). As mentioned earlier, these values mainly include time to detection and affected population in CWS design.
Figs. 4. The values of normalized, (a) $VOI_1(j)$, (b) $VOI_{45}(j)$, (c) $TE(1,j)$, and (d) $TE(45,j)$, which are shown by blue circles and $\forall j$.

After calculation of the VOI and TE matrices, the optimization model is executed.

The obtained pareto-optimal solutions contain 335 CWS layouts with 5 to 114 nodes.
for placement of sensors. Also, the solutions include 4 sets with 113 and 114 nodes with VOI equal to 74.24, albeit their TE are equal to 65.16 and 65.09, respectively. This implies that these sets of nodes are capable of almost perfectly representing the entire WDS nodes with respect to the considered states and costs. A randomly selected set of pareto-optimal solutions and their performance metrics including probability of detecting a contamination event ($P_d$), minimum, maximum and average time to detection ($T_{d_{min}}, T_{d_{max}}$ and $T_{d_{ave}}$, respectively), as well as average affected population ($P_{a_{ave}}$) are provided in Table 4.

**Table 4.** A randomly selected set of pareto-optimal solutions with their respective values of objective functions, probability of detection ($P_d$), minimum, maximum and average time to detection ($T_{d_{min}}, T_{d_{max}}$, and $T_{d_{ave}}$, respectively ), and average affected population ($P_{a_{ave}}$).

<table>
<thead>
<tr>
<th># Pareto point</th>
<th>Number of sensors</th>
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<th>$T_{d_{ave}}$ (min)</th>
<th>$T_{d_{min}}$ (min)</th>
<th>$T_{d_{max}}$ (min)</th>
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<td>2886.67</td>
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The selected sets of potential locations for placement of sensors by Bazargan-Lari (2014) and Naserizade et al. (2018) contain 20 (SN20) and 19 (SN19) nodes, respectively (Fig. 5). As shown in Table 4, three of the pareto-optimal solutions also have the same number of sensors. These solutions are pareto-point number 273 (P120), 274 (P220), and 275 (P119). Although, 19 and 20 sensors seem too many for the case study, these solutions are only presented for comparison purposes. However, the obtained solutions are satisfactory to prove the need to use more efficient approaches compared to the traditional methods to enhance the decision space of a CWS optimization problem. The VOI, TE, probability of detection, minimum, maximum and average time to detection, and average affected population for those sets of nodes are calculated using the simulated scenarios (Table 5).

<table>
<thead>
<tr>
<th>Set of nodes</th>
<th>Number of sensors</th>
<th>VOI</th>
<th>TE</th>
<th>Pd</th>
<th>Td_{ave} (min)</th>
<th>Td_{min} (min)</th>
<th>Td_{max} (min)</th>
<th>Pa_{ave} (persons)</th>
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<td>8.80</td>
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<td>19</td>
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<td>0.9923</td>
<td>43.32</td>
<td>3</td>
<td>179</td>
<td>770.14</td>
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<tr>
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<td>67.31</td>
<td>7.01</td>
<td>0.9963</td>
<td>13.87</td>
<td>2</td>
<td>201</td>
<td>388.66</td>
</tr>
<tr>
<td>P220</td>
<td>20</td>
<td>67.27</td>
<td>6.85</td>
<td>1</td>
<td>15.64</td>
<td>2</td>
<td>201</td>
<td>54.29</td>
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<tr>
<td>P119</td>
<td>19</td>
<td>66.95</td>
<td>6.23</td>
<td>0.9963</td>
<td>15.64</td>
<td>2</td>
<td>201</td>
<td>388.66</td>
</tr>
</tbody>
</table>
As shown in Fig. 5, 18 nodes are common among the pareto solutions. However, there is only one common node among the all five sets of nodes (one complete circle in this figure). For more clarity, the probability distribution of detecting a contamination event during the first 60 minutes from the injection in 2 minutes intervals are depicted in Fig.6 for these five sets of nodes. While P119, P120 and P220 provide a probability of ~0.35 for contamination detection under 2 minutes, this probability is about 0.08 and 0.00 for the SN19 and SN20, respectively. The probability of detecting contamination under 60 minutes are 0.976, 0.971, and 0.982
for SN20, P120, and P220, respectively, while it is 0.949, and 0.971 for SN19, and P119, respectively.

Figs. 6. Comparing the probability distribution of time to detection for, (a) SN20, P120, and P220, and (b) SN19, and P119.

To provide a deeper understanding about the coverage of the sets of nodes, the VOI of all sets of nodes are divided by the VOI of the 122 nodes of the WDS and the relative VOIs are visualized in Fig. 7. The only difference between P120 and P220 is the location of two nodes, and the remaining 18 nodes have similar locations. Only P220 is plotted in this figure.
Figs. 7. The VOI of, (a) SN20, and (b) P220, with 20 nodes; and (c) SN19, and (d) P119, with 19 nodes, relative to VOI of the all nodes.

Obviously, the economic factor (i.e. budget limitation) is the main constraint in deployment of CWS in WDS. Berry et al. (2005b) pointed out this obstacle and argued that even different locations for placement of sensors would have different cost. However, using the third objective function in the proposed model in this study,
we implicitly assumed that placing sensors at any point would have equal costs. Also, the proposed model could be modified by replacing the third objective function with a constraint on the number of selected nodes to find more CWS layouts which are economically justifiable. To evaluate the performance of the proposed model against that of TEVA-SPOT, we have considered CWS designs with fewer number of sensors (i.e. CWS designs with 3, 4, 5, 6, 7, 8 and 9 sensors).

We will now present a brief comparison between the results of the proposed VT model and TEVA-SPOT for the Lamerd WDS. An extensive report on the performance comparison of the VT model against TEVA-SPOT from computational efficiency and results’ accuracy viewpoints is provided in Supplementary Materials. Both models were executed on a desktop PC (CPU: Intel® Core™ i7-4500U; RAM: 12GB DDR3). Also, the parameters of both models’ hydraulic engine (i.e. EPANET v2) have been set up identically. Simulating the 216,000 scenarios by TEVA-SPOT is, however, not possible on a desktop PC. According to Janke et al. (2017), when the size of WDS (i.e. number of nodes) and/or number of simulation scenarios are very large, execution of TEVA-SPOT would not be possible on a typical PC. Hence, only about 12,000 scenarios (about 5% of the scenarios used earlier) were simulated by both models for CWS design. Single-node injection and simultaneous injection from two and three nodes are considered in contamination injection scenarios. The details of the simulated scenarios are provided in Table S3 of Supplementary Material. Both models were constrained to provide at least 80% probability of detection of contamination events (i.e. \( P_d \geq 0.8 \)).
Since TEVA-SPOT uses a single-objective optimization scheme, it should be executed for every objective separately, providing a single solution each time. To evaluate the robustness and accuracy of the solutions of VT model against TEVA-SPOT, two objectives were defined for TEVA-SPOT; i.e. minimization of the Value-at-Risk (VaR) of time to detection \( (T_{d_{VaR}}) \) and minimization of average of time to detection \( (T_{d_{ave}}) \). VaR is the point on a pdf where cumulative probability of the pdf exceeds a certain level (Sarykalin et al., 2008). Hence, for each number of sensors, there would be two solutions from the TEVA-SPOT, one for \( T_{d_{ave}} \) and one for \( T_{d_{VaR}} \) which are denoted by TSM and TSV, respectively. On the other hand, the multi-objective optimization module of VT model was executed only once for each number of sensors, which in turn provided a pareto front. Each solution on the paeto front is denoted by VT followed by a number. Only one selected solution from VT model is provided here (Table 6) for each CWS design, and the complete pareto front of VT model is provided in Table S5 in Supplementary Material.

The results show that VT model is overall significantly more efficient than TEVA-SPOT. While, the simulation module of VT is 23% slower than that of TEVA-SPOT, its VOI+TE and optimization modules are 350% and 177% faster than impact assessment and optimization modules of TEVA-SPOT, respectively. Also, TEVA-SPOT occupied 5 Giga Bytes (5120 Mega Bytes) of disk space, while VT only occupied 244 Mega Bytes.

**Table 6.** The results of the TEVA-SPOT and VT models for design of CWS with 3, 4, 5, 6, 7, 8 and 9 sensors in Lamerd WDS. The bold items indicate superiority of the solution among others for each specific number of sensors.
In Table 6, values of VOI, TE, $Td_{min}$, $Td_{max}$, $Td_{ave}$ and probabilities of detecting contamination events under 60 and 2 minutes from injection ($Pd_{60}$ and $Pd_2$, respectively) are provided for comparison. It is clear from the table that nearly in all cases the CWS designs from VT model are superior to those of TEVA-SPOT. Although the optimization module of TEVA-SPOT uses a single-objective algorithm, the results of VT are more accurate than those of TEVA-SPOT even for the objective that TEVA-SPOT is calibrated for. We attribute this behavior to the inefficacy (low efficacy) of the GRASP algorithm which is used in TEVA-SPOT.

### Table 6: Comparison of Contamination Event Detection Times and Probabilities

<table>
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<th># of sensors</th>
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<th>$Td_{min}$ (min)</th>
<th>$Td_{ave}$ (min)</th>
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<th>$Pd_{60}$</th>
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<td>3</td>
<td>49.10105</td>
<td>192</td>
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<td>TSV</td>
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<td>0.224353</td>
<td>3</td>
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<td>192</td>
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5. **Conclusion**
In this paper, an information theoretic approach is used for designing Contamination Warning System (CWS) in Water Distribution System (WDS), which can either be used to determine the best possible potential locations for placement of sensors to be in turn employed in an optimization framework, or to single-handedly devise ultimate sensor placements. The Value Of Information (VOI) and Transinformation Entropy (TE) techniques are utilized to determine different sets of nodes. Former warrants maximum achieved information value and latter examines uniqueness of acquired information (which in turn manifests itself in minimizing the number of required sensors and maximizing the probability of detection). The advantage of the proposed framework lies in its cost-effectiveness and objectivity. It is noted, however, that the simulation part is similar for all offline-simulation based methods. The proposed method summarizes results of the simulation scenarios in two square matrices with the size of the WDS nodes. The optimization run-time in the information theoretic framework, unlike the traditional approaches, is independent of the number of simulation scenarios. In this study, a large number of contamination scenarios (over 216,000 scenarios) are simulated, which in turn greatly enhanced the decision space and warranted more accurate and robust results. Comparisons efforts show that the proposed information theoretic model is capable of outperforming selected models from the literature including TEVA-SPOT, both from computational efficiency and results’ accuracy viewpoints.

Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.watres.2018.06.050.
References


Yokoyama, K., 2007. Our recent experience with sarin poisoning in Japan and pesticide users with references to some selected chemicals. Neurotoxicology., 28 (2), 364e373.
