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Optimal and Objective Placement of Sensors in Water Distribution Systems 25 **Using Information Theory** 26

Mohammad S. Khorshidi¹, Mohammad Reza Nikoo², Mojtaba Sadegh³ , 27

Abstract 28

24

Optimization-based deployment of contamination warning system in water 29 distribution systems has been widely used in the literature, due to their superior 30 performance compared to rule- and opinion-based approaches. However, 31 optimization techniques impose an excessive computational burden, which in turn is 32 compensated for by shrinking the problem's decision space and/or using faster 33 optimization algorithms with less accuracy. This imposes subjectivity in 34 interpretation of the system and associated risks, and undermines model's accuracy 35 by not exploring the entire feasible space. We propose a framework that uses 36 information theoretic techniques, including value of information and 37 transinformation entropy, for optimal sensor placement. This can be used either as 38

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1. Introduction 60

Delivering safe drinking water through water distribution systems (WDSs), although 61 seemingly straightforward, is a challenging task. A WDS is a distributed and easy- 62 to-access infrastructure, which could deliver contaminants along with potable water 63 to a large population (Shafiee and Zechman, 2013). Water quality in WDSs can be 64 easily compromised by accidental or intentional incidents (Berry et al. 2008; Hu et 65 al. 2018; Janke et al. 2017), and severely damage public health. Such incidents can 66 claim thousands of civilian lives, create horror in a community, harm public 67 confidence in the water supply system, and potentially impair the economy of an 68 entire nation due to post-incident expenses and long-term impacts. Literature 69 corroborates the WDSs' vulnerability to contaminant intrusion, and its catastrophic 70 impacts (Gavriel et al., 1998; Yokoyama, 2007; Forest et al., 2013). For example in 71 May 2000, contaminated drinking water in Walkerton, Ontorio, Canada, severely 72 affected 2,300 people and claimed 7 lives (Hrudey et al. 2003). 73

In the face of preventive actions being near impossible to protect an entire WDS, 74 deploying online and real-time monitoring system of water quality has been regarded 75 as the best alternative to mitigate risks of delivering contaminated water to consumers 76 (Hart and Murray 2010; Rathi and Gupta 2014). This is also referred to as 77 Contamination Warning System (CWS), and is a network of multiple quality sensors 78 placed at different locations of a WDS with a centralized monitoring system that 79 detects the time and location that safe drinking water is compromised. Due to 80 expensive costs of purchase, installation and maintenance, placement of sensors at 81 every node in WDS is not economically justified (Zeng et al., 2016). For example, 82 PSA 10.255 analyzer is an online water quality sensor for measuring various elements 83

such as Mercury, Selenium and Arsenic in potable water with 1 micro-grams/Liter 84 accuracy, which costs between 3000\$ to 5000\$ (P. S. Analytical Co., 2018). 85 Therefore, optimal deployment of CWS has been the focus of several studies (e.g. 86 Berry et al. 2005a; Krause et al. 2006; Shastri and Diwekar, 2006; Berry et al. 2008; 87 Ma et al. 2010; Afshar and Marino, 2012; Berry et al. 2012; Zhao et al., 2016; 88 Mukherjee et al., 2017; Janke et al. 2017); in which, an optimization model is 89 developed to find a layout of sensors with objective functions such as minimizing 90 affected population, time to detection, and volume of contaminated water, and 91 maximizing probability of detection. 92

Although optimization approach is proven to be superior to rule- and opinion-based 93 approaches, one of the obstacles in its application for real-world large utility networks 94 is computational burden. Due to expensive computational nature of optimization 95 algorithms, usually a fixed set of nodes or a fixed number of sensors is selected to 96 reduce the number of decision variables (e.g. Weickgenannt et al., 2010; Bazargan- 97 Lari, 2014; Afshar and Khombi, 2015; Tinelli et al., 2017; Janke et al. 2017). Another 98 approach is to use more computationally efficient optimization algorithms that search 99 for a near optimal solution, at the expense of lower accuracy, objectivity and 100 robustness (Berry et al. 2008; Berry et al. 2012; Janke et al. 2017). While most of the 101 literature have focused on developing different optimization techniques to solve for 102 subjectively selected decision variables, Diao and Rauch (2013) argued a more 103 precise and objective method to pre-determine the potential locations of sensors in 104 WDS should be adopted. In an effort to tackle this issue, they proposed an approach 105 inspired by the concept of control theory for complex system analysis (Liu et al., 106

2011). The goal of the proposed controllability analysis is to reduce the number of 107 decision variables of the CWS deployment optimization problem by eliminating 108 nodes with "redundant signals" from the set of potential locations, i.e. decision 109 variables. Therefore, the retained nodes would provide unique signals. The advantage 110 of this method lies in being merely dependent on the hydraulic of the WDS. However, 111 the focus of their study was only to determine a minimum number of nodes by virtue 112 of which any contamination event could be detected (full coverage). Fast detection 113 of contamination or minimum health impact cannot be guaranteed by this method 114 (Diao and Rauch, 2013). To account for these factors, possible contamination events 115 should be simulated and a complementary optimization model should be used along 116 with this method. 117

Due to the stochastic nature of how, when and where a contamination could be 118 injected in WDS, simulation of possible contamination events seems an inevitable 119 part of optimization approach to deploying CWS (Hart and Murray 2010). Therefore, 120 this will lead to obtaining a series of probability distribution functions (pdfs) of time 121 to detection, and affected population, among others, for any potential node for 122 placement of sensors. Previous studies have focused on optimization of one or a few 123 signatures of these pdfs as objective function. These signatures were usually the pdfs' 124 mean (e.g. Berry et al. 2003, 2005a, 2008 and 2012; Preis and Ostfeld, 2008; 125 Bazargan-Lari, 2014; Janke et al. 2017), or the pdfs' tale (e.g. Berry et al. 2008 and 126 2012; Janke et al. 2017; Naserizade et al. 2018). However, aggregating the pdf into a 127 few signatures, which are in turn used in the optimization process, loses a lot of 128 valuable information and might not be good representative of the entire system 129

functionality. These pdfs can deliver paramount inferable information, and different 130 nodes of a WDS provide diverse information regarding exposed population to 131 contaminated water, and time interval between injection and detection of 132 contaminant, among others. This information describes the behavior of the WDS and 133 the consequences of a contaminant intrusion, and details a corresponding action by 134 decision maker. Obviously, equipping every node of a WDS with sensors would 135 provide maximum information about the WDS. However, the installation and 136 maintenance costs of such comprehensive CWS is prohibitive. Moreover, 137 information provided by different nodes may not be unique. Even similar information 138 obtained from a given pair of nodes may have different value to the decision maker, 139 which can be defined by a utility as a function of objective pdfs. Such perception 140 would lead to definition of a WDS as a finite set of probabilistic curves, and hence, 141 provide the ability to compare the locations for placement of sensors in a more 142 objective way. Consequently, choosing an optimal subset from the original space not 143 only warrants objectivity but also would hugely reduce the runtime and memory 144 requirements of the optimization model. This allows for searching for global 145 optimum in a complete and comprehensive decision space, as opposed to the 146 traditional sensor placement studies that either had shrunk the decision space or 147 employed more-efficient but less-accurate optimization algorithms to tackle the 148 computational burden (Berry et al. 2008; Berry et al. 2012; Janke et al. 2017). 149

We adopt an information theory-oriented approach to determine potential and/or 150 optimal locations for CWS in WDS. In doing so, we use the concept of Value of 151 Information (VOI) to determine a set of nodes which can provide maximum 152

information values, and can be used as closest representatives of other nodes. 153 Grayson (1960) developed the VOI technique to assess the value of received 154 information for decision-making. Ever since, the VOI technique has found a wide 155 range of applications in water science, including groundwater quality assessment 156 (Wagner et al., 1992; Shaqadan, 2008), designing groundwater quality monitoring 157 systems (Reichard and Evans, 1989; Ammar et al., 2009; Khader et al., 2012; 158 Hosseini and Kerachian, 2017), designing CWS in agricultural systems (Roberts et 159 al., 2009), and flood monitoring and impact assessment (Alfonso, 2010; Verkade and 160 Werner, 2011; Alfonso and Price, 2012; and Alfonso et al., 2016). The literature has 161 used VOI for time-series analysis, while in this study we use the technique for event- 162 based analysis. 163

Different sets of selected nodes with similar count (but with different locations) may 164 yield a comparable VOI, but with different level of information redundancy. 165 Following a practical assumption in the literature, this study is based on using sensors 166 which are 100% reliable above a certain concentration (e.g. Janke et al. 2017; 167 Naserizade et al. 2018). This assumption warrants avoiding information redundancy. 168 Therefore, Transinformation Entropy (TE) is used to minimize the mutual 169 information of selected nodes. Also, the level of TE for any given pair of nodes 170 depends on the spatial distance of the nodes. Hence, minimizing TE would also result 171 in obtaining more spatially distributed CWS which yields a higher probability of 172 detecting contamination across the WDS. Shannon (1948) introduced the Entropy 173 theory to quantify the information content of a data set (Harmancioglu., 1981). TE is 174 a special form of entropy that quantifies mutual information of two data sets, and has 175 been applied in many monitoring network design problems, including reservoir 176 quality monitoring stations (Lee et al., 2014; Nikoo et al., 2017), groundwater quality 177 monitoring stations (Caselton and Husain, 1980; Mogheir and Singh, 2002; Mogheir 178 et al., 2004a, 2004b, 2005 and 2009; Masoumi and Kerachian, 2008 and 2010; and 179 Mondal and Singh 2012), and river quality monitoring stations (Harmancioglu and 180 Yevjevich, 1987; Ozkul et al., 2000; Karamouz et al., 2006 and 2009; Salark and 181 Sorman, 2006; Mahjouri and Kerachian, 2011; and Memarzadeh et al., 2013). 182

In this study, we use the VOI and TE techniques to quantify the value and uniqueness 183 of information of different nodes in WDS. Then, a multi-objective optimization 184 model, namely NSGA-II (Deb et al., 2000 and 2002), is formulated with a potential 185 to be used as a pre-selection or ultimate selection method for design of a CWS in 186 WDS. The objectives of the proposed model are, 1) maximizing VOI, 2) minimizing 187 TE, and 3) minimizing number of selected sensors. This method is applied for design 188 of CWS in Lamerd City's WDS which is a large scale WDS and previously used in 189 other studies (Bazargan-Lari, 2014; Naserizade et al., 2018). Finally, results are 190 compared to the potential locations for placement of sensors in previous studies and 191 optimal CWS designs from TEVA-SPOT model (Janke et al. 2017) to assess the 192 proposed method's capabilities. 193

2. Methodology 194

2.1. Value of Information (VOI) 195

Grayson (1960) first introduced the concept of VOI, and Hirshleifer and Riley (1979) 196 presented it for monitoring network design (Alfonso and Price, 2012). A decision 197 maker can update their perception about the state of a system, which can be quantified 198

in discrete form as the vector of prior probability, $P(s)$, of having a particular state 199 s. When new information comes to light, based on Bayes' theorem, the updated belief 200 can be represented as, 201

$$
P(s|m) = \frac{P(m|s) P(s)}{P(m)},
$$
\n(1)

where, $P(s|m)$ is the updated belief following the receipt of message m; $P(m|s)$ is 202 the conditional probability of receiving message m when the state of the system is s ; 203 and $P(m)$ is the probability of receiving message m. 204

If one assumes that the time interval between injection and contaminant detection at 205 a given node is the state, s, of that node (which we call "detection state" hereafter), 206 vector $P(s)$ is the belief of the WDS's utility about the possibility of contamination 207 being detectable at any given node in each detection state, s. This belief may be 208 determined roughly by utility's experience or more precisely by simulation of random 209 scenarios. The message m can be inferred as the received contamination warning 210 after passage of *time units from injection of contamination from a sensor placed 211* at any node. Once $P(s)$ is determined, before calculation of updated belief, $P(s|m)$, 212 one has to calculate evidence probabilities, $P(m)$, and evidence conditional 213 probabilities, $P(m|s)$, by gathering new information independent from prior belief. 214 To calculate $P(m)$ for any given node, we assume a sensor is placed at that node and 215 probabilities of receiving contamination warning in m time unit from injection is 216 calculated by the new information. Then, the selected node should be paired with 217 another node and conditional probabilities of receiving contamination warning from 218 the selected node and detection states of the other node should be calculated to obtain 219 $P(m|s)$. In this study, the results of simulation of random scenarios are used for 220 calculation of both prior and evidence probabilities. Since the scenarios are generated 221 randomly, they are independent from Bayesian viewpoint (Alfonso and Price, 2012 222 and Alfonso et al. 2016). 223

Presume a utility manager uses a sensor placed at node i to warn consumers of node 224 ݆ about water contamination. Timely warnings would preclude major consequences, 225 while early or late warnings would result in panic or exposure of consumers, 226 respectively. Hence, an action (warning), a , with various lag times and its associated 227 consequences, $C(a, s)$, when the detection state of node *j* is *s*, can be defined. The 228 utility of those actions based on received message, u_m , and without the received 229 message, u_s , can be calculated as, 230

$$
u_m = \sum_{S} C(a, s) P(s|m), \qquad (2.a)
$$

$$
u_s = \sum_{s} c(a, s) P(s), \qquad (2.b)
$$

where, S , is the total number of detection states, S . Therefore, the value of a chosen 231 action a_m based on a received message, m, would be $u_m - u_s$. A rational decision 232 maker will choose the action with maximum utility, therefore, the VOI of node i for 233 determining the detection state of node j , $(VOI_i(j))$ can be calculated by, 234

$$
VOI_i(j) = \sum_{M} P(m) \left[\max_{a} \left(\sum_{S} C(a, s) P(s|m) \right) - \max_{a} \left(\sum_{S} C(a, s) P(s) \right) \right], \quad (3)
$$

in which, M is number of messages. For a given node i , VOI can be calculated for all 235 j nodes, which will result in a VOI_i curve. The area below the VOI_i curve is equal to 236 the VOI of node \dot{i} for detecting the states of the entire system. Obviously, the VOI of 237 node i is always maximum for determining the detection state of node i . When more 238 than one node is selected to determine the detection state of node j, $VOI_{i_{1,2}}(j)$ would 239 be maximum of $VOI_{i_1}(j)$ and $VOI_{i_2}(j)$; i.e. the VOI of set of nodes which is selected 240 for placement of sensors would be union of VOI curves of the selected nodes. Hence, 241 the VOI of all nodes of a WDS is maximum, and can be used as a benchmark to 242 assess the behavior of any set of selected nodes for placement of sensors. A numerical 243 example is provided in Supplementary Material which explains the steps involved in 244 calculation of $VOI_i(j)$. 245

246

2.2. Transinformation Entropy (TE) 247

In discrete form, the mutual information which also called transinformation, $TE(i, j)$, 248 of two selected nodes (i and j) for placement of sensors can be calculated as (Mogheir 249 et al., 2004a, 2004b), 250

$$
TE(i,j) = -\sum_{s} \sum_{ss} P(i_s, j_{ss}) \ln \left[\frac{P(i_s, j_{ss})}{P(i_s)P(j_{ss})} \right],
$$
 (4)

where, $P(i_s, j_{ss})$ is the joint probability of having detection state s at node i, while 251 the detection state of node *j* is ss; and $P(i_s)$ and $P(j_{ss})$ are probabilities of having 252 detection states s and ss at nodes i and j , respectively. Similar to VOI, TE of a set of 253 nodes is union of their TE curves. Also, the area below the TE curve of a set of nodes 254 is dependent on the spatial distribution of those nodes. Hence, minimizing TE of the 255 selected nodes set for placement of sensors would warrant uniqueness of information 256

3. Case Study 272

 $minimize Z_2 = \frac{1}{\max\{[TE]\}}\sum_{\forall j\neq i}\max\{b_i \times b_j \times TE(i, j)\}$ $(5.b)$

$$
minimize Z_3 = \sum_{\forall i} b_i.
$$
 (5.c)

 (5.a)

2.3. Optimization model
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\nin two square matrices, where the elements in
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$$
 row and j^{th} column are equal to
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TE (Z_2) . The VOI and TE matrices are normalized by their maximum values, 267

respectively (eqs. 5.a and 5.b). By including the third objective, it is implicitly 268

assumed that the costs associated with placement of sensors have a monotonic 269

relationship with number of placed sensors in WDS. 270

of nodes, and consequently selection of nodes with the highest spatial distribution in 257 WDS. Higher spatial distribution of CWS warrants greater possibility of detecting 258

contamination. 259

13

1

 $maximize Z_1 = \frac{1}{\max\{[VOI]\}}\sum_{\forall j} \max_i \{b_i \times VOI_i(j)\},$

The proposed methodology is applied for designing CWS in WDS of Lamerd City 273 (Fig.1), Fars province, Iran. This WDS consists of 185 links (pipes), 122 junctions, 274 23 hydrants, 2 reservoirs and a tank, and supplies water demands of about 81,000 275 people. The base demand of Lamerd's population is approximately 260 LCD (liters 276 per capita per day), and the daily demand pattern is depicted in Fig.2. 277

Fig. 1. Water Distribution System (WDS) of Lamerd City. The locations of 280 potential contamination injection: hydrants, reservoirs and tank are marked with 281 symbols as defined in legend. 282

Fig. 2. Daily pattern of base demand's hourly multipliers. 284

283

Arsenic is used as the contaminant of choice in this study as commonly used in the 286 literature (e.g. Shafiee and Zechman, 2013; Bazargan-Lari, 2014; Naserizade et al. 287 2018). Arsenic is a cheap, accessible and well-known poisonous substance, which 288 could be deadly at very low dosages. The critical dose, C_d (milligrams), of this 289 substance depends on the weight, W_p (kg), of the exposed person and can be 290 calculated as follows (Shafiee and Zechman, 2013), 291

$$
C_d = 5.0 \times 10^{-8} \times W_p.
$$
 (6)

A person with an average 70 kg weight could be critically affected by ingesting 3.5 292 mg of Arsenic. Throughout this study, the term "affected population" refers to the 293 population who ingested equal to or greater than 3.5 mg Arsenic due to consuming 294 contaminated water. Furthermore, it is assumed that ingesting contaminant could 295 only occur through drinking, and each person drinks 0.93 L/day (Shafiee and 296 Zechman, 2013). 297 A contamination event is associated with various uncertainties. Monte Carlo 298 Simulation (MCS) is used to incorporate the uncertainties emanating from 299 contaminant intrusion. For this purpose, mass and duration of injection are considered 300 as stochastic variables; and time of injecting, location of injection and number of 301 simultaneous injections are considered as scenario-based inputs in MCS. Then, all 302 combinations of these parameters (Table 1) are generated and used as contamination 303 injection scenarios. It is worth mentioning that 14 hydrants, of the available 23, 304 together with the tank and two reservoirs are selected as the potential locations for 305 contaminant intrusion. Due to hydraulic of the WDS, contamination injection from 306 the other 9 hydrants has very low impact on the affected population. Therefore, these 307 hydrants are not considered in contamination scenarios (Naserizade et al., 2018). The 308 injection scenarios are simulated using EPANET (Rossman, 2000) model of Lamerd 309 WDS, which was previously calibrated by Bazargan-Lari (2014). The simulation 310 period is 48 hours and both hydraulic and quality time-steps are 60 seconds. Since 311 Arsenic cannot react with materials on the pipe wall in short time, only bulk flow 312 reaction is considered in the quality simulation (-0.05 day^{-1}) . The results of the 313 simulated scenarios are then used to calculate the time to detection at each node, the 314 ingested mass of Arsenic, and consequently the affected population. It is assumed 315 that the detection limit of available sensors is 0.01 (mg/L) (Naserizade et al. 2018). 316 Assumption is that sensors are 100% reliable in detecting contamination with 317 concentration above the detection limit, and will fail for concentrations below this 318 threshold. These results are used to derive the VOI matrix for all pairs of nodes. 319

Table 1. Characteristics of scenarios considered in MCS. 321

In order to show the method's capability, the obtained solutions are compared to the 323 sets of potential nodes for placement of sensors from Bazargan-Lari (2014) and 324 Naserizade et al. (2018) studies, as well as the results of TEVA-SPOT model (Janke 325 et al. 2017). Similar to this study, the researchers use offline simulation of 326 contamination scenarios. However, deriving the objective functions from a large set 327 of simulation results requires huge amount of logical operations, and consume a large 328 volume of memory, especially in an optimization process. Therefore, Bazargan-Lari 329 (2014) and Naserizade et al. (2018) have determined a set of potential locations for 330 placement of sensors, through sensitivity analysis, to reduce the number of decision 331 variables. On the other hand, TEVA-SPOT model requires the user to specify the 332 number of sensors to be placed in WDS. Also, its Graphical User Interface version, 333 TEVA-SPOT GUI, uses a Greedy Randomized Adaptive Search Procedure (GRASP) 334 optimization algorithm to provide near-optimal solutions with lower memory 335 requirements and in quickest possible way. The offline calculation of VOI and TE, 336 however, summarizes all scenarios in two matrices; hence, the decision space could 337 be significantly enhanced. Moreover, VOI and TE approaches hugely reduce the 338 memory requirements and provide faster runtimes, and hence, more accurate 339 optimization algorithm (NSGA-II — a multi-objective version of GA) is used in our 340 framework. Ultimately, we compare our solutions with the same number of nodes to 341 that of the decision space (potential sensors' locations) of Bazargan-Lari (2014) and 342 Naserizade et al. (2018) and the results of the TEVA-SPOT model. 343

344

4. Results and Discussion 345

A total of seven detection states are considered to derive VOI for each node (Table 346 2). These states are considered based on the criterion of fast detection of 347 contamination, given a comprehensive pre-analysis of simulated scenarios (Fig.3). 348 When an intrusion occurs, contaminated water could not necessarily reach all nodes 349 of the WDS due to its hydraulic characteristics. From the set of nodes which 350 contaminated water can reach, almost none is affected by contaminated water in the 351 first 5 minutes (average affected population $\approx 0.5\%$). In the next 10 minutes, the 352 affected population increases smoothly (\approx 2%). During the first 60 minutes from 353 injection, the contamination reaches majority of the possible nodes, however, with 354 relatively low concentration which affect only about 6% of the population. In the next 355 240 minutes the affected population rapidly increases (\approx 96%) due to accumulation 356 of contamination in the consumers body and relatively high concentration of 357 contamination. 358

359

360

361

Detection state	Time interval between injection and
	detection of contaminant (min)
	0 to 5
	5 to 15
3	15 to 30
	$30 \text{ to } 60$
5	60 to 120
	120 to 300
	More than 300 min

Table 2. Detection states of each node in contaminant intrusion events. 363

365

Fig. 3. Temporal change of affected population from the time of injection at each 366 node in all simulated scenarios. The nodes that remain immune from contaminated 367 water in each event are removed from the calculation. 368

Calculation of VOI requires determining the cost matrix,
$$
C(a, s)
$$
, whose elements are
the cost of releasing "no consuming" warning after receipt of message *m* from *i* 371
node, when the detection state at node *j* is *s*. For example, assume in an intrusion 372
event, contaminated water reaches nodes *i* and *j* in the 4th and 2nd detection state, 373

respectively, and a sensor is placed at node i while there is no sensor at node j . A 374 utility manager warns the consumers at node \dot{j} following the receipt of contamination 375 warning from the sensor i , which is late, because consumers were already drinking 376 contaminated water for about 45 minutes. The average affected population of node $j = 377$ in the 45 minutes interval is the cost of action a_4 when the state is s_2 , i.e. $C(a_4, s_2)$. 378 On the other hand, early warning would result in a widespread panic among the 379 population of node *j*. Although a more specific approach for cost assessment of panic 380 among consumers could be considered, the costs of early warnings are considered 381 equal to those of late warnings in this study. Using these assumptions, Fig.3 is used 382 to determine the mean affected population of nodes in each detection state. The 383 difference between the average fraction (%) of affected population multiplied by 384 mean population of nodes (≈ 663 persons) for each pair of time intervals (the 385 average of affected population at the beginning and at the end of the interval) with 386 negative sign (Table 3) is set as $C(a, s)$. 387

388

Table 3. The cost of actions, $C(a, s)$, in each detection state for the calculation of 389

From over 216,000 simulated contamination scenarios, about 80% are used for 392 calculation of prior probabilities, and the remaining 20% are used for calculation of 393 evidence probabilities. The VOI and TE are then calculated for every pair of nodes. 394 The normalized VOI and TE of all 122 nodes are equal to 74.24 and 68.82, 395 respectively. For example, the normalized VOI and TE for nodes 1 and 45 are shown 396 in Fig. 4. It is vivid in this figure that both VOI and TE are reliant on spatial distance. 397 Although, TE quantifies the mutual information of a pair of nodes, their information 398 content may have different value to a decision maker (the neighbors of node 1 in Figs. 399 4a and c). As mentioned earlier, these values mainly include time to detection and 400 affected population in CWS design. 401

Figs. 4. The values of normalized, (a) $VOI_1(j)$, (b) $VOI_{45}(j)$, (c) $TE(1, j)$, and (d) 404 $TE(45, j)$, which are shown by blue circles and $\forall j$. 405

After calculation of the VOI and TE matrices, the optimization model is executed. 407 The obtained pareto-optimal solutions contain 335 CWS layouts with 5 to 114 nodes 408

for placement of sensors. Also, the solutions include 4 sets with 113 and 114 nodes 409 with VOI equal to 74.24, albeit their TE are equal to 65.16 and 65.09, respectively. 410 This implies that these sets of nodes are capable of almost perfectly representing the 411 entire WDS nodes with respect to the considered states and costs. A randomly 412 selected set of pareto-optimal solutions and their performance metrics including 413 probability of detecting a contamination event (Pd) , minimum, maximum and 414 average time to detection (Td_{min} , Td_{max} and Td_{ave} , respectively), as well as 415 average affected population (Pa_{ave}) are provided in Table 4. 416

417

Table 4. A randomly selected set of pareto-optimal solutions with their respective 418 values of objective functions, probability of detection (Pd) , minimum, maximum 419 and average time to detection (Td_{min} , Td_{max} , and Td_{ave} , respectively), and 420

average affected population (Pa_{ave}) . 421

Table 5. Comparing VOI, TE, probability of detection, minimum, maximum and 436 average time to detection, and average affected population of different sets of 437

nodes. 438

Fig. 5. The decision spaces of Bazargan-Lari (2014) (SN20) and Naserizade et al. 441 (2018) (SN19) with 20 and 19 nodes, respectively. The pareto-points #273 (P120) 442 and, #274 (P220) with 20 nodes, and #275 (P119) with 19 nodes are also shown. 443

444

As shown in Fig. 5, 18 nodes are common among the pareto solutions. However, 445 there is only one common node among the all five sets of nodes (one complete circle 446 in this figure). For more clarity, the probability distribution of detecting a 447 contamination event during the first 60 minutes from the injection in 2 minutes 448 intervals are depicted in Fig.6 for these five sets of nodes. While P119, P120 and 449 P220 provide a probability of \sim 0.35 for contamination detection under 2 minutes, this 450 probability is about 0.08 and 0.00 for the SN19 and SN20, respectively. The 451 probability of detecting contamination under 60 minutes are 0.976, 0.971, and 0.982 452

for SN20, P120, and P220, respectively, while it is 0.949, and 0.971 for SN19, and 453 P119, respectively. 454

Figs. 6. Comparing the probability distribution of time to detection for, (a) SN20, 456 P120, and P220, and (b) SN19, and P119. 457

To provide a deeper understanding about the coverage of the sets of nodes, the VOI 458 of all sets of nodes are divided by the VOI of the 122 nodes of the WDS and the 459 relative VOIs are visualized in Fig. 7. The only difference between P120 and P220 is 460 the location of two nodes, and the remaining 18 nodes have similar locations. Only 461 P220 is plotted in this figure. 462

Obviously, the economic factor (i.e. budget limitation) is the main constraint in 467 deployment of CWS in WDS. Berry et al. (2005b) pointed out this obstacle and 468 argued that even different locations for placement of sensors would have different 469 cost. However, using the third objective function in the proposed model in this study, 470 we implicitly assumed that placing sensors at any point would have equal costs. Also, 471 the proposed model could be modified by replacing the third objective function with 472 a constraint on the number of selected nodes to find more CWS layouts which are 473 economically justifiable. To evaluate the performance of the proposed model against 474 that of TEVA-SPOT, we have considered CWS designs with fewer number of sensors 475 (i.e. CWS designs with 3, 4, 5, 6, 7, 8 and 9 sensors). 476

We will now present a brief comparison between the results of the proposed VT 477 model and TEVA-SPOT for the Lamerd WDS. An extensive report on the 478 performance comparison of the VT model against TEVA-SPOT from computational 479 efficiency and results' accuracy viewpoints is provided in Supplementary Materials. 480 Both models were executed on a desktop PC (CPU: Intel® Core™ i7-4500U; RAM: 481 12GB DDR3). Also, the parameters of both models' hydraulic engine (i.e. EPANET 482 v2) have been set up identically. Simulating the 216,000 scenarios by TEVA-SPOT 483 is, however, not possible on a desktop PC. According to Janke et al. (2017), when the 484 size of WDS (i.e. number of nodes) and/or number of simulation scenarios are very 485 large, execution of TEVA-SPOT would not be possible on a typical PC. Hence, only 486 about 12,000 scenarios (about 5% of the scenarios used earlier) were simulated by 487 both models for CWS design. Single-node injection and simultaneous injection from 488 two and three nodes are considered in contamination injection scenarios. The details 489 of the simulated scenarios are provided in Table S3 of Supplementary Material. Both 490 models were constrained to provide at least 80% probability of detection of 491 contamination events (i.e. $Pd \geq 0.8$). 492

Since TEVA-SPOT uses a single-objective optimization scheme, it should be 493 executed for every objective separately, providing a single solution each time. To 494 evaluate the robustness and accuracy of the solutions of VT model against TEVA- 495 SPOT, two objectives were defined for TEVA-SPOT; i.e. minimization of the Value- 496 at-Risk (VaR) of time to detection (Td_{VaR}) and minimization of average of time to 497 detection (Td_{ave}) . VaR is the point on a pdf where cumulative probability of the pdf 498 exceeds a certain level (Sarykalin et al., 2008). Hence, for each number of sensors, 499 there would be two solutions from the TEVA-SPOT, one for Td_{ave} and one for 500 Td_{VaR} which are denoted by TSM and TSV, respectively. On the other hand, the 501 multi-objective optimization module of VT model was executed only once for each 502 number of sensors, which in turn provided a pareto front. Each solution on the paeto 503 front is denoted by VT followed by a number. Only one selected solution from VT 504 model is provided here (Table 6) for each CWS design, and the complete pareto front 505 of VT model is provided in Table S5 in Supplementary Material. 506

The results show that VT model is overall significantly more efficient than TEVA- 507 SPOT. While, the simulation module of VT is 23% slower than that of TEVA-SPOT, 508 its VOI+TE and optimization modules are 350% and 177% faster than impact 509 assessment and optimization modules of TEVA-SPOT, respectively. Also, TEVA- 510 SPOT occupied 5 Giga Bytes (5120 Mega Bytes) of disk space, while VT only 511 occupied 244 Mega Bytes. 512

Table 6. The results of the TEVA-SPOT and VT models for design of CWS with 3, 513

4, 5, 6, 7, 8 and 9 sensors in Lamerd WDS. The bold items indicate superiority of 514 the solution among others for each specific number of sensors. 515

In Table 6, values of VOI, TE, Td_{min} , Td_{max} , Td_{ave} and probabilities of detecting 518 contamination events under 60 and 2 minutes from injection (Pd_{60} and Pd_{2} , 519 respectively) are provided for comparison. It is clear from the table that nearly in all 520 cases the CWS designs from VT model are superior to those of TEVA-SPOT. 521 Although the optimization module of TEVA-SPOT uses a single-objective algorithm, 522 the results of VT are more accurate than those of TEVA-SPOT even for the objective 523 that TEVA-SPOT is calibrated for. We attribute this behavior to the inefficacy (low 524 efficacy) of the GRASP algorithm which is used in TEVA-SPOT. 525

5. Conclusion 527

In this paper, an information theoretic approach is used for designing Contamination 528 Warning System (CWS) in Water Distribution System (WDS), which can either be 529 used to determine the best possible potential locations for placement of sensors to be 530 in turn employed in an optimization framework, or to single-handedly devise ultimate 531 sensor placements. The Value Of Information (VOI) and Transinformation Entropy 532 (TE) techniques are utilized to determine different sets of nodes. Former warrants 533 maximum achieved information value and latter examines uniqueness of acquired 534 information (which in turn manifests itself in minimizing the number of required 535 sensors and maximizing the probability of detection). The advantage of the proposed 536 framework lies in its cost-effectiveness and objectivity. It is noted, however, that the 537 simulation part is similar for all offline-simulation based methods. The proposed 538 method summarizes results of the simulation scenarios in two square matrices with 539 the size of the WDS nodes. The optimization run-time in the information theoretic 540 framework, unlike the traditional approaches, is independent of the number of 541 simulation scenarios. In this study, a large number of contamination scenarios (over 542 216,000 scenarios) are simulated, which in turn greatly enhanced the decision space 543 and warranted more accurate and robust results. Comparisons efforts show that the 544 proposed information theoretic model is capable of outperforming selected models 545 from the literature including TEVA-SPOT, both from computational efficiency and 546 results' accuracy viewpoints. 547

Appendix A. Supplementary data 548

Supplementary data related to this article can be found at 549 https://doi.org/10.1016/j.watres.2018.06.050. 550

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