Economic Growth under Weak and Strong Sustainability Scenarios

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Abstract

Romer (1990) introduced endogenous growth by placing emphasis on human capital as a major driver of innovation. Barbier (1999) expanded this framework to include a single exhaustible resource, natural capital. These developments added to the discussion of economic growth, innovation, and the production process. This project extends Romer and Barbier’s endogenous growth framework to include natural resource utilization in the consumption function in order to open the discussion to weak and strong sustainability. This project compares the growth rates, steady state, and dynamics across these models so that we more effectively capture non-linearities and feedback loops. Finally, we explore the question of whether a weak or strong sustainability viewpoint in an endogenous growth framework leads to economic collapse or prosperity. Keywords: technological change, endogenous economic growth, natural resources, sustainable development, optimal control

Global damage and irreversible effects from climate change motivate the development of sustainable approaches to economic growth. Weak sustainability allows for limitless substitution between types of capital (e.g., natural to manufactured) while strong sustainability requires avoiding crossing planetary boundaries. The use of non-renewable resources has increasing costs due to depletion and pollution. This has significant effects on the total value of natural capital and climate change. The risk is very high in allowing the non-renewable resource base to deplete since natural capital is essential to the production process.

In this project’s model, natural capital is central in production and innovation. New technologies, while costly to implement as resources become scarce, are essential as substitutes for non-renewables in the long run as we face uncertainty from global change. We examine the implications of various natural resource scenarios in the endogenous economic growth framework. Sustainable approaches and economic growth models are critical to our current economic situation because this gives us a holistic approach to understanding the interactions between economies, environmental systems, and social structures.

Despite accepted conventional wisdom that we can grow our GDP without bound, the research clearly shows that there is threat for environmental and/or social collapse if we continue things as they are. If we want prosperity or survival in the future, then we have to consider the role of the environment in economic growth models. This article will discuss ways to implement natural resources, endogenous technological change, and population scenarios in macroeconomic growth models.

This project offers a model which will account for the theoretical developments of technological change and environmental resources while being underpinned by sustainability considerations. Although these frameworks have been considered extensively, as well as variations within them, comprehensive models/frameworks have not been fine-tuned to explicitly allow for full interactions between these systems. The goal of this project is to (1) synthesize these frameworks and explore their implications, (2) set up and solve the model under various conditions, and (3) showcase the differences between results of the models. This will be concluded with possible extensions and considerations to this model.

Literature Review

Economies are dynamic and evolving. Political, social, financial, organizational, and environmental landscapes have to constantly adjust for change within and between these systems. A sustainable perspective allows us to think about how economic systems interact on a broader scale in the era of the Anthropocene. Innovations have
attributed to productive expansion, increase in wellbeing, and wealth of knowledge. However, technological advances have expanded our ability to consume faster than some natural processes can replenish.

Creating useful, realistic, and tractable models of economic growth and change has been the goal of economists, such as David Ricardo and Thomas Malthus (1809), who were often given credit for laying the foundations for such models, for over 100 years. These foundations were made more concrete and unified with frameworks presented by Ramsey (1928) and Solow (1956, 1957). Both had confounding problems but simple and powerful results. Since then, these problems have been reformulated in a variety of contexts.

Economic growth theory has its roots in Ramsey, Schumpeter, and Solow. Ramsey brought capital accumulation into the model and found “optimal growth paths” along with a way to plan for the future. Schumpeter made observations to the changing nature of the economy and brought the idea of an evolving economy and creative destruction. Finally, Solow gave us a model to examine technological change analytically and empirically.

**Technology driven growth**

Technology change can be an exogenously and endogenously determined phenomenon. Solow pioneered both the theoretical foundations and empirical methods in economics to measure these changes. Instead of taking technology as something that is external to the economic process, we can internalize it. Here we use endogenous economic growth as a foundation for our framework. We have to answer the question—where does technology come from.

Arrow (1971) proposed that we learn in the process of production. We learn better techniques to produce goods and deliver services. This does not take any additional investment outside the initial capital used to produce the goods and supply wages for labor utilized in the process. This is only part of the story. There is also a learning by searching aspect. This is what we can call Research and Development (R&D). There are whole sectors of industry devoted to developing and refining new production processes and techniques.

In the production process, forms of capital accumulate, and thus the economy and these capital inputs increase over time. Lucas (1988) talked about the formulation and development of human capital. Human capital is considered the primary factor in making labor a productive input in the economy. What we observe are large variations in living standards, rates of growth, and human capital inputs between economies. As a result, he finds that rich countries stay rich, poor countries stay poor, and the distributions of wealth will remain constant in the long run. This is due to marginal productivity of capital stock being equal to time preference in the steady state. As a consequence, even in an open world economy there is no incentive to move productive capital inputs across borders because rates of return are the same in all economies. Essentially, availability of human capital determines long run potential output and rate of growth so that countries who start out with more capital, end up at a higher level.

**Endogenous technology.** Romer takes a similar approach to evolution in the economic system where human capital drives the process of innovation in the economy. Contrary to Lucas, he defines the growth rate of productivity of inputs to grow endogenously to the system. So we don’t just result in an economy with ever increasing levels of accumulated capital, but we get technology that evolves in time with the system. The rate of technology growth is thus dependent upon human capital inputs and existing technologies, but there is no bound on it. Romer’s results are significant. Unbounded growth is possible, but as he notes, it is more of an assumption of the model rather than result so that the marginal product of human capital grows linearly with technology. Romer (1990) says:

There is little doubt that much of the value to society of any given innovation or discovery is not captured by the inventor, and any model that missed these spillovers would miss important elements of the growth process. Yet it is still the case that private, profit-maximizing agents make investments in the creation of new knowledge and that they earn a return on these investments by charging a price for the resulting goods that is greater than the marginal cost of producing the goods. (p. 89)

The most significant implication of Romer's model is that consumption, and thus population size, is not sufficient for sustained output and technology growth. The essential aspect of growth is pushed by the level of human capital. Thus countries with a large population and low levels of human capital would do best by opening up trade to other nations to integrate into markets with large amounts of human capital, especially onto the world markets where levels of human capital are highest and the highest amount of benefits can be captured.

A second significant result of Romer’s model is the possibility of increasing returns. With a relaxation of restraints of the parameters, increasing returns can be achieved, allowing innovation to feed on itself generating a
positive feedback loop. With endogenous growth models, sustained innovation can be achieved through multiple avenues allowing multiple strategies for economies to avoid stagnation in the long run. A final result of Romer’s model is the mechanism for the production of new technologies. Individual firms have incentives to produce new, rival knowledge because of their profit maximizing interests. Thus, the balance of payments to human capital in both output and research sectors drive innovation.

The demographic transition. Thomas Malthus (1809) gave an early, thorough treatment on how limited land would constrain our agricultural output, limiting population growth, and in turn slow economic expansion. He did not consider how powerful the technological change would open up possibilities for economic expansion and increases of wellbeing. The most notable characteristic of his model was the demographic transition.

Galor developed Unified Growth Theory (UGT) in a series of publications (Galor & Weil, 2000; Galor, 2011) to illustrate the process of economies in transition and Malthus’s observations. The UGT framework brings together a multi-sector economy, an evolving technology, and intergenerational agents to model production and evolution of factor inputs into the economy. The driving force for growth in these types of models comes from accumulation of capital, education, and technology similar to Lucas’s results. The models are useful because they show us how evolving technologies act as signals for the economy to shift regimes and find new steady states and growth paths. Technology determines the type of output/growth regime and thus produces phase shifts in the system. Labor and technology can grow but land is in a fixed quantity. This means that there are strictly decreasing returns to land while the other factors of production can change. The level of capital in this economy helps determine interest rate and thus determines households’ human capital investment, utility curves, and fertility choices.

This Unified Growth Theory framework and their model results reflect historical data and developmental leaps with reasonable assumptions. A good example of this modeling and use of historical evidence is in the paper “From Malthus to Solow” (Hansen and Prescott, 2002). This paper examines data of the transition from pre-industrial, stagnant, Malthusian styled economies to post-industrial growth characterized in Solow growth. This paper shows the consistency between UGT and real-world data.

It seems that Galor and Malthus were onto something. Over time, we learned how to use the stored value of assets, both natural and man-made, to create value along with productive capacity, and found that even though land par capital fell our output increased over time thanks to a cornucopia of inventions. These inventions ranged from the Arabic numbering systems, the printing press, to the cotton gin, steam engines, electricity, radio, and most recently, the internet (Clark 2008). These models clearly show that there are limiting factors to economic growth and technological change whether it is availability of human capital or population growth as drivers for innovation. One avenue to further explain these limits is to explicitly define the physical and natural capital as factor inputs.

Natural resource driven growth

Research shows that natural resources are a main contributor in our economy. The methods for implementing natural resources into the production and optimum planning problem in economics was pioneered by Hotelling (1931), Stiglitz (1974), and Dasgupta (1974), among others (Heal, 2000). As in most optimal planning problems, their results showed there was an optimal path even though the resource could not regenerate. Since the inclusion of natural resources in an economic growth model has been considered, economists have also wondered what kind of an effect it has on development in an economy.

Limits to growth. Limits to growth provides us with foundational information and a model needed to show the dynamics of physical growth in a finite world. This book and model use a systems perspective to build a framework of our physical economy function. It focuses on how humans, the economy, and earth systems interact to generate throughput which models the physical economy. It is evidence that while these scenarios may be completely realistic, there are interactions and shifts that can have severe interdependencies, feedback loops, and collapse. This is accomplished with the World3 model which contains: the food system (dealing with agriculture and food production); the industrial system; the population system; the non-renewable resources system; and the pollution system. With this model, many behaviors are assumed to give us an array of “scenarios” for a simplified version of our earth system. They conclude that we are in overshoot. Failure to act will throw our economy and population system into collapse.

They also posit a glimmer of hope to our outcome as a society if we change our actions to reverse the momentum to delay or undo the destruction of our world. It fails to assume that the earth system changes when going past thresholds (i.e., when total ecological footprint exceeds one earth, how does the earth system respond to
that stress?). It also fails to exhibit changing functions and composition when differing stocks and or systems face demographic change. For example, as the economy develops, it could use fundamentally different processes to produce throughput than it did at an earlier time. More importantly their model shows interaction between economic, environmental, and social systems. Economic through total throughput of the economy. Environmental through natural resource depletion, renewable resources, and accumulation of pollution. Social through the dynamics of population and demographics. But the shortcomings of this model is that it is a physical model. Our concern is the economic results and so while models like this can show us how our world potentially looks in various scenarios, it does not give us a clear picture in how the economy functions or changes.

The natural resource curse. Guilló and Perez-Sebastian attempted to answer this question in Neoclassical growth and the natural resource curse puzzle (2015). Natural capital is used in a dynamic Heckscher-Ohlin neoclassical growth model with international trade and growth where a two-sector approach is used, primary and final goods sectors. In this model, a more capital-intensive primary sector with more capital-intensive inputs implies that there is slower growth in natural-resource abundant economies, and that those economies take longer to converge to the adjustment path. Evidence is provided that natural resource endowment affects average income, amount of growth, and the convergence speed of an economy. Although with this evidence it is unclear if the natural capital adds to the transitional effects, it may help to explain why poor countries tend to stay poor for long periods of time.

Similar to the Wolrd3 model, Ayers (1995) and Ayres and van den Bergh (2005) suggest that it is not just resource scarcity that restricts the growth of technology and output, but rather the accumulation of pollution. In this case, we must develop technologies to be able to mitigate the restrictive environment and costs of externalities generated by pollution. In his model of the physical economy, even a constrained economic growth rate requires more and more effort from output and technology to mitigate the effects of pollution buildup and total entropy in the physical-economic system. This also requires a closer look at the implications of how fast we increase levels of output and population because of the high levels of waste that can be generated in the economic process. Since none of these components of the economy and natural world exist in a vacuum, sustainability allows us to view our situation.

Sustainability

Sustainability economics is economics from a holistic perspective. It is built upon the three pillars of sustainability: economy, society, environment. This includes aspects of our economy, the natural world, and social spheres. To realize their full effects on each other requires us to reformulate where productive inputs come from, how we derive utility, and what we value. From the opening of the conclusion of the Brundtland Report in 1987 at WCED, sustainable development was best defined as, “development that meets the needs of the present without compromising the ability of future generations to meet their own needs.”

Sustainable development and sustainability can be characterized into two distinct types, weak and strong sustainability. Weak sustainability is centered on maintaining economic value. This is reinforced by the belief that both the market and technology will provide solutions without having to place limits on our consumptive path. Strong sustainability is about preserving ecological value. This stems from a strong sustainable perspective and belief that the economy and environment are complements, not substitutes. We must consider all valuable contributions to the system and model them in a realistic manner to capture the dynamic interaction between all factor inputs. If the system is modelled realistically, the tradeoffs and choices that must be made are clear. While some may argue that this is normative in nature, as any social planning problem is, considering the health and value derived from every contributor allows us to account for more costs and externalities.

We derive utility and value from a wide range of ecosystem services. These services can be somewhat intangible and hard to measure, such as the enjoyment that the wildlife preserves provide for recreation. Other services are more concrete and measurable, such as the pollination services bees provide for agriculture. The total global value of ecosystem services from plants and animals is staggering, for a crude estimate of $145 trillion per year (Costanza R., et al., 2014) where US GDP in 2014 is $17.4 trillion (USD 2014 dollars). This adds a significant portion of value to generation of total output as we derive use of these ecosystem services.

In Reconciling “weak” and “strong” sustainability by Hediger (1999), a new model is developed that tries to include ecosystem resilience and basic human needs for minimum sustainability in what is called a “sustainably based social value function.” They emphasize that trade-offs are impossible to avoid because economic, social, and ecological goals are opposing forces and intertwined. There is also the introduction of very weak and very strong sustainability. Very weak sustainability consists of a Solow style, consumption based desire to transfer wealth, consumption levels, and standard of living to future generations. The very strong sustainability is considered total
protection of environmental assets to limit human’s reach into the ecological sphere and to limit growth. This is feasible for a steady state, but does not foster development in any economic sense. In turn, even in sustainable goals, moderation has to be put into place to allow for development and wellbeing of current and future generations as well as the health of the earth.

Recall the models of UGT where the economy changes structure from a Malthusian land based-regime to a modern Solow regime with capital input driven growth. This illustrates how the economy fundamentally changes as the needs for inputs changes giving new potential for output. We can also consider how the earth system fundamentally changes as we approach planetary boundaries and progress into the era of the Anthropocene. This would be a complete treatment of sustainability in a modeling framework.

This still is not satisfactory. If we want a true strong sustainability in our model, we must balance the needs of the present and future evenly. Graciela Chichilnisky (1993) developed mechanisms to measure and balance pure preservation of resources and their consumption. This balances health of the resource stock and utility we derive from it in the future with what we must consume to get there.

In any case, the work to develop sustainability as a consistent framework, and a subfield of economics has not gone without criticism. The goals of sustainability have to be continually redefined, but the goals are becoming clearer. We know that balance between economic, social, and ecological systems has to be put into place. Also, tools for the valuation of goods and services outside (or weakly connected to) the economy must be developed to capture the proper externalities that permeate the economic and value system. Furthermore, some think the work in sustainability is just seen as an extension of natural resource and ecological economics. It is still a nascent field, and its distinctions will be developed in time.

Summary of literature review

The implication of much of the existing literature is that increasing our consumption spending, or spending habits (MPC or MPS) can have strong, long-lasting effects on the growth rate or steady state of our economy. The literature also shows that changing population growth rate directly affects the growth of the economy in general. To Malthus, limiting factors were key to the trajectory of the economy. His main warning was that of collapse, and while we haven’t made it there yet, he makes a strong case.

Even in the broad cross section of literature reviewed, each one of these topics could be placed in another subsection. What I mean to say is that all of these issues are interrelated, and so we must consider them as such. This is exactly the approach of sustainability. Barbier (1999) formulated a model, and I will use his approach as a springboard for my model.

Model Setup

Endogenous growth theory has rekindled interest in the role of innovation in determining long-term economic growth. Generally, this body of literature has ignored the contribution of natural resources to growth or the role of innovation in overcoming resource scarcities. The latter problem has been a focus of resource economics for many years, but innovation is usually modelled as exogenous rather than endogenous technological change.

The goal is to build an economy with endogenous technology, non-renewable environmental resources, distinguished capital inputs, population, and see how that economy interacts with its environment. The model considered here is a reformulation of the Romer-Stiglitz model presented in Endogenous growth and natural resource scarcity (Barbier 1999) where the scarcity of a natural resource restricts innovation.

First, I will define how production in the economy will be identified. Next, I will specify the state variables to describe evolution of inputs in the economy. Then, I will define endogenous technology and utility in the sustainable framework. Finally, I will show how different depleted levels of the resource stock impacts the steady state.

Production

I will use the standard Neoclassical growth model in Cobb-Douglas form for aggregate output of the economy. Let \( Q \) be the aggregate productive output in the economy so that

\[
Q = F(A, x, H_Q, L, \sigma) = H_Q^a L^\beta \sigma^\gamma A x^{1-(a+\beta+\gamma)}
\]
where: $H_Q$ is human capital used in the process of production; $L$ is the supply of unskilled labor in the economy; $\sigma$ is the rate of use of a natural, non-renewable resource; $A$ is assumed to be the stock of technology (designs) in the economy; $x$ are durable goods used in production.

Let $K$ be the total capital stock. I will follow the same formulation as Romer (1989) and Barbier (1999) for $K$. If it takes $\eta$ units of forgone capital to produce one unit of some durable good $x$, and $A$ is the embodiment all of the designs that the economy can produce, then the amount of capital used in the production process $K = \eta A x$.

Aggregate output in the economy can be redefined as

$$Q = A^{a+\beta+\gamma} H_Q^a L^\beta \sigma^\gamma K^{1-(a+\beta+\gamma)} \eta^{a+\beta+\gamma-1}. \quad (1)$$

Evolution of economic inputs

The labor force $L$ evolves at the same rate as population. We will assume that the labor force grows and shrinks at the same rate of population growth so the evolution of the labor force in the economy is $L = n L$ where $n$ is the growth rate of the population. At any point in time, the labor inputs of the economy have evolved from the initial amount so that $L = L_0 e^{nt}$. For ease of analysis let $L_0 = 1$. This will not qualitatively change the results which allows for a more tractable model.

Human capital is divided into distinct types. $H_Q$ are the human capital inputs involved in production while $H_A$ are the human capital used in the process of innovation. In this model, we take human capital as some fixed amount so that total human capital in the economy is

$$H = H_Q + H_A. \quad (2)$$

Let $s$ be some nonrenewable, exhaustible resource stock. It is assumed that $\sigma$ is the rate at which the natural resource stock is exploited. Because this resource cannot be replenished, the current state of the resource is the initial level of the resource $s_0$ less the total amount of resource extracted so that $s_t = s_0 - \int_0^t \sigma \, dt$ or in other words $s_t \geq s_0 - \int_0^t \sigma \, dt$. Thus, the state of the resource evolves (depletes) at the rate in which it is consumed

$$s = -\sigma, \text{ where } \sigma > 0. \quad (3a)$$

Using the common formulation, capital accumulates according to the existing stock of capital minus aggregate consumption so that growth of capital is defined as

$$\dot{K} = Q - c L. \quad (4)$$

Endogenous technology

Technology evolves endogenously in this model. Like Romer (1990), technology grows through the activities of the R&D sector thanks to the already existing state of technology $A$ and human capital inputs devoted towards innovation $H_A$. Barbier proposes that as natural resource stocks shrink, this restricts the rate of growth of technology and the new product development process in a multitude of ways. Three examples illustrate ways in which natural resource scarcity has a negative effect on the rate of technology growth.

Recent investigations in political economy have additionally suggested that the ‘supply’ of innovation may itself be constrained by resource scarcities, especially in the developing world (Robinson & Acemoglu, 2012; Guilló & Perez-Sebastian, 2015). The increased social tension for lack of resources is enough to cause friction in production of new technologies because these resources could instead be used for consumption and production of capital.

Rare earth metals are essential to the production of high tech components. If rare earth metals continually become harder to find or their process becomes increasingly expensive to produce, these costs (economic constraints and pollution) can hinder the technological process.
Knowledge is also stored in natural capital, so biodiversity is a key source of new forms of technology, such as biomimicry, as it adds to the total stock knowledge. Thus, as natural capital is used up, this cuts the potential gains from technologies derived from ecological sources.

Technology is restricted by the ratio of the rate of extraction of the renewable resource to the availability of the resource \( \frac{\sigma}{s} \). Thus, technological evolution is a function of available technologies, human capital devoted to research, consumption of the natural resource, and health of the non-renewable resource stock \( \dot{A} = f(A, H_A, \sigma, s) \).

If the function is linear in these variables, the level of technology and human capital adds to the growth of technology \( G(A, H_A) = hAH_A \), where \( h \) is some productivity parameter of generating new technologies. Scarcity of natural resource stock and their consumption inhibit technological change \( R(\sigma, s) = -\frac{\omega}{s} \), and \( \omega \) is a parameter showing the rate at which the availability of resources constricts technological growth. Finally, the linear evolution of technology is as follows:

\[
\dot{A} = hAH_A - \frac{\omega}{s}
\]

Utility

The standard utility function is \( u(c) \). This means we base our utility strictly upon consumption of manufactured goods. This is at best weak sustainability because it is based upon the consumption of manufactured capital. Throughout sustainability literature we can define utility in a new way where it is not only based on consumption, but the health of our resources. A comprehensive and concise overview of this is supplied in Heal (2000). This is defining utility, value from the stocks and flows from resources. This has to include benefits from consuming the produced economic good, consumption of natural resource stocks, and the benefits from having some maintained stock of environmental assets.

Let \( u(c, s) \) be the sum of utility derived from the consumption of manufactured goods as well as the state of environmental/ecological capital good. We will use a standard logarithmic utility function:

\[
u(c, s) = \ln c + \ln s.
\]

This is especially useful because just like the standard CRRA utility, logarithmic utility has positive first derivative and negative second derivative, so these functions behave very similarly. To be clear, utility derived from consuming a natural resource stock is implicitly defined in \( u(c, s) \) through \( \ln c \). This is the case because the natural resource is needed in the production function. So indirectly, as we consume manufactured capital, we gain benefit from consuming the natural resource as well.

Now, the utility from the stock of the non-renewable natural resources is clear. Why should we develop a utility function as the level of stocks of resources? For planning the use of such resources. If a stock falls, then there is less potential output to be derived from such a stock. It benefits agents to have a healthy stock of capital (natural and manmade) for consumption in the future. Furthermore, the ecosystem services provided by resource stocks are essential to adding welfare to agents. While sources such as forests are renewable, at first glance many systems and services are from non-renewable sources: once a stock of biodiversity is depleted, it does not regenerate. This is a qualitative difference: preservations of stocks are essential for health of the economic system because production processes cannot persist without such things. Thus, this is our first major avenue towards sustainability in the model.

Social welfare

Our objectives are to maximize (i) consumption of manufactured goods, (ii) natural resource goods, and (iii) the allocation of human capital towards production and development of new technologies. This is restricted by the evolution of inputs to production, innovation, and the natural resource stock. Formally, social welfare \( W \) over the infinite time horizon is

\[
W = \int_0^\infty u(c, s)L_0e^{nt} e^{-\delta t} dt
\]

such that \( \delta \) is the discount rate.
Let \( \rho \in (0, 1) \) be some fraction of the resource stock that is to be consumed (the preserved level of the resources are \( (1 - \rho)s_0 \)). Since social welfare is partially a function of the level of resource stock that is to be consumed, then we will describe welfare as \( W(\rho s_0) \). We can divide the model by \( L_0 \), (remember we set \( L_0 = 1 \)) to get per capita consumption and combine like terms so that the planning problem is finally

\[
\max_{c,\delta,H} \{ W(\rho s_0) \} = \max_{c,\delta,H} \{ \int_0^\infty u(c, s) e^{-(\delta-n)t} \, dt \}.
\] (8)

**Sustainable optimal paths and steady states**

There are multiple scenarios. With respect to the natural resource, we can consume all of it and preserve none \( \rho = 1 \), or save a fraction of it going into the future \( 0 < \rho < 1 \). We want to maximize the welfare with respect to the state variables \( K, A, \) and \( S, \) and their respective adjoint variables. Let \( \lambda \) be the shadow price of accumulating capital stock \( K, \) \( \mu \) is the shadow price of developing technology \( A, \) and \( \psi \) is the shadow price of using the non-renewable resource \( S, \) such that \( \lambda(t), \mu(t), \psi(t) > 0, \forall t. \) In this following section, we will analyze the possible scenarios.

**No preservation of resource stocks.** Without trying to preserve the resource stock, our value \( \rho = 1 \) so that the social welfare function is \( W(s_0). \) We maximize (8) with respect to (4), (5), and (3a), and shadow prices \( \lambda, \mu, \) and \( \psi, \) respectively. The current value Hamiltonian is then constructed as:

\[
\mathcal{H} = u(c, s) + \lambda \dot{K} + \mu \dot{A} + \psi \dot{s}.
\] (9)

(Details for solving the model appears in the appendix.) The first order conditions yield:

\[
\lambda = \frac{1}{c},
\] (10)

\[
\mu = \lambda \alpha \frac{K}{H_0 hA} K
\] (11)

\[
\psi = \lambda \gamma \frac{K}{s} - \mu \omega s
\] (12a)

\[
\dot{\lambda} - (\delta - n)\lambda = -\lambda (1 - (\alpha + \beta + \gamma)) \frac{Q}{K},
\] (13)

\[
\dot{\mu} - (\delta - n)\mu = -\lambda (\alpha + \beta + \gamma) \frac{Q}{A} - \mu h A,
\] (14)

\[
\dot{\psi} - (\delta - n)\psi = -\frac{1}{s} - \mu \omega \sigma s
\] (15)

The key difference between Barbier’s model and the one presented here is that the cost of using the resource stock also depends on the marginal utility derived from that resource. This is because utility in our model (6) requires resource stocks, while Barbier’s model does not. Also, in the steady state of technological growth as \( s \to 0, \) we need \( \frac{\dot{A}}{A} = 0 \Rightarrow hAH_A = \frac{\omega\sigma}{s} \) where increases in resource use must be offset by increases in technology.

**Preservation.** Suppose only a portion of the resource stock can be consumed \( 0 < \rho \leq 1. \) We would have consumed \( \rho s_0 \) leaving \( (1 - \rho)s_0 \) preserved. The previous example allows \( \rho = 1 \) and nothing preserved. Thus, under preservation where \( \rho \in (0, 1) \) we will consume a fraction less leaving the depletion of the resource stock under some preservation level so that for some preserved level of natural capital \( s_t = \rho \left( s_0 - \int_0^t \sigma \, dt \right) \) we have

\[
\dot{s} \equiv \dot{s}_\rho = -\rho \sigma.
\] (3b)

This time, we maximize (8) with respect to (4), (5), and (3b), and shadow prices \( \lambda, \mu, \) and \( \psi, \) respectively. The modified current valued Hamiltonian is
\[ H = u(c, s) + \lambda R + \mu A + \psi(-\rho s). \] (9)

Under this new formulation we have higher costs of utilizing additional units of the resource

\[ \psi(\rho) = \frac{1}{\rho} \left( \lambda \gamma K - \mu \frac{\omega}{s} \right) \] (12b)

and taking the difference between the current costs and the previous costs it follows

\[ \psi(\rho) - \psi = \frac{\rho - 1}{\rho} \left( \lambda \gamma K - \mu \frac{\omega}{s} \right) > 0 \] (18)

that there is an increase in costs since this expression is positive.

Towards the steady state \( hAH_A \) continues to grow, while \( \omega \frac{s}{s} \rightarrow \omega \frac{s}{1-\rho s} \). So that as other economic inputs approach the steady state we have \( hAH_A > \frac{\omega s}{(1-\rho s)} \). Unlike the previous formulation, technology can grow without bound with \( s \rightarrow (1 - \rho)s_0 \) and costs of resource exhaustion are offset by advances in technology. This is highly optimistic assuming we don’t exhaust the reserves of new technologies.

**Model results.** Barbier’s results state that innovation can be substituted for resource use through an increase to the amount of human capital devoted to innovation. In our model without preservation of the resource, innovation must increase to offset resource scarcity while at the same time the loss of utility from that resource continues to grow in the steady state:

\[ \frac{d}{ds} u(c, s) = \frac{1}{s} = \psi(\delta - n) - \frac{K \omega s}{s^2 cAH_Q}. \] (19)

There is a major problem. As \( s \rightarrow 0 \), we see that \( s^2 < s \rightarrow 1 \). Thus, at a certain point, the costs associated with resource depletion grow faster than technological growth in the steady state. To maintain consumption and utility levels in the steady state, capital must continue to grow and this cannot happen with stagnant technological growth. There is no way to offset the utility losses from the destruction of resources as they near depletion.

In the second version of our model, resources only deplete to the level \((1 - \rho)s_0\). Approaching the steady state, technology continues to develop, growing faster than the costs associated with resource depletion:

\[ \frac{\dot{A}}{A} \rightarrow 0 \Rightarrow hAH_A > \frac{\omega s}{(1-\rho)s_0}. \]

Let \( P = (1 - \rho)s_0 \) be the preservation level of the resource stock. So when the stock of the resources depletes to preserved levels \( s \rightarrow P \) it follows

\[ \frac{1}{P} = \psi(\delta - n) - \frac{1}{P^2 cAH_Q}. \] (20)

Then the effective discount rate \((\delta - n)\) approaches

\[ (\delta - n) = \rho \left( \frac{\sigma P}{\lambda \gamma KP - \mu \omega} \right) \frac{1}{P^2 cAH_Q} \frac{K \omega s}{s^2}. \] (21)

where \( \rho \) is some fixed consumption level of the resource.

Strong sustainability in this context means capping consumption of the natural resource. This means we can continue to derive utility indefinitely because new innovations allow for capital growth. This makes sense because after some the resources become depleted, hidden costs build up. Either extraction costs become too high, or the
natural system undergoes some unpredictable change in behavior disallowing use of the resource as we once knew it.

**Discussion and Conclusions**

Strong sustainability means that we must recognize the role of natural capital and ecosystem services in human wellbeing (utility). The paper therefore demonstrates that endogenous growth can overcome resource scarcity, but the outcome in the long run critically depends on preserving natural capital above catastrophic levels. If the natural resource is not preserved then not only do we experience costs from over-extraction, but there are unpredictable costs to the changes in natural and social systems as well.

While this exercise has been theoretical in nature, this model hopes to explain the processes in a more realistic manner as well as give avenues to be able to estimate the empirical relationships between these variables. The usefulness of using Cobb-Douglas functional forms is evident because it allows for a testable hypothesis. Furthermore, as ways to value natural capital become more refined, the data will better reflect its role in the generation of output.

For any discussion on sustainability we have to ask—what is the role of renewable resources. It can be assumed that anything that is manufactured is some form of a renewable resource. While for some this may be satisfactory, manufactured capital is more “grey” infrastructure while renewable resources are “green” infrastructure. Therefore, a proper representation of renewable resource must include both renewable and non-renewable resources, and their individual roles in the production function. What this model does not capture is switching between use of resources in growth regimes because it is hard to model and not fully understood. It is hard to formulate and capture signaling in the economy for regime shifts from the use of non-renewable capital to renewable capital inputs.

Where is the rest of world economy in this economic growth model? While this can model the economy on a world-wide scale, the factor input varies greatly between countries. If we open up this model to world trade, considering this economy as a country taking prices and trade flow from the world economy as an influence is only part of the story. So when we think of this economy interacting with an outside economy, we can consider factors and flows from other economies. Other countries, big or small, bring their advantages and disadvantages with them. Insights from production theory may help us understand better ways to represent these functions and the production process. This means that microeconomics can lend a hand to macroeconomics again, similar to how Romer used micro-foundations to build incentives to produce technologies into the model.

If we wish to extend our definition of sustainability further, we might have to re-approach the social planning problem. While the principal agent framework has been a powerful tool in economic analysis and application of policy, many including myself want to ask why only this agent. If the representative agent is the controller in this system, then we have a conflict of interest in the welfare of the population in question. Other approaches could use heterogeneous agents. One approach I would like to pursue is to divide the agents into social planners, consumers, and producers. This is a question of true sustainability. If the social planner is maximizing their utility, what about the health of the consumers and producers? Are they best off in this scenario as well? Do the social strata stay intact in the long run here?

Many have also proposed that we have collected all of the low-hanging fruit on the tree of technology. Thus, the technological advancements may dry up and at best we could try to sustain a growth of human capital to avoid stagnation and push economic evolution. The linear representation of endogenous technology may not be accurate and we instead must think about outcomes of a model with nonlinear technological growth. With threat of the approach of the steady state, there is also evidence that the great divide of prosperity and wealth distribution across the world does not change in the steady state (Lucas, 1988). Poor nations tend to stay poor. This can further cause social unrest and eat away at our social capital, causing a collapse outside the scope of our model. Thus, preserving natural resources is only part of the answer in finding equitable, sustainable ways to grow.

**References**


Appendix

Notation

For some variable $X$ a dot over the top denotes a time derivative: $\frac{dx}{dt}(X) = \dot{X}$. Variables at specific dates can be denoted with a time subscript $X(t) = X_t$. For readability the time subscript will be forgone unless specifically needed. Initial values are distinguished from values at other times by a subscript of 0 such as $X(0) = X_0$. Say that $Z$ is a function of $X$ so that $Z = f(X)$, then the partial derivative of $Z$ with respect to $X$ is denoted with a subscript so that $\frac{dZ}{dX} = Z_X$.

Generalized form of the model

I chose functions that are consistent with both economic theory and empirical research for this paper. For those that are hesitant to specify functional form, supplied below is a more generalized form of the model presented in this paper. The sustainable resource stock at any time $t$ is the difference between the initial stock and the total amount consumed from time 0 to time $t$. The evolution of the natural stock is defined in terms of the rate of resource consumption and a fraction at any date derived as follows $s_t = s_0 - \int_0^t \sigma \, dt$. Since we want to consume only a portion of this stock at any date $\rho s_0 \geq 0, \forall t$. When the resource is used to the desired level, $(1 - \rho) s_0$ is left over. The amount consumed at any date becomes a fraction of what it was before such that

$$s_t = \rho \left( s_0 - \int_0^t \sigma \, dt \right),$$

$$s = -\rho \sigma.$$

The evolution and definitions of the rest of the variables in the economy are:

$$\dot{K} = Q - cL,$$

$$Q = F(A, K, H_0, L, \sigma),$$

$$H = H_0 + H_A,$$

$$K = \eta A x,$$

$$\dot{A} = G(A, H_A) - R(\sigma, s),$$

$$L = nL.$$

The first order conditions are as follows:

1. The current value Hamiltonian

$$H = u(c, s) + \lambda \dot{K} + \mu \dot{A} + \psi \dot{s}$$

2. Derivative of the system w.r.t. the control variables are equal to zero

$$H_c = 0 \implies \lambda = u_c$$

$$H_{H_A} = H_{H_Q} = 0 \implies \mu G_{H_A} = \lambda F_{H_Q}$$

$$H_\sigma = 0 \implies \psi = \lambda F_{Q} - \mu G_A$$
3. Derivative of the system w.r.t. the negative of the state variables is equal to the time derivative of the present value of their respective shadow prices

\[ \mathcal{H}_K = -\frac{d}{dt}(\lambda e^{-(\delta-n)}) \implies \dot{\lambda} - (\delta - n)\lambda = -\lambda F_K \]

\[ \mathcal{H}_A = -\frac{d}{dt}(\mu e^{-(\delta-n)}) \implies \dot{\mu} - (\delta - n)\mu = -(\lambda F_A + \mu G_A) \]

\[ \mathcal{H}_s = -\frac{d}{dt}(\psi e^{-(\delta-n)}) \implies \dot{\psi} - (\delta - n)\psi = -(u_s - \mu R_s) \]