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M&M's®, Sampling, and X-Bar/R Control Charts: A Demonstration and Validation

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ABSTRACT

Relaying statistical concepts to students is difficult without “real-world” examples and corresponding data. However, it is difficult to obtain data for student usage, let alone have the data mean something to the student. This paper attempts to provide students with a hands-on demonstration to aid in their understanding and application of control charts using the tangibility of M&M’s®. During the demonstration, students gain an understanding of how to perform sampling, evaluate basic statistics, and leverage tangible data to create x-bar & R charts. The demonstration’s effectiveness is then assessed using a post-hoc data analysis of student examination data to evaluate the demonstration’s effect on student performance.

Keywords: control charts, M&Ms, SPC, x-bar chart, R chart

INTRODUCTION

Statistical process control (SPC), first introduced by Shewhart (1926), provides the ability of a firm to monitor and control the production process. Many companies have leveraged SPC in the form of control charts, and in particular the x-bar and R charts. When used effectively, a firm has the ability to monitor a process during normal operations to determine if the process is “in control” or to determine if corrective action is needed (Sanders and Reid, 2013; Swink et al., 2017). As a result, control charts have become important cornerstones of quality initiatives, such as Six Sigma (Goh and Xie, 2003), and have been used in a variety of industries. Given the importance of the topic, it is necessary for students to leverage data, construct control charts, and interpret the results. This makes experiential learning a vital part of the learning process.

Experiential learning provides a “real-life,” dynamic, learning environment where students can learn concepts and apply them in a controlled manner. Of note, it has been demonstrated that experiential learning through classroom demonstration can assist student learning and application of statistical process control (SPC) – in particular, control chart concepts (e.g. Fish, 2009; Fish and Braunscheidel, 2012). To this end, there have been several teaching activities that have used candy, such as M&M’s®, in demonstrations that have explained the construction of c-charts (Fish, 2009; Fish and Braunscheidel, 2012) and provided process capability demonstrations (Lembke, 2016). However, there has been a limitation in activities that directly address the construction of x-bar and R charts in the literature (e.g., Fish, 2007; Hill and Schvaneveldt, 2011), and the associated validation of the demonstrations’ impact on student performance. The purpose of this paper is to provide instructors a method for incorporating an inexpensive, experiential-learning activity/demonstration to enhance student learning of x-bar and R control charts; and then assess the effectiveness of the demonstration using a statistical analysis using examination data.

Next, the paper will address what has been previously done in the literature to demonstrate the construction of control charts. This will be followed by the design of the classroom demonstration of using M&Ms® to construct control charts; which will also provide a justification of why M&Ms® are a reasonable, normally-distributed product appropriate for constructing x-bar & R charts. Finally, an analysis will be presented to show that participation in the M&M® control chart demonstration improved student performance on x-bar & R chart exam questions.

PREVIOUS RESEARCH

Several authors have constructed demonstrations and activities that utilize x-bar & R charts. Rada and Hu (2002) demonstrated the application of evaluating student-student comments utilizing x-bar & R charts to detect variation in student responses. Coy (2016) evaluated the steps of plan, do, check, act (PDCA) with students constructing x-bar & R charts to evaluate the length of time required to construct paper squares. Additionally, several authors have supplemented their class projects of building catapults (Mitchell et al., 2013) or paper helicopters (Johnson, 2011) to explore the value of x-bar and R charts. However, in many of these activities, multiple class periods or time outside of class are required for students to fully learn the SPC concepts and the control charts are used to supplement another activity.

Fish (2007) provides a method of direct instruction that is intended to explicitly teach x-bar and R chart construction – using string length. However, their instruction is not validated with data and limits the number of participants. The demonstration presented in this paper provides a modification that allows for complete classroom participation while additionally assessing the impact on student exam performance. The demonstration presented in this paper does the following: 1) provide a low cost, interactive demonstration toward the construction of x-bar & R charts and 2) evaluates the effectiveness of the demonstration through the usage of exam data.

DEMONSTRATION DESIGN

The normality of fun-sized M&M's®

This demonstration requires the usage of an individually packaged snack item. Preferably uniform in nature; this demonstration used "fun-sized" packets of M&M's®. Given that there will be a variation in the number of M&M's® present in the fun-sized bag, the instructor started by sampling the fun-sized M&M® packets to estimate the mean and standard deviation for the population. At the time of this paper being submitted, the mean (μ) was estimated to be roughly 15.25 M&M's® per fun-sized package with a standard deviation (σ) of 0.96. The population estimation is best done by purchasing a set of fun-sized M&M's®, counting the number of M&M's® in each fun-sized packet, and then calculating the mean and the standard deviation. While the counting of M&M's® is generally discrete (i.e., associated with the Poisson distribution), many fun-size packets of M&M's® contain broken or malformed M&M's®. This allows students to estimate partial numbers of M&M's® and obtain an approximated, continuous measurement of the "count." However, it is important to inform students that traditional forms of continuous measurement (e.g., weight, mass, length) would be better suited for x-bar and R charts.

Given that x-bar & R charts assume a normal (i.e., Gaussian) distribution, the normality of the M&Ms® population contained in a fun-sized packet was estimated by sampling 128 fun-sized packets. Based upon the Shapiro-Francis ($p = 0.82$) and Shapiro-Wilk ($p = 0.42$) tests, the assumption of normality cannot be rejected. Additionally, skewness was not a concern with the population. The assumption of normality is also supported by Lembke (2016), who demonstrated in their activity that the weight varies with each fun-sized packet, the weight of the fun-sized packets appears to be normal, and the M&M's® themselves vary in mass.

Classroom Demonstration

Prior to the class demonstration, the class received instruction on the basics of SPC via a traditional lecture. Based upon their required coursework, they were additionally familiar with the concept of a statistical distribution, how to calculate the mean and standard deviation of the distribution, and how to calculate the sample standard deviation from the population standard deviation. For the purposes of this demonstration, the instructor relied on the texts of Sanders and Reid (2013) and Swink et al. (2017) for the basis of the control chart formulations.

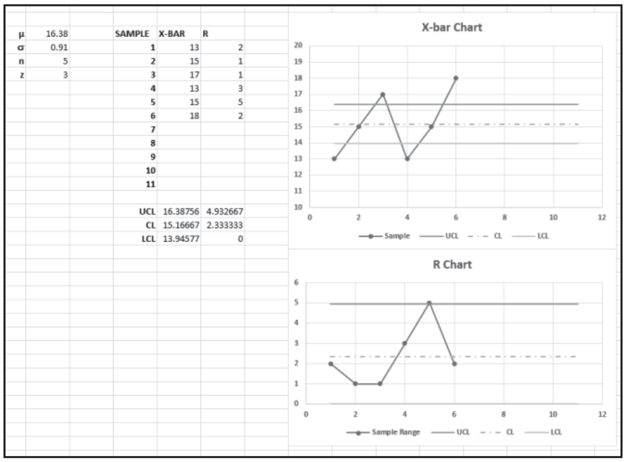
Each student is given a packet of M&M's® and assigned to a group. Each group is comprised of five students. Students were informed that malformed and partial M&M's® should be estimated as a fractional/decimal amount. Each group will be a sample (i) of five packages (n). The student will count the whole and fractional number of M&M's® within their packet ($\mathrm{x_n}$) and then calculate the sample mean ($\mathrm{\bar{x}_i}$) with their group – see equation (1). Additionally, each group will also determine their sample range ($\mathrm{R_i}$) – see equation (2).

$$\mathrm{\bar{x}_i = \frac{\Sigma(x_n)}{n}} \tag{1}$$

$$\mathrm{R_i = max\{x_n\} - min\{x_n\}} \tag{2}$$

When they have calculated the sample mean and range, they will enter their values on a predesigned spreadsheet (or comparable medium) that is available for the class to see via an overhead projector (See Figure 1).

Figure 1: Example of spreadsheet used by the classes



μ	16.38
σ	0.91
n	5
z	3

SAMPLE	X-BAR	R
1	13	2
2	15	1
3	17	1
4	13	3
5	15	5
6	18	2
7		
8		
9		
10		
11		
UCL	16.38756	4.932667
CL	15.16667	2.333333
LCL	13.94577	0

Once all groups (k) have reported their sample means and ranges, have all the students calculate the grand mean ($\bar{\bar{x}}$) and the average sample range ($\bar{R}$). These will be the center lines (CL) for their control charts:

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k} \bar{x}_i}{k} \tag{3}$$

$$\bar{R} = \frac{\sum_{i=1}^{k} R_i}{k} \tag{4}$$

Utilizing the spreadsheet in Figure 1, the answers can be hidden from students until they have calculated grand mean and the average range. Validate that they have properly calculated the grand mean and the average sample range. Next students construct their control charts. The students will calculate the upper control limit (UCL) and the lower control limit (LCL) for the x-bar chart using three sample standard deviations (σ_x); however, this could be substituted by using A_2 from the control chart constants along with the average range ($\bar{R}$).

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{5}$$

$$UCL_{\bar{\bar{x}}} = \bar{\bar{x}} + 3\sigma_{\bar{x}} \cong \bar{\bar{x}} + A_2\bar{R} \tag{6}$$

$$LCL_{\bar{\bar{x}}} = \bar{\bar{x}} - 3\sigma_{\bar{x}} \cong \bar{\bar{x}} - A_2\bar{R} \tag{7}$$

Similarly, the students will construct the R-bar chart control limits using the table of control chart constants D_3 and D_4 [see equations (8) and (9)].

$$UCL_{\bar{R}} = D_4\bar{R} \tag{8}$$

$$LCL_{\bar{R}} = D_3\bar{R} \tag{9}$$

The students should then be instructed to build the upper and lower control chart limits and draw the control charts. For this demonstration, the classroom had white boards along the walls of the classroom allowing students to build up their control charts publicly. This allowed the instructor and their fellow students could see what they had done.

RESEARCH DESIGN AND ANALYSIS

In past there has been a lack of analysis evaluating whether x-bar & R chart demonstrations/activities have an effect on student performance. This presented the following, simple research question: does participation in the M&M® control chart demonstration result in higher exam performance than nonparticipation? To evaluate this research question, data were collected and analyzed using multiple semesters of exam data to determine if there is a greater likelihood of students correctly answering x-bar & R charts questions.

Data Collection

The student data were collected over five semesters. All sections were given a lecture on the construction of control charts and how to assess process capability. However, unlike the first three semesters of instruction, the demonstration was incorporated into the lecture for the latter two semesters. The outcome data were then collected from each section's examinations post-hoc. This resulted in a total of 477 students with an average class size of 53 students. Given the prerequisites of the course, the students were juniors and seniors that were admitted to the business college. Based upon the students answering of the control chart questions on their second exam, 462 students answered the x-bar chart question, while only 459 students answered the R chart question. Correct answers were coded as "1" and incorrect answers were coded as "0." Table 1 shows a breakdown of students (as a percentage) based upon their participation in the activity and their respective performance on x-bar and R chart questions.

Table 1: Comparison of M&M® demonstration participation and percent correctly answering exam questions

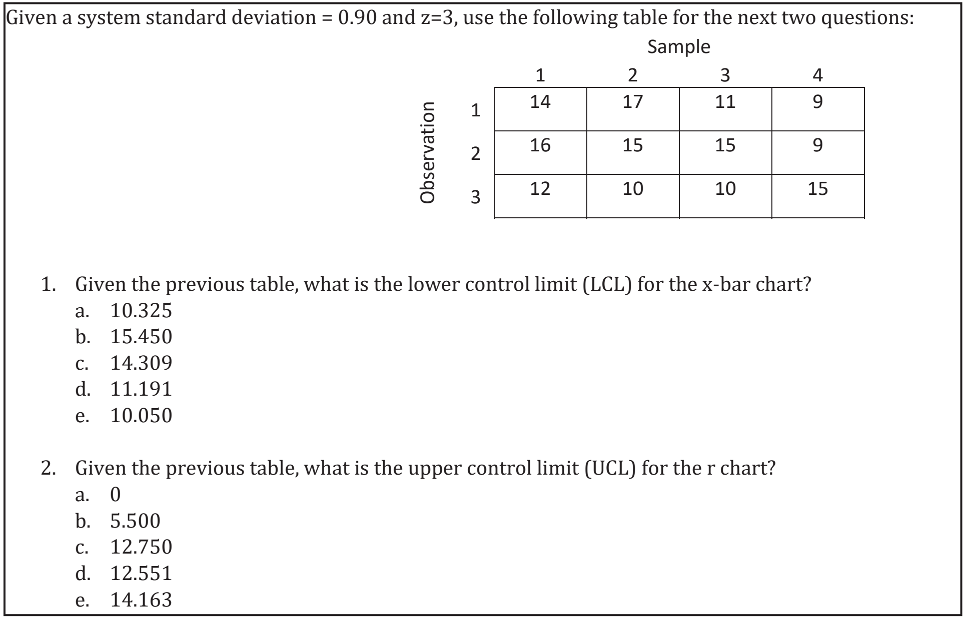
	% Correct on x-bar chart	% Correct on R chart
Class with no M&M (MM = 0)	33.66 (n = 305)	19.93 (n = 305)
Class with M&M (MM = 1)	46.50 (n = 157)	48.70 (n = 154)

Variables and Controls

Independent variable: The variable MM was coded as "1" for those students who participated in the M&M's® demonstration and coded as "0" for those students who did not participate in the M&M's® demonstration. This acted as the treatment for the analysis.

Dependent variables: Each semester students were asked similar questions on their exams – with varying numbers – regarding the x-bar (x-bar Chart) and R (R Chart) charts – see Figure 2. Their propensity to answer each question correctly was extracted from their Scantron exam reports and coded as a "1" if they answer the corresponding question correctly, and "0" if they did not.

Figure 2: Sample exam questions given to students each semester



Given a system standard deviation = 0.90 and z=3, use the following table for the next two questions:

		Sample			
		1	2	3	4
Observation	1	14	17	11	9
	2	16	15	15	9
	3	12	10	10	15

1. Given the previous table, what is the lower control limit (LCL) for the x-bar chart?
    a. 10.325
    b. 15.450
    c. 14.309
    d. 11.191
    e. 10.050

2. Given the previous table, what is the upper control limit (UCL) for the r chart?
    a. 0
    b. 5.500
    c. 12.750
    d. 12.551
    e. 14.163

Control variables: given that these data were collected over multiple semesters, semester (Semester) was controlled for starting with "1" = fall semester 2016 and adding a subsequent number for each semester taught thereafter until "5" = fall semester 2018; summers semesters were not taught, and thus not included in the count. The sex (Female) of the student was also controlled for with female students coded as "1" and male students coded as "0." This was done since the literature indicates male and female students may have differing levels of performance during participation-based, math activities (Peterson and Fennema, 1985). Additionally, Riegle-Crumb and Humphries (2012) identify that within math instruction there is the potential for bias towards female students, even if implicit or unintended. Finally, since past student performance is typically an indicator of future performance, the exam grade (Prior Exam Score) from their first exam was used as a control.

Data Analysis & Results

Given the dependent variables (x-bar Chart and R Chart) are binary in nature (i.e., incorrect/correct), the data were analyzed using a logit regression. Logit regression is preferred over a linear probability model (i.e., linear regression) since the analysis specifically accounts for the binary outcome (Wooldridge, 2015). Additionally, since the data were collected over multiple semesters, and also differing sets of students, the data were considered a pooled cross-sectional data. To account for the pooled cross section, the time variable (i.e., Semester) was included in the logit regression to account for any changes in the outcome due to time (Wooldridge, 2015).

Table 2 shows the results of the logit regression. As shown in Table 2, there is a positive and significant effect of MM on students correctly answering the x-bar ($b = 1.292$, $p < 0.01$) and the R ($b = 1.686$, $p < 0.001$) chart questions. By calculating the odds ratios (O/R) from the logit regression coefficients, it is shown that participants in the M&M® demonstration were, on average, much more likely to correctly answer the x-bar question (O/R = 3.640, 95% CI = [1.64, 8.09]) and the R chart question (O/R = 5.396, 95% CI = [2.21,13.16]). Based upon the odds ratio, students that participate in the demonstration were, on average, 3.64 times more likely to answer the x-bar chart question correctly, and 5.40 times more likely to answer the R chart question correctly. This demonstrates that there is positive and significant effect on student exam question performance when students participated in the M&M® control chart demonstration.

Table 2: Logit regression results of M&M's® demonstration on x-bar and R chart questions

	x-bar Chart (Model 1)	R Chart (Model 2)
Female	-0.061	0.319
	(0.203)	(0.222)
Semester	-0.267	-0.073
	(0.150)	(0.174)
Prior Exam Score	0.586***	0.646***
	(0.119)	(0.130)
MM	1.292**	1.686***
	(0.407)	(0.455)
Constant	-0.222	-1.520***
	(0.333)	(0.398)
AIC	580.642	497.825
N	462	459
Psuedo R^2	0.069	0.122

Two-tailed test; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

DISCUSSION

The M&M's® x-bar & R chart demonstration is a hands-on, experiential exercise that allows for a tangible example in the construction of control charts. The result of the analysis shows a positive and significant relationship between the usage of the demonstration and the propensity for a student to correctly answer exam questions related to the x-bar & R control charts. This demonstration also expands the work of Fish (2009) allowing for instructors to utilized fun-sized packets of M&M's® for both x-bar/R charts in addition to the construction of c-charts. Furthermore, with the analysis showing that participation in the M&M® control chart demonstration is associated with higher exam question performance, there is likely a similar benefit to other activities also constructing x-bar and R charts, such as those presented by Fish (2007) and Hill and Schvaneveldt (2011).

Limitations

The greatest issue when attempting to generate data for x-bar and R charts is the need for continuous data. While fun-sized packets of M&Ms® provide malformed and broken pieces, M&Ms® are generally viewed to be discrete. Fish (2007) attempted to create an opportunity to construct control charts with string lengths. This ultimately requires the instructor to artificially create data and use a medium that may not stimulate the students in the activity. Additionally, Hill and Schvaneveldt's (2011) usage of baseball statistics leverages real data, but the data might intimidate students. However, these more continuous mediums (i.e., string and baseball), might provide students a better understanding of when it is appropriate to use x-bar and R charts. To remedy the issue with count data in this demonstration, it is recommended to use scales to measure the mass of the M&Ms® packets (cf., Lembke, 2016). This would allow for a continuous measurement of the M&Ms® packets while minimizing the change to the demonstration.

While the effect size of the M&M's® demonstration was lower for the x-bar question, further exploration of the exam questions was performed. It was noted that several students used the population standard deviation (σ), instead of the sample standard deviation ($\sigma_{\bar{x}}$), when responding to the x-bar chart question. This may account for the lower impact the M&M's® demonstration had on that particular outcome. However, this difference could also be explained by the difference in the populations. For example, nonparticipants generally scored lower in the R chart question. The larger effect size may instead be the result of the gap between participants and nonparticipants being larger for the R chart question.

Finally, given that the pseudo r-squared is low, there is the potential for additional student activities or supplements (e.g., homework) to be assessed in conjunction with the demonstration. It also further exemplifies the difficulty in achieving student success in the classroom even in the presence of class activities and demonstrations.

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