4-21-2014

Bose-Einstein Condensation in a Double-Well System

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BOSE-EINSTEIN CONDENSATION IN A DOUBLE WELL SYSTEM

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TRiO
RONALD E. McNAIR POST-BACCALAUREATE ACHIEVEMENT PROGRAM

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ABSTRACT

The two-site Bose-Hubbard model is a simplified model of ***N*** bosons confined to two quantum potential wells (Double Quantum Well, **DQW**s). We study systematically the behavior of low-temperature Bose-Einstein condensates(BECs) as a function of the strength of the repulsive interactions (***U***) between the bosons and the amount of quantum-mechanical tunneling between the wells (***t***), employing both analytical and computational methods.

In particular, we study the transition between a single BEC comprised of bosons in both wells, and a fragmented condensate consisting of two distinct BECs, one for each well.

Bose-Einstein Condensates

A Bose-Einstein condensate (BEC) is a "state of matter" associated with quantum behavior which manifests macroscopically:

Very nearly all the particles occupy the same quantum mechanical state, producing a "quantum condensed" system with many strange properties (superfluidity or superconductivity, for example).

We focus on the macroscopic quantum properties of the system near the absolute zero of temperature (experimental BEC systems have been produced with temperatures below one-millionth of a degree Kelvin above absolute zero)**.**

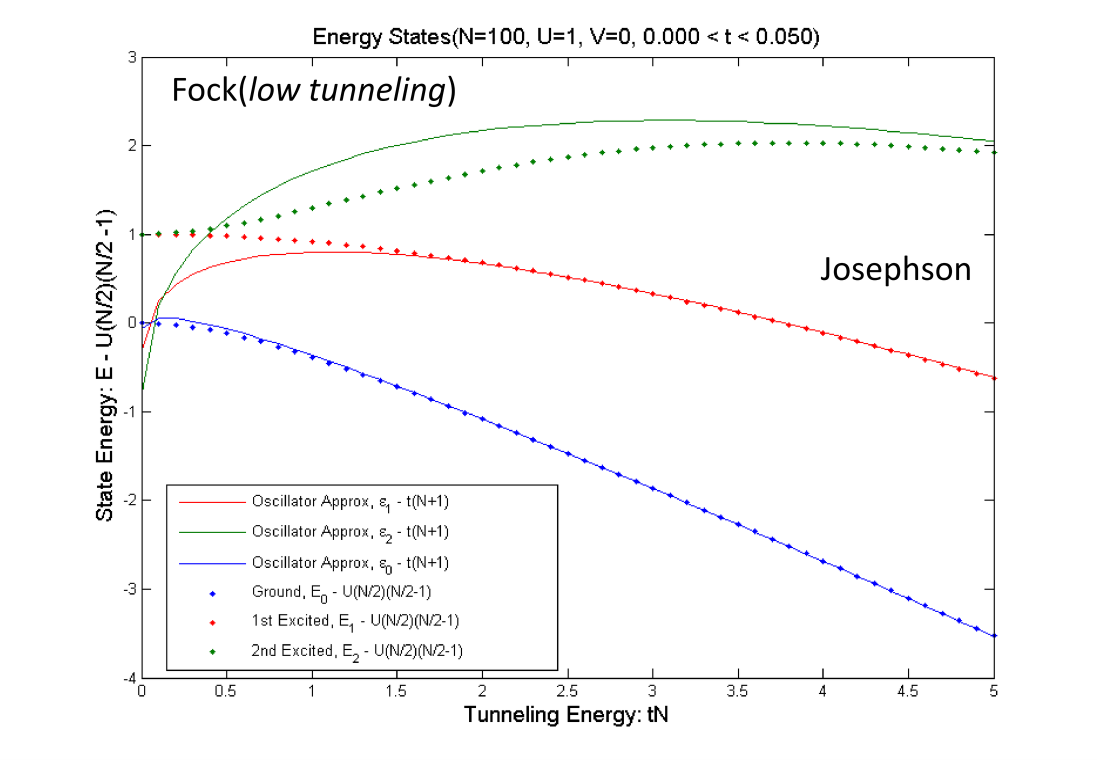

Fig. 1

Fig. 1 compares numerically calculated energy values from a Hamiltonian matrix solution with a quantum harmonic oscillator (QHO) approximation that improves as *tN* increases. Note that the first and second excited energy states are **degenerate** (have the same energy) for systems in the extreme Fock regime (*t*=0).

Fig. 2

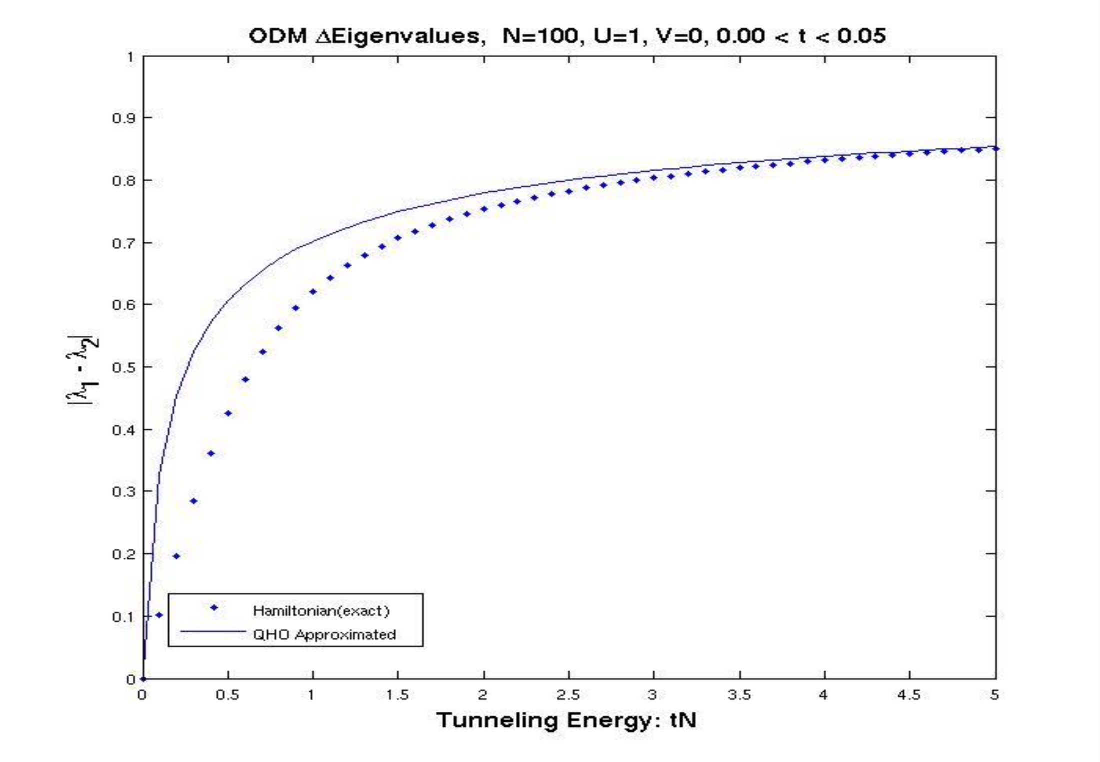

Figure 2 shows the degree of BEC (computed as the difference of the eigenvalues of the One-Particle Density Matrix); it would be 0 for fragmented condensates and 1 for a singly condensed systems.

Fig. 3

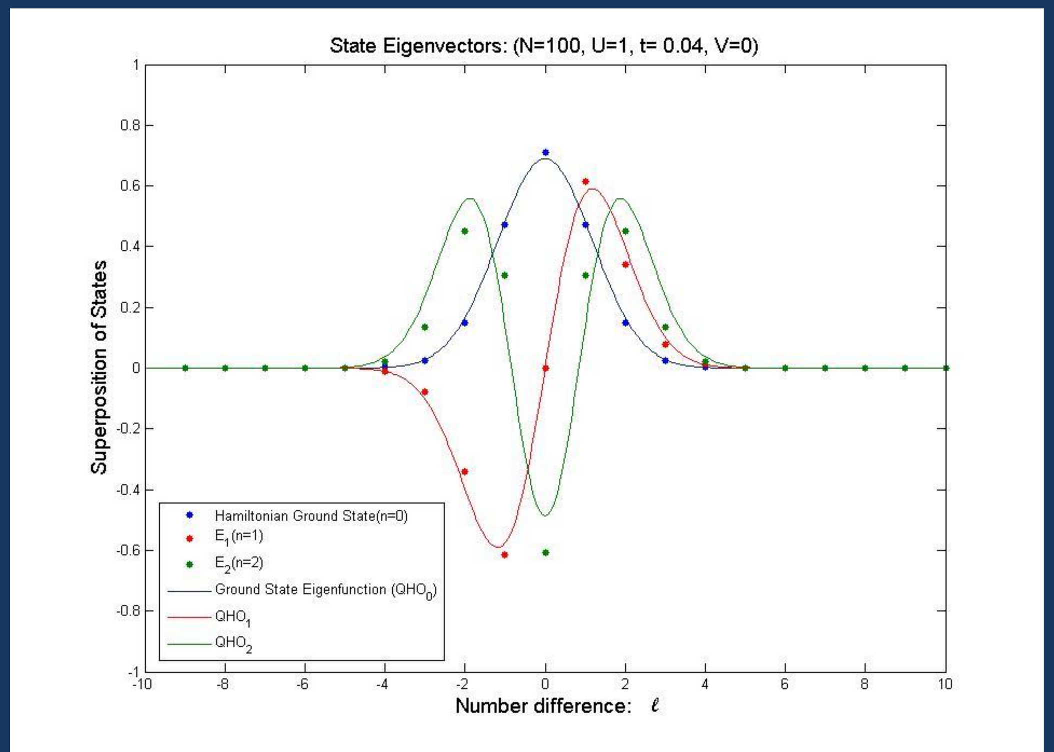

Figure 3 shows the first three energy eigenstates: the ground through 2nd excited state for *tN*=4. A peak occurs at the ground state for $\ell = 0$; an even distribution of particles in each well is most probable.

Two-Site Bose-Hubbard Model

$$\hat{\mathcal{H}} = -t(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) - \frac{V}{2}(\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2) + \frac{U}{2}\sum_{j=1}^{2} \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j$$

Our analysis diagonalizes the Hamiltonian matrix to solve the Schrödinger equation for the DQW, obtaining the energies and states of the system.

- Our study assumes a **repulsive** inter-particle potential (*U*>0).
- We typically generate results for the balanced case, without a bias or voltage (*V*=0).

For a fixed number of particles, *N*, we study particles in various regimes: interested in systems in the **Fock** (*tN/2 << U)*, **Josephson** (*2U/N << t << UN/2*), and **Rabi** (*U << 2t/N*)

Preliminary Results, Future Investigations

We observe that even very small amounts of tunneling lead to coherence (entanglement) between wells, producing a single condensate from a fragmented one. Future results will quantify this effect more carefully as a function of the tunneling.

In the future we will use the DQW to examine the amount of tunneling necessary to produce interlayer coherence in more complex systems, such as bilayer quantum Hall systems.

Acknowledgements
Boise State University McNair Scholars Program
NSF grant DMR-0906618