2018 Graduate Student Showcase

Graduate Student Showcases

April 2018

Proportional Reasoning in Middle School

Katie Paulding
Boise State University

Proportional Reasoning in Middle School

Abstract

The ability to reason proportionally is a foundational concept for students to master while in middle school, leading to increased understanding and deeper conceptualization of higher level mathematics and science in the later grades (Heller et al., 1989; Johnson, 2015; Lesh, Post, & Behr, 1988; Lobato & Ellis, 2010; Ellis, 2013; Carney et al., 2015). Yet mathematics education lacks research-based teaching strategies on developing proportional reasoning skills in middle school students. This study seeks to determine the influence that specific ratio relationships and models (diagrams) have on student thinking regarding a given proportional reasoning problem and its solution via cognitive interviews with 29 middle school students in Idaho. Results may inform teaching practices on proportional reasoning.

BOISE STATE UNIVERSITY

Proportional Reasoning in Middle School

Katie Paulding

Education

I. Introduction

The Common Core State Standards Initiative in Mathematics (2010) place an increased emphasis on proportional reasoning in middle school (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). However, research has shown that many students struggle in their understanding and use of proportional reasoning. This research study addresses the influence of mathematical models and ratio relationships in students' ability to reason proportionally as follows:

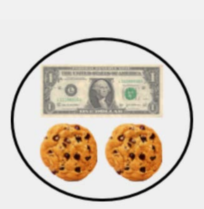
- How does changing the mathematical model influence student strategies and student conceptions?
- How does shifting the numerical quantities of the given ratio change both student strategies and student conceptions?
- In what ways does the given mathematical model or ratio relationship press students to specifically solve via composed unit or multiplicative comparison?

According to National Council of teachers of Mathematics (NCTM), a foundational component of proportional reasoning is one's ability to create a ratio by "attending to two quantities simultaneously" (Ellis, 2013, p. 1), either by forming a composed unit or creating a multiplicative comparison (Lobato & Ellis, 2010).

Sample Proportional Reasoning Task

"Bailey can buy 6 cookies for $3. How many cookies can she buy for $1? How much does 1 cookie cost? How much would it cost to buy 12 cookies?

Sample Composed Unit Solution



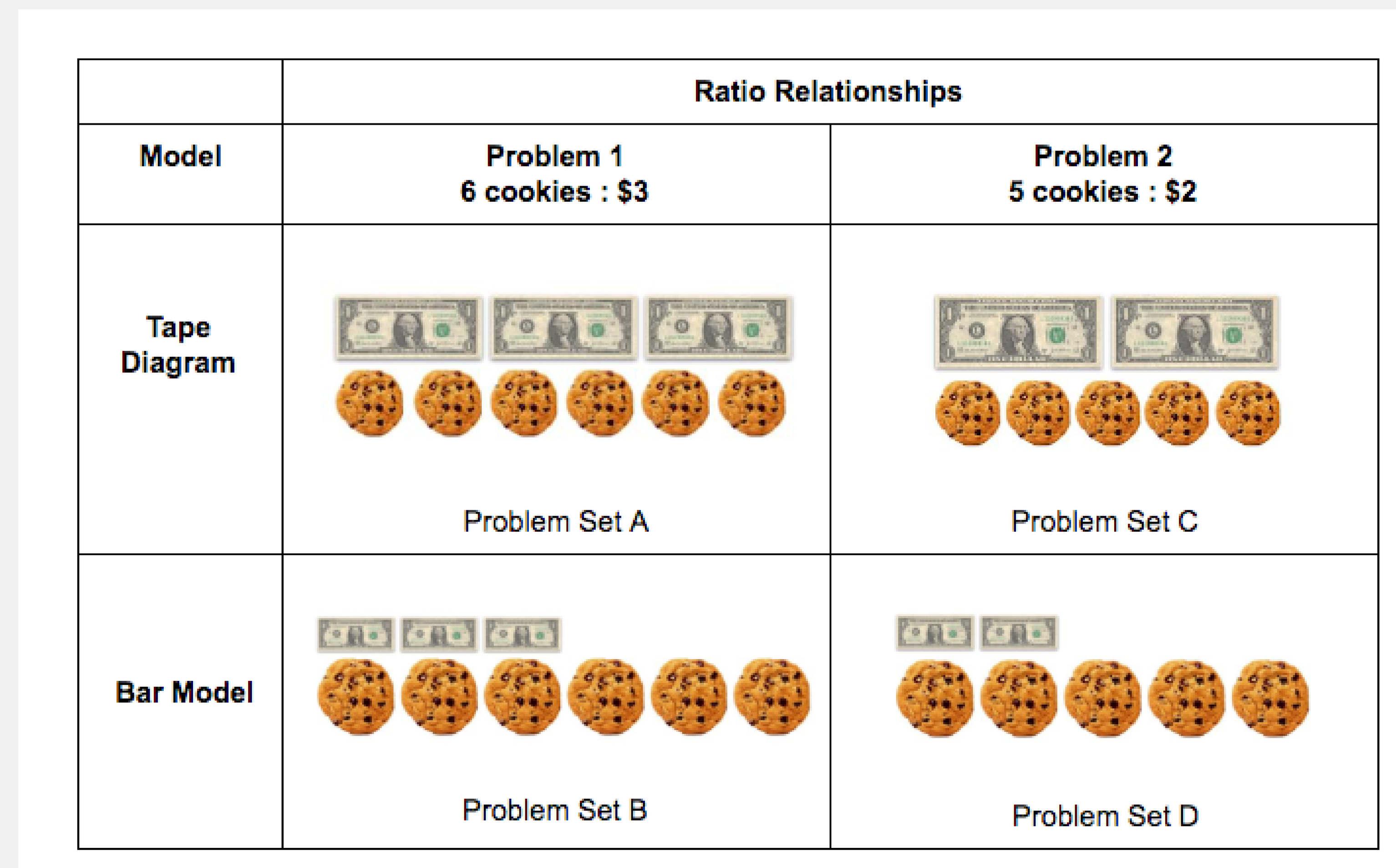
$$\frac{\$1}{2 \text{ cookies}} \times \frac{6}{6} = \frac{\$6}{12 \text{ cookies}}$$

Sample Multiplicative Comparison Solution

$$\frac{\$3}{6 \text{ cookies}} \times \frac{1}{2} \quad \text{therefore} \quad \frac{\$6}{12 \text{ cookies}} \times \frac{1}{2}$$

II. Methods

In Winter 2018, I conducted cognitive interviews with 29 middle school students in the state of Idaho. Students were asked the same six questions while being provided with one of four problem sets. The problem sets maintained a constant context (cookies and cost) but differed in the given mathematical model (tape diagram; bar model) and/or ratio relationship (6 cookies to $3; 5 cookies for $2) as the following graphic displays. I analyzed and coded student responses based upon their understanding of the context and statements of the relationship between the two given variables.



Questions	Sample Student Strategy	Sample Student Response
Q1: Given 6 cookies for $3 (or 5 cookies for $2) and this diagram, what can you tell me about the relationship between cookies and cost?	Multiplicative Comparison	"That if you were to **multiply each of the factors by two you would still have the same ratio**, for five to two, so if you had ten cookies, it would cost four dollars in that relationship. And then, divide by two, um, point five, I think, yeah, that would be a dollar for two point five of a cookie, so that would be the relationship between there." Interview 012008D52BM
Q2: How much does it cost for 1 cookie?	Unit Rate	"Fifty cents. Three goes into six twice, so I got, so two cookies, **so these two cookies would equal one dollar** [indicating first two cookies on the left of the diagram]…Each individual cookie, like this one [indicating last cookie on the right], would be fifty cents." Interview 011407B63BM
Q3: How many cookies can you buy for $1?	Composed Unit	"Two cookies? I just did two dollars, er, **two cookies take away and then that's one dollar out**, and then two cookies take away and that's one dollar out, and then I have one last dollar." Interview 012208B63BM
Q4: How much money would 12 cookies cost?	Scale Up by Doubling	"Six dollars, because you are **multiplying six by two**, which would equal twelve, that means **you are still multiplying the cost also by two**, which would equal six dollars." Interview 010906A63TD
Q5: How would you describe the relationship between the cookies and cost?	Multiplicative Comparison	"Each cookie is two point, you get two point five cookies, okay. **Two point five cookies times every dollar,** so it's [student writing "2.5 x" on paper] er, well, **this can just be N [student writing 2.5 x N on paper] cause it's the amount of dollars**." Interview 011507C52TD
Q6: Do you think a different diagram would help you interpret the relationship more easily? Would you mind drawing it for me?	Changes Bar Model into a Tape Diagram	"Okay, I would draw it, cause this is a little bit confusing cause they only have a dollar in front of three cookies, so that's what you would think when you see it, so **I would draw three dollars and then underneath it I would draw the six cookies. So that way you know that overall these cookies [indicating all six cookies] are three dollars.**" Interview 010608B63BM

III. Results

Mathematical Models

- The **bar model** aided more students in solving via multiplicative comparison than did the tape diagram.
- The **tape diagram** was more influential in aiding students to see dollar and cookie unit rates.

Ratio Relationships

- The **6 cookies : $3** ratio relationship influenced students to use the composed unit strategy and identify the dollar unit rate and cookie unit rate.
- The **5 cookies : $2** ratio relationship was more challenging as students struggled to identify a relationship between the variables. Students also found it easier to scale up to a larger ratio than scale down to the unit rates.

IV. Conclusions

While I gained valuable insight into the influence of bar models, tape diagrams, and ratio relationships in these tasks, more research is needed to confirm these findings.

Further research with larger sample size will enhance the validity and generalizability of these results.

I would also like to study the implications of the order sequence of my interview questions as well as the effects on a different population of students.

References

Ellis, A. (2013). Ratio and proportion research brief: Teaching ratio and proportion in the middle grades. National Council of Teachers of Mathematics.

Lobato, J., & Ellis, A. (2010). Developing essential understanding of ratios, proportions, & proportional reasoning in grades 6-8. Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards (mathematics)*. Washington D.C: National Governors Association Center for Best Practices, Council of Chief State School Officers.