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Optimal Bayesian Estimation in Sensor Networks

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Abstract

In environment monitoring,data from multiple sensors are critical. Transmitting all sensors data in timely fashion is difficult due to bandwidth constraint. So, it is critical to best utilize the bandwidth to transmit the data . We consider sensor networks where each sensor quantizes its local observation into one bit, and transmit it to a central node through the channel where the estimate of parameter is made based on the received data. We design optimal distributed system in Bayes framework under different channel quality setting. We demonstrate the robustness and efficiency of our designs via both theoretical derivation and numerical validation.

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1. Abstract

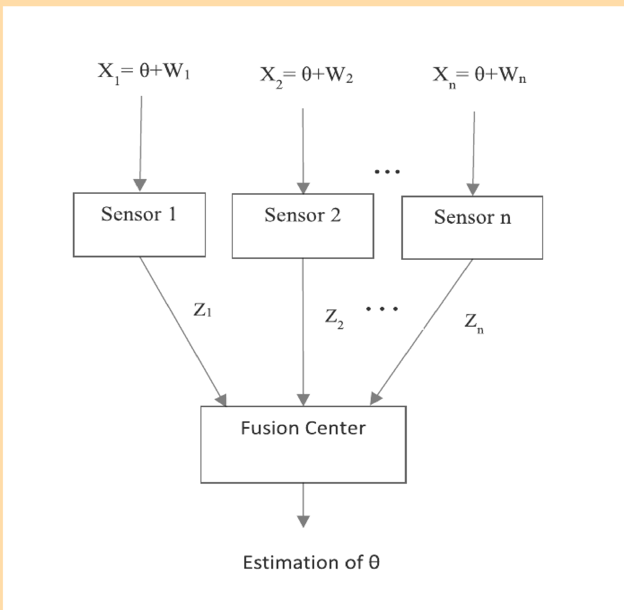
Data from multiple sensors require large bandwidth and data are critical to virtually all the applications. Transmitting data is difficult due to the bandwidth constraint so it requires to best utilize the bandwidth to transmit the data. We consider distributed sensor network where each sensor quantizes its local observation into one bit and transmit it through the channel and estimate parameter of observation based on the received data. We design the optimal distributed estimation system for given observation in Wireless Sensor Networks(WSN) under Bayesian framework. Further we investigate robustness of our design i.e. compare design results under different settings of channel.

2. MOTIVATION

- WSN nodes: limited energy, power, lifetime, computing, and communication capabilities
- Different quantization schemes exist from which an optimal system is to design for given observations while meeting stringent power and bandwidth budgets..

3. DISTRIBUTED ESTIMATION SYSTEM

- Parallel Network Structure
  - Local sensors send compressed observations to a fusion center (FC) & FC makes an estimate
  - In particular, we consider the most stringent case where sensor observations are quantized into single bits based on quantization rules (probabilistic & deterministic)
- Distributed Estimation System Diagram



- Research Problems and Goals
  - Unlimited choices of quantizer rules and estimators
    - Deterministic -Single threshold employed for qunatization
    - Probabilistic Noise added to signal before quantization, e.g., dithered quantizer
  - Identify how much information changes as a result of adjusting sensor decision rule under different channel settings
- Performance Matrices
  - Mean square error (MSE)
  - MSE lower bounded by its Cramer-Rao lower bound (CRLB)
  - Maximum likelihood (ML) and Maximum a Posteriori (MAP) estimator achieves CRLB asymptotically

4. DISTRIBUTED BAYESIAN ESTIMATION WITH ONE-BIT QUANTIZATION

4.1 Estimation Problem

- Estimate θ from N sensors: $X_i = \theta + W_i$, where θ is parameter to be estimated with prior pdf $p_\theta(\theta)$, X_i : sensor observations, W_i : i.i.d noise with pdf $f_w(.)$
- Observation model

$$f(\mathbf{X}/\theta) = \prod_{i=1}^{N} f(X_i/\theta)$$

- Quantizers
  - Observation quantized to one-bit (0 or 1) with quantization rule $\gamma_i(X_i) = Pr(Z_i = 1/X_i)$
  - Identical quantizers $\gamma = \gamma_1 = \gamma_2 = ... = \gamma_N$

4.2 Estimation Performance

- Determined by $k(\theta) = Pr(Z/\theta) = E_{X/\theta}[Pr(Z_i = 1/X_i)] = \int_X [\gamma(x) f(x/\theta)]dx$ [2]
- For an unbiased estimator $\hat{\theta}$,MSE is lower bounded, i.e., $MSE = E(\hat{\theta} - \theta)^2 \geq CRLB(\theta, k, f) = \frac{1}{I(\theta,k,f)}$ [3] , where $I(\theta, k, f)$:Fisher Information (FI)
- Normalized asymptotic MSE: $N.MSE \rightarrow \epsilon(\theta, k, f) = E_\theta(\frac{1}{I(\theta,k,f)}) = \int_{-\infty}^{\infty} \left[p_\theta(\theta)\frac{1}{I(\theta,k,f)}\right] d\theta$
- Optimization problem: $\gamma_0(x) = \arg min_{\gamma(.)} \epsilon(\theta, k, f)$
  - determine an optimal $\gamma_0(x)$ that minimizes the error ϵ for any given $p_\theta(\theta)$ and $f(X/\theta)$

4.3 Performance Limit

- Full-Precision PL
  - Unquantized CRLB , $Z_i = X_i$
  - Too loose (power and bandwidth constraints)
- Distributed Bayesian PL
  - $X_i = \theta$,noiseless, deterministic
  - Perfect observation$\rightarrow$best performance
  - Monotone increasing increasing quantization rule

$$\gamma(X_i) = Pr(Z_i = 1/\theta) = k(\theta) = \frac{1 + sin(g(\theta))}{2}$$

  - FI in close form with adjusting sensor decision rule with channel of error, (ρ):$I_i(\theta) = \frac{(1-2\rho)^2 cos^2 g(\theta)(g'(\theta)^2}{(1-(1-2\rho)^2 sin^2 g(\theta))}$ $g'(\theta) = c.p_\theta^{\frac{1}{3}}(\theta)$ where $c = \frac{\pi}{(2\pi)^{\frac{1}{3}}3^{\frac{1}{2}}}$
  - FI in close form with adjusting sensor decision rule with channel of error, (ρ): $I_i(\theta) = \frac{\left[\frac{d}{d\theta}k(\theta)\right]^2}{k(\theta)[1-k(\theta)]} = (g'(\theta))^2$, $g'(\theta) = c.p_\theta^{\frac{1}{3}}(\theta)$ where $c = \frac{2sin^{-1}(1-2\rho)}{(2\pi)^{\frac{1}{3}}3^{\frac{1}{2}}}$
  - optimization problem [1]

$$g_I(\theta) = \arg\min_{g(\theta)}\{\int_a^b p_\theta(\theta)(g'(\theta))^{-2}d\theta\},\ s.t., g(\theta)\epsilon\left[-\frac{\pi}{2}, \frac{\pi}{2}\right],\ g'(\theta) \geq 0$$

  - In full precision case, MSE for $\theta \sim N(0, 1)$ and $W_i \sim N(0, \sigma_w^2)$ is σ_w^2
  - In genie-aided MSE for $\theta \sim N(0, 1)$ and $W_i \sim N(0, \sigma_w^2)$ equals to $\frac{\pi\sigma_w^2}{2}$

5. RESULTS

- Without adjusting sensor decision rule with channel
  - $PL = \frac{\left(\int_a^b p_\theta(\theta)^{\frac{1}{3}}d\theta\right)^3}{\pi^2}$, when $\int_a^b p_\theta(\theta)^{\frac{1}{3}}d\theta$ exists ,PL is achieved by $k_0(\theta) = \gamma_0(X) = \frac{1}{2}\left(1 - cos\left(\pi\frac{\int_a^\theta p_\theta(t)^{\frac{1}{3}}dt}{\int_a^b p_\theta(\theta)^{\frac{1}{3}}d\theta}\right)\right)$[1]
- With adjusting sensor decision rule with channel of error probability ρ
  - $PL = \frac{\left(\int_a^b p_\theta(\theta)^{\frac{1}{3}}d\theta\right)^3}{4[sin^{-1}(1-2\rho)]^2}$, when $\int_a^b p_\theta(\theta)^{\frac{1}{3}}d\theta$ exists ,PL is achieved by $k_0(\theta) = \gamma_0(X) = \frac{1+\left[\frac{2sin^{-1}(1-2\rho)\int_a^\theta p_\theta(t)^{\frac{1}{3}}dt}{\int_a^b p_\theta(\theta)^{\frac{1}{3}}d\theta} - sin^{-1}(1-2\rho)\right]}{2}$

6. SIMULATION RESULTS

-Prior: Gaussian $\theta \sim N(0, 1)$
-θ range: $(-3, 3)$

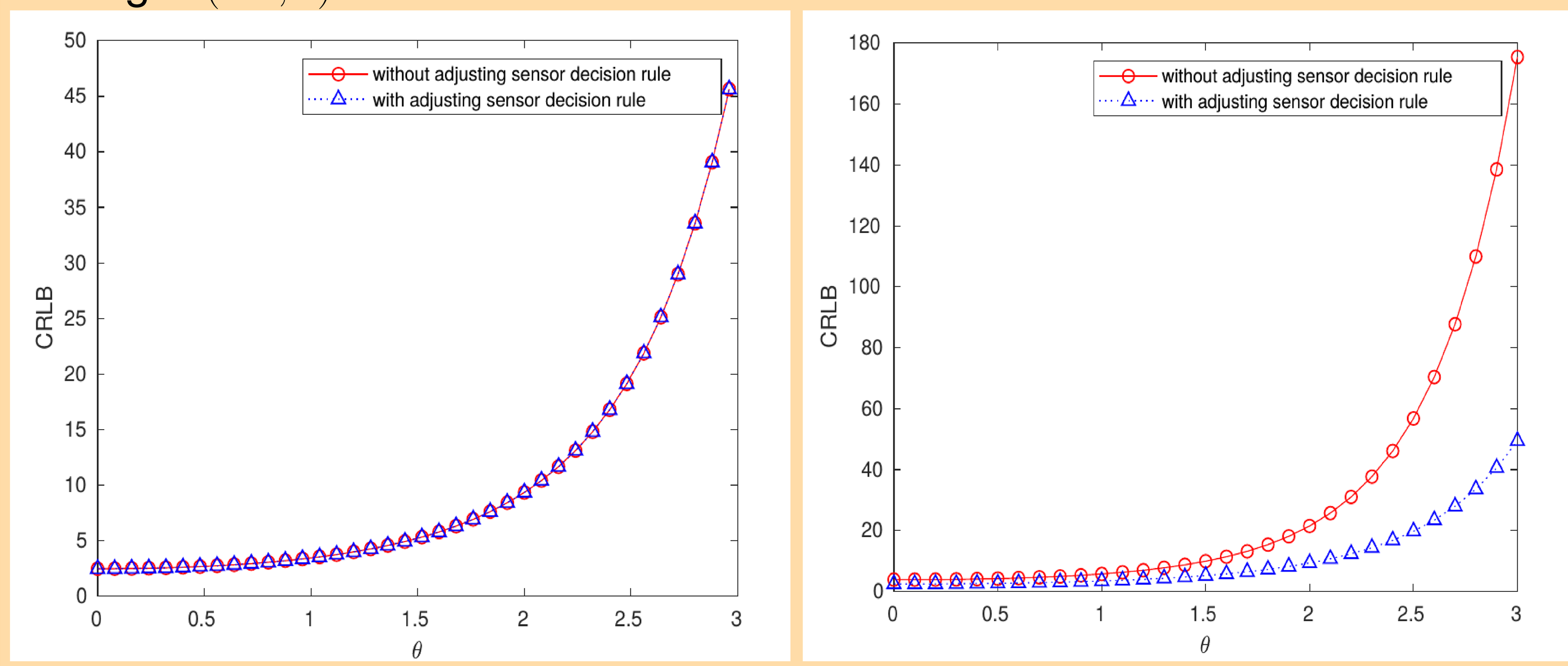


Figure 2: *Performance comparison under $\rho = 0$, and $\rho = 0.1$ channel settings with/without adjusting sensor decision rule*

7. CONCLUSION

- Bandwidth and power efficient parameter estimation in distributed system under the Bayesian criterion is considered
- Identified how system performance changes as a result of adjusting sensor decision rule under different channel settings in terms of CRLB

References

[1] Xia Li, Jun Guo, H. Chen and U. Rogers, "The performance limit for distributed Bayesian estimation with identical one-bit quantizers," 2015 IEEE Signal Processing and Signal Processing Education Workshop (SP/SPE), Salt Lake City, UT, 2015, pp. 19-24.

[2] H. Chen and P. Varshney, “Performance limit for distributed estimation systems with identical one-bit quantizers,” Signal Processing, IEEE Transactions on, vol. 58, no. 1, pp. 466–471, Jan 2010.

[3] H. L. Van Trees, Detection, estimation, and modulation theory. John Wiley & Sons, 2004.