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Economics and Game Theory

Jeremiah Patrick Prenn
Boise State University

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Game Theory and Nash Equilibrium

A game has players, outcomes, and strategies. The question that games raise in the world of both math and economics is how best to define the players, the moves, and the strategies. Games are a fundamental part of economics. The science of economics is often said to be the study of how people allocate scarce resources to suit various needs. Economics can also be thought of as the study of many games being played at once, and how the strategies of players affect the outcome. A useful concept in the realm of economics is "utility." Utility is a quantitative measure of how much use an economic agent gets out of the results of any particular decision. Games can arise when multiple players have conflicting wants, and when both seek to maximize their utility. Nash equilibrium is a state where no players in a game can expect to get a higher utility by changing their strategies so long as the strategies of the other players do not change. Below are several areas in which game theory is applied to the science of economics.

Duopoly Theory

A duopoly occurs when only two firms sell a product and control its price. This case is opposed to the case of perfect competition, in which innumerable firms offer an identical product whose price is determined by the average total cost of the most cost-efficient producer. A duopoly presents an interesting case of game theory, as the actions of each firm affect the welfare of the other firm.

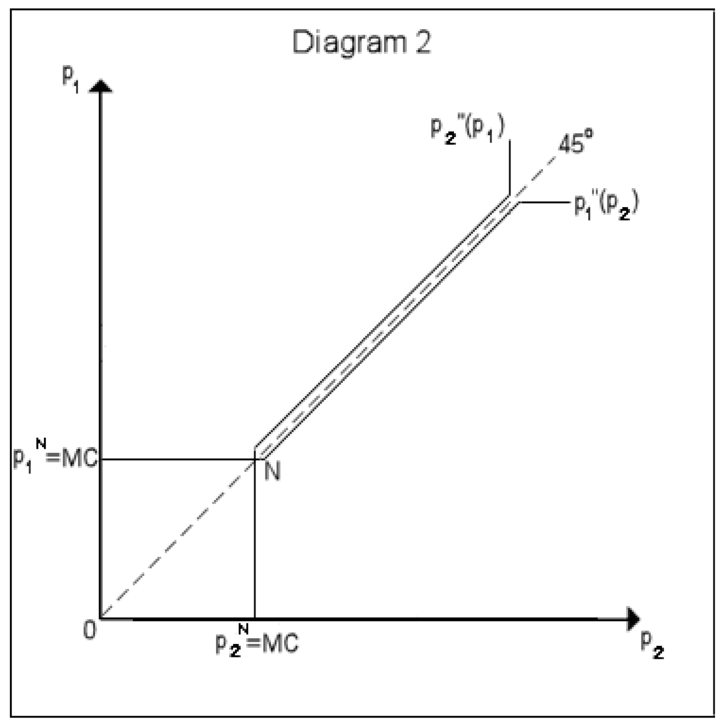

Fig. 1: Bertrand Price Competition

Duopolies can compete in quantity or price. Cournot quantity competition occurs when the firms change the quantity they sell based on the quantity sold by the other firm. Each firm has a reaction function whose independent variable is the quantity sold by the other firm and whose dependent variable is their own quantity sold. Where these functions intersect determines the equilibrium quantity each firm will produce.

Bertrand price competition occurs when one firm gains all market demand by undercutting the price of the other firm. The other firm will respond by undercutting the first firm. This will eventually lead to both firms selling at a price equal to their marginal costs.

Ordering of Preferences, Voting, and Organizations

Given three preferences labeled A, B, C and three or more voters who rank these preferences in different orders, it is impossible to determine the best overall preference for all three voters when preferences are voted on two-by-two. If person a has preference ranking $A > B > C$ and b has preference ranking $B > C > A$ and c has preference ranking $C > A > B$, then in a vote between A or B, A wins. In a vote between A or C, C wins. In a vote between B or C, B wins. This violates the principal of transitivity: A is preferred to B is preferred to C is preferred to A. This means that it is impossible to rank preferences on a society-wide level when voting two-by-two.

A cartel consists of members who agree to output a certain amount of a good at a certain price, in order to receive as much profit as a monopoly would. Each member of the cartel has an incentive to "cheat". Any member supplying the product at a lower price than the other members would reap all the profits. The incentives to break the agreement are so high that penalties must be put in place to ensure members do not "cheat."

Producers are much more easily organized into a coalition than consumers are. The ratio of producers to consumers of a good is such that the costs of organization are higher for consumers than for producers. It's extremely rare to see a "union" of consumers whose objective would be to demand a lower price of a good, because the size of this union would be so large that each individual member will make the choice to behave as if he wasn't in the union, trusting the other members to take up the cause of lowering the price of a good. This is one example of what is known as the "free-rider problem."

Pareto Optimality

Pareto Optimality is a condition in which among a set of moves from a given position, there is no move that will increase the welfare of one player without decreasing the welfare of another player. A set of possible outcomes is called a *Pareto Region*. In his paper *The Relevance of Pareto Optimality*, James M. Buchanan shows that the set of possible Pareto optimal outcomes is dependent on the "rules" applied to the Pareto Region in question.

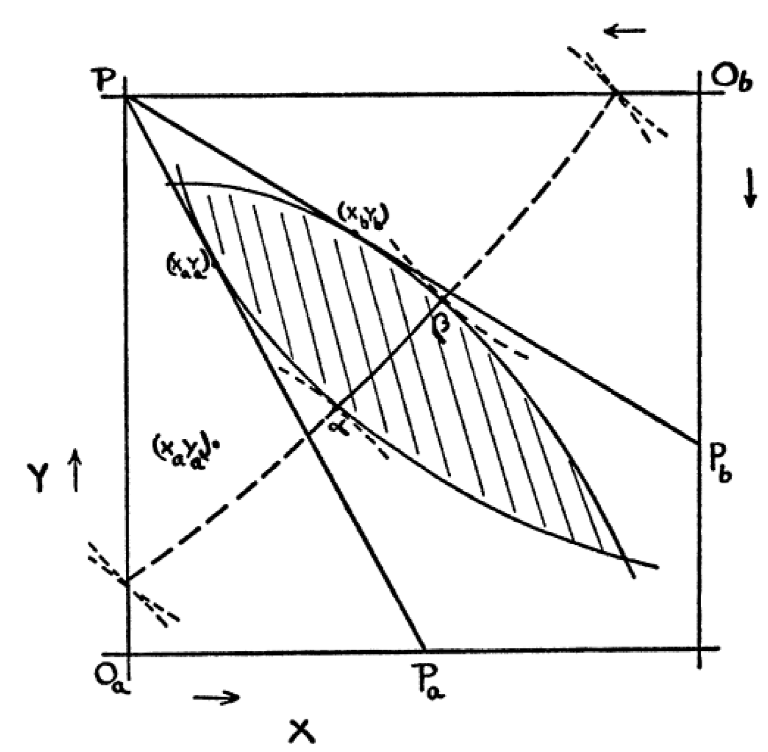

Fig. 1. Pareto point-sets under varying rules.

Unanimity in rule changes will result in a limited Pareto region, where the optimal outcomes have meaning. Changes in the rules defining a Pareto region can be optimal or not.

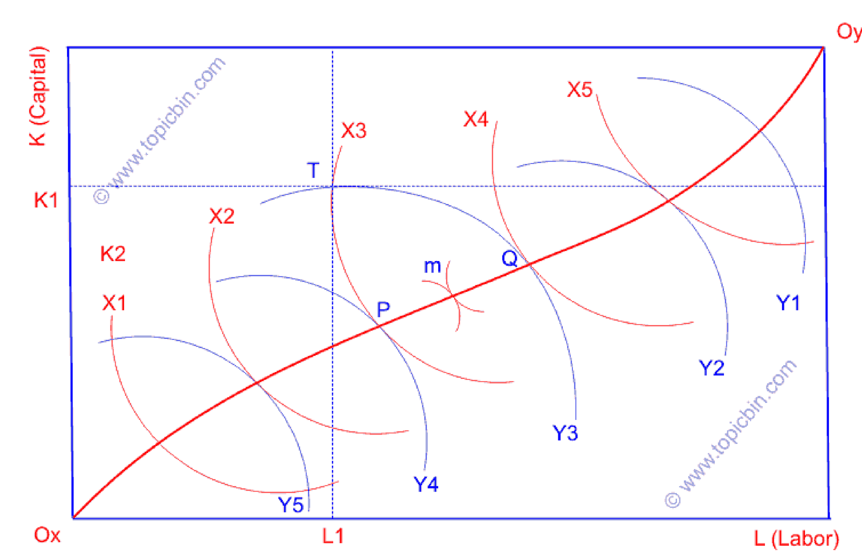

Fig. 3: Two Firms and Pareto Optimality

One instance occurs when two firms A and B share a pool of fixed amounts of labor L and capital K between them. The amount of a resource used by one firm is equal to the total minus the amount used by the other firm. Assume that each firm wants to produce as much output as possible. Each firm has several isoquants, all combinations of labor and capital to produce a given level of output.

Each firm seeks to position themselves opposite their origin. Given any pre-determined allocation of labor and capital to A and B, the set of all possible Pareto Optimal outcomes lies on the line whose points are the tangential intersections of all pairs of opposing isoquants. At any point on this line, A cannot move to a higher isoquant without B moving to a lower isoquant, and vice versa. However, at a point not on this line, a firm can move to the line and produce the same output while increasing the output of the other firm. The diagram above is known as an Edgeworth Box.

Henry S. Farber's Theory of Final Offer Arbitration

In his paper An Analysis of Final-Offer Arbitration, Henry S. Farber created a mathematical model that outlined the strategies of two parties in submitting a final offer to an arbitrator during a contract dispute. Final-offer arbitration (FOA) occurs when each of the two parties submits a final offer to the arbitrator, and the arbitrator chooses which offer will be enforced. The arbitrator must choose one or the other offer and enforce it. The arbitrator knows what is "fair" and will choose the offer that is closest to what he considers to be "fair." Each party's strategy will involve guessing what the arbitrator's sense of "fair" is (labeled y_f) and making a final offer that is closer than the other party's final offer to y_f while maximizing their utility. Suppose the dispute concerns the distribution of some good between party A and party B. y is the amount A receives and 1-y is the amount B receives. y_A is the offer of party A and y_B is the offer of party B. If we assume $y_A > y_B$, then the probability that B's offer is enforced is $Pr(chB) = F(\frac{y_A+y_B}{2})$, where F denotes the cumulative distribution function of the random variable y_f. If the average of both offers is larger than y_f, then B's offer will be enforced, and if the average of both offers is smaller than y_f, then A's offer will be enforced. The expected utility of each party is

$$E(U_A) = [1 - F(\frac{y_A + y_B}{2})]U_A(y_A) + F(\frac{y_A + y_B}{2})U_A(y_B)$$

and

$$E(U_B) = [1 - F(\frac{y_A + y_B}{2})]U_B(1 - y_A) + F(\frac{y_A + y_B}{2})U_B(1 - y_B)$$

."Given that the parties are manipulating their respective final offers so as to maximize their respective expected utilities, the Nash equilibrium set of final offers is that pair of final offers which has the property that neither party can achieve a higher expected utility by changing its final offer."

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