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An Exploration of Metacognition and Its Effect on Mathematical Performance in Differential Equations

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Abstract: Research suggests that students in certain contexts who are "metacognitively aware learners" demonstrate better academic performance (Shraw & Dennison, 1994; Md. Yunus & Ali, 2008). In this research, the metacognitive levels for two classes of differential equations students were studied. Students completed a survey adapted from the Metacognitive Awareness Inventory (MAI) (Shraw & Dennison, 1994) at the start of the course. The questions chosen from the MAI were aimed at three components concerning the students' knowledge about their cognition: declarative knowledge, procedural knowledge, and conditional knowledge. Analysis shows student performance, as measured by the course grade, cannot be predicted by metacognitive awareness levels.

Key Words: Metacognition, MAI, differential equations

I. Introduction.

Metacognition is a person's ability to understand how s/he thinks, reflect upon one's thinking, and control one's learning based upon the understanding of and reflection on one's thinking (Shraw & Dennison, 1994). Flavell (1976) considered metacognition remembering to remember, and then monitoring and updating knowledge. Cross and Steadman (1996) believe that, though a person knows his or her strengths and weaknesses, s/he needs to take the next steps—the metacognitive steps—of planning how to approach a problem, checking for understanding, and testing for learning. These steps have been described by Nelson and Narens (1990, 1994) as a metacognitive system with two basic levels—the meta-level and the object-level. Information flows between the levels to monitor a person's learning process (object-level to meta-level) and control what a person does to enhance their learning (meta-level to object-level). The information flow process in the metacognitive system could also be thought of as knowledge about cognition (monitor) and regulation of cognition (control) (Shraw & Dennison, 1994).

In the concept of self-regulated study, as a student studies, s/he monitors his/her learning by using a variety of metacognitive judgments (for example, Ease-of-Learning (EOL), Judgments of Learning (JOL), Feeling of Knowing (FOK)) (Dunlosky, Serra, & Baker, 2007). Then the student decides (or uses controls) to continue to study based on the judgments that were made. In the hierarchical model proposed by Theide and Dunlocky (1999) students regulate their learning through planning and goal-setting. In this model, students set a high or low goal in terms of how much they feel they need to master and pre-plan their study time to accomplish their goal. In another model, Region of Proximal Learning, by Metcalfe and Kornell (2004) students regulate their learning through first choosing what items to study or not study, then stop studying an item when they feel that they are no longer learning anything about that item. Other researchers have determined that not all students use metacognitive knowledge to control their

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learning (Son & Metcalfe, 2000) or that the metacognitive knowledge may not be consciously used to enhance learning (Reder & Schunn, 1996; Siegler & Shipley, 1995).

Some research suggests that students who are "metacognitively aware learners" demonstrate better academic performance. Schraw and Dennison (1994) generated an inventory (Metacognitive Awareness Inventory--MAI) that examined metacognitive awareness and was deemed suitable for use with young people and adults. They tested their instrument on undergraduate students in several introductory educational psychology courses at a midwestern university. Schraw and Dennison concluded, among other things, that students with high scores on the inventory also scored high on reading comprehension exams. Other researchers, Md.Yunus and Ali (2008), used the MAI to test mathematics education students in their final year at a university in Malaysia. Their research studied the relationship between metacognition and gender and cumulative GPA of the students. Their findings indicated that procedural, declarative, and conditional knowledge (that is, components of the knowledge about cognition (Shraw & Dennison, 1994)) have "significant correlation to performance in mathematics and overall academic performance" (p. 129) and that "metacognitive scores can predict students' achievement to a certain extent" (p. 130).

Other research, however, has shown that higher metacognitive awareness did not lead to higher scores academically. Pressley and Ghatala (1990) studied students at the university level and at the elementary school level with respect to reading comprehension. Students at both of these levels were asked to read something and judge their performance on questions about the reading. In a variety of experiments at both elementary and university levels they found that "a lot of cognition is not very efficient" (p. 31) and though students may be monitoring their studying, poor monitoring contributed to poor performance. Their research suggests that students must be both metacognitively aware and proficient at applying this awareness in their studying.

Studies also indicate that few students engage in the processes of metacognition in a manner that would help them be successful at problem solving. Schoenfeld (1992) examined how students worked through problems that were not familiar to them and found that many spent very little or no time on planning during problem solving. Students would read the problem, consider some method of solution and use it without regard to whether it was leading them to a solution. Stillman and Galbraith (1998) identified the same lack of planning by students in their research. Schoenfeld (1985) also found that students, when they saw that a strategy was not working, threw it completely away and did not consider what they learned from that strategy as they moved on to another strategy.

Various other researchers have also noted the importance of planning in problem solving. Lester (1982) notes that to be a successful problem solver a person needs to know and understand his/her own cognitions and be able to monitor the thought processes throughout the solving of the problem. Silver (1982, 1987) suggests that the strategies used to solve a problem are metacognitive and guide how the process proceeds. In particular a person's beliefs about learning and mathematics can greatly impact how a person solves a mathematical problem. Veenmen (2006) believes that initially in the process of solving a mathematics problem, metacognitive skills are more important than intellectual ability and that a student needs an "adequate repertoire" of metacognitive skills to insure success in problem solving.

II. Inquiry.

For many years the author has observed some students who approach their learning very intentionally, systematically using their time and effort. A portion of these students performed very well, while other students did not. Conversely, some students who did well appeared to have a very haphazard approach to their learning. These anecdotal observations led the author to investigate how her students' knowledge about their cognition related to their performance, as measured by course grades, in a differential equations course. In particular, the author was interested in their declarative knowledge (knowing what is important to learn), their procedural knowledge (understanding how to use strategies from past experiences), and their conditional knowledge (knowing when what strategy is most effective). The author felt that perhaps successful students, however they approached their learning, were using metacognitive tools that less successful students were not.

The study is focused on students enrolled in differential equations, a course which requires students to have completed two prerequisite semesters of calculus. The author has taught courses in differential equations for 25 years, most of which have involved sections of approximately 35 students. In the spring of 2010 and the spring of 2011, trial versions of "large sections" (89 students and 120 students, respectively) of differential equations for engineering majors were offered. These courses became the focus of this study.

This study focused on the following research question:

- Do a student's metacognitive levels predict his/her course performance, as measured by course grade, in a differential equations course?

III. Methods.

The subjects for the study were students in two university junior level differential equations courses, from different semesters, with the majority of students majoring in engineering. As Nelson and Narens (1990) suggest, the main tool to generate data about a person's metacognition is from their own reports about their thinking. Hence, students' metacognitive levels were assessed using a survey. The survey was adapted from the Metacognitive Awareness Inventory or MAI, (Shraw & Dennison, 1994) which is considered a reliable measure of knowledge of cognition (Sanchez-Alonso & Vovides, 2007). The survey used a subset of 17 questions that were specific to the three areas of knowledge the author intended to study. Each question asked students to indicate their perception of the truth of the statement along a continuous scale from 0 to 100, with zero corresponding to completely false and 100 corresponding to completely true. Scores on the survey were translated to a scale of 0 to 10 for the analyses. The survey is attached (see Appendix) with notations added to show how the questions related to the areas of knowledge being tested (as indicated in the MAI). The survey was administered in the first two weeks of the semester and was counted as one homework assignment.

In the analyses below, only data from students who finished the courses are included (70 from spring 2010 (Class 1) and 105 from spring 2011 (Class 2)). Students consented to have their data included in the study and the methodology was approved by the university's Institutional Review Board.

To answer the research question, the author compared summative scores on the revised MAI instrument using equal interval division to indicate low, moderate, and high metacognition. Data were compared using descriptive statistics. Unpaired Student's t-tests were used to determine any significant differences in means of the data. Cohen's d was calculated to determine effect size for means that were significantly different. Mann-Whitney tests were used to determine any statistical significance between the medians of the data. The two-sample F-test was used to determine any differences between variances of the data.

IV. Results.

To get a sense of students' levels of metacognition when they enter a differential equations course, the survey data from the classes were combined and the metacognitive scores were considered.

The summative metacognition scores were broken into three subintervals indicating low, moderate, and high metacognition for the three categories of metacognition being examined. Table 1 gives the percentage of students, in the classes combined, with scores in each of the subintervals.

Table 1. Subintervals of Low, Moderate, and High Summative Metacognition Scores with Percentage of Students in each Subinterval (175 students).

	Low	Moderate	High
Declarative	40-52	53-66	67-79
Percentage	13.1	56.5	30.3
Procedural	14-22	23-31	32-40
Percentage	5.7	49.7	44.6
Conditional	20-29	30-40	41-50
Percentage	11.4	49.7	38.9

Research Question: Do a student's metacognitive levels predict his/her course performance, as measured by course grade, in a differential equations course?

This question was studied within classes to account for any differences in the grading of the two classes. Students who received an F for the course and completed surveys were not considered because of the small number of them (4 in Class 1 and 2 in Class 2).

Figures 1 through 3 give the percentage of students, by grade received in the class, in the low, moderate, and high subintervals for each category of metacognition studied for both classes. (The ranges of scores for the subintervals are as noted in Table 1.)

For each class individually, the Student's t-test was used to compare the means of the metacognitive scores between students with differing grades. No significant differences were found in Class 1. In Class 2 significant differences were found between students receiving a B in the course and those receiving a C in the course in the categories of declarative and conditional metacognition. Class 2 also showed significant differences in means in all metacognitive categories when comparing C students to D students. Tables 2 and 3 give those results.

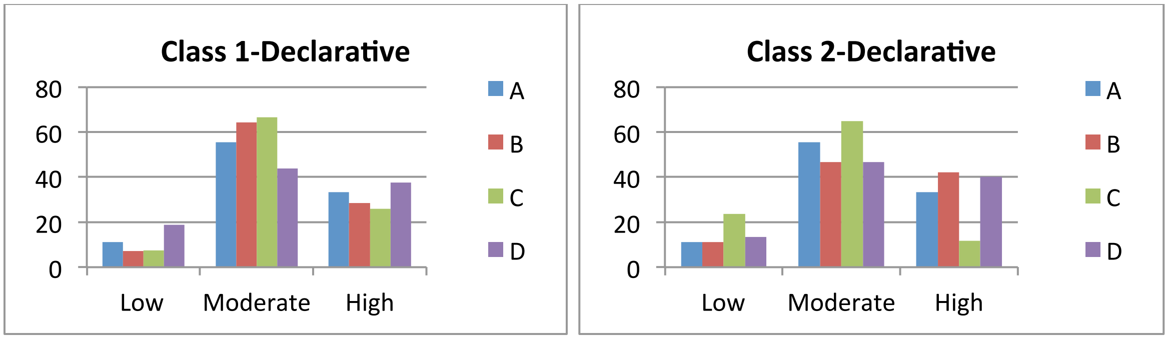

Figure 1. Percentage of students in the low, moderate, and high subintervals for declarative metacognition, by grade in class.

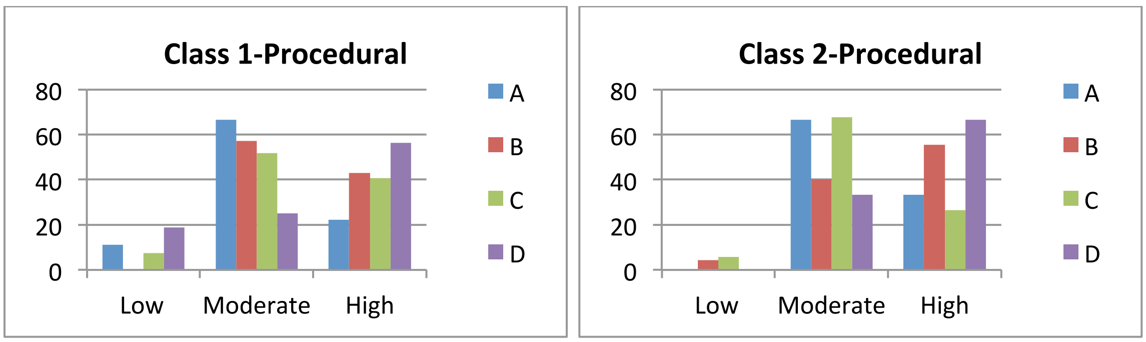

Figure 2. Percentage of students in the low, moderate, and high subintervals for procedural metacognition, by grade in class.

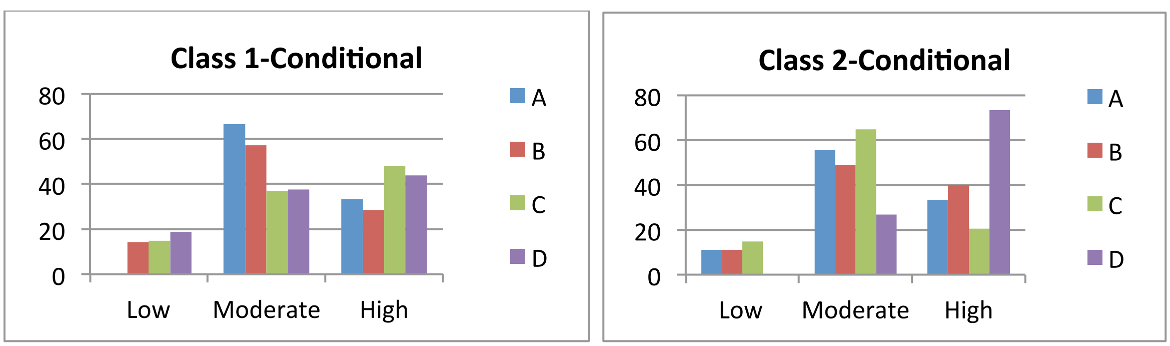

Figure 3. Percentage of students in the low, moderate, and high subintervals for conditional metacognition, by grade in class.

Table 2. T-test and Cohen's D Information for Mean Metacognitive Scores, for B and C students, Class 2.

	Declarative		Conditional	
Grade	B	C	B	C
Mean	63.8	58.0	38.6	35.7
p	0.00155		0.036481	
d	0.74061		0.47439	
Effect Size	0.34726		0.23079	

Table 3. T-test and Cohen's D Information for Mean Metacognitive Scores, for C and D students, Class 2 (34 C students and 15 D students).

	Declarative		Procedural		Conditional	
Grade	C	D	C	D	C	D
Mean	58.0	63.4	29.3	32.3	35.7	41.1
p	0.028988		0.047864		0.007015	
d	-0.72913		-0.66258		-0.90727	
Effect Size	-0.34251		-0.31448		-0.41311	

Mann-Whitney tests run to compare the medians of the metacognition scores for each class between students with differing grades in the course showed no significant differences in Class 1. Class 2 had significant differences in all metacognition categories between C and D students. Table 4 gives those results.

Table 4. Mann-Whitney U Test Information for Median Metacognitive Scores, for C and D students, Class 2 (34 C students and 15 D students).

	Declarative		Procedural		Conditional	
Grade	C	D	C	D	C	D
Median	56	63	30	33	36	43
p	0.006934		0.033408		0.033462	
U	352.5		353.0		389.5	

Because D students had higher metacognitive levels than C students in Class 2, the calculus I and calculus II grades for C and D students were compared to determine whether C students came into differential equations with higher content knowledge than D students, thus indicating that perhaps prior knowledge had a greater effect on their performance than their metacognitive levels. It was found that there were statistically significant differences in the mean scores which C and D students received in these courses. However, D students had a statistically higher mean in calculus I, while C students had a statistically higher mean in calculus II. Table 5 shows the results. In the calculation of the mean grade for C students in calculus I, three students were not included as they received credit for this course using the results of an AP exam or something similar. The mean grade is based on a 4 point grade scale.

Table 5. T-test and Cohen's D Information for Mean Metacognitive Scores, for C and D students, Class 2, in Calculus I (31 C students and 15 D students) and Calculus II (34 C students and 15 D students).

	Calculus I		Calculus II	
Grade	C	D	C	D
Mean	2.49	2.95	2.52	2.24
p	0.0354		0.0309	
d	-0.59641		0.56497	
Effect Size	-0.28577		0.27184	

Because Class 1 had no significant differences in mean or median metacognitive scores between students with differing grades in the course and Class 2 did, the classes were compared to see if any insights concerning that would arise. When comparing the classes by grade received in the classes and the students' calculus preparation, no significant differences were found. When the classes were compared by grade in the class and their metacognitive scores, the only significant difference occurred between C students and their declarative metacognition ,with Class 2 having the higher metacognitive scores in that area ($p = 0.0147$, $d = 0.659$, Effect size = 0.2669).

V. Discussion.

The metacognitive levels for two classes of differential equations students were studied. The classes were combined to get a sense of the metacognitive levels of students as they enter differential equations. The classes were considered on their own to determine whether the metacognitive scores could predict course performance.

The metacognition scores for all students in the study were broken into three subintervals indicating low, moderate, and high metacognition for the three categories of metacognition being examined. Though the majority of students were in the moderate subinterval for all categories, over 40% of the students were in the high subinterval for procedural metacognition (understanding how to use past strategies) and nearly 40% of the students were in the high category for conditional metacognition (knowing which strategy is most effective).

It would appear that students came into the differential equations classes with very reasonable levels of metacognition, in general.

No significant differences were found in the mean and median metacognitive scores for students in Class 1 with differing grades. However, the statistical analyses show B and C students in Class 2 differed significantly in their mean metacognitive scores in knowing what was important to learn (declarative knowledge), and in knowing what strategy is most effective (conditional knowledge), with B students having higher means.

The C and D students from Class 2 also had significantly different mean and median metacognitive scores in all categories of metacognition, with D students having the higher means and medians. While trying to account for this seeming inconsistency, content knowledge from calculus I and calculus II, based on the grade received in each course, was considered and it was found that students receiving a D in the differential equations course had significantly higher scores in calculus I and students receiving a C in the differential equations course had significantly higher scores in calculus II.

When considering Class 1 versus Class 2 the C students were significantly different in the declarative metacognition scores. All other comparisons, by grade received in the course with metacognition scores or by grade received in the course with calculus preparation, were not significantly different. With the analyses done, no evidence arises that would indicate why the classes had the differing metacognitive scores.

It is apparent from the findings above that it would be difficult to predict a student's course performance, as measured by the grade in the course, by considering their scores in declarative, procedural, and conditional metacognition. While it seems reasonable that B students would have higher metacognitive levels than C students (as seen in Class 2), the pattern does not hold with other students in Class 2. In fact, the D students (Class 2) had higher metacognitive levels than C students. This is not consistent with the results from other studies that used the MAI instrument with undergraduates in introductory education psychology (Shraw & Dennison, 1994) and mathematics (Md. Yunus & Ali, 2008) which indicated that students with higher metacognitive awareness have higher academic performances.

This analysis indicates that being "metacognitively aware" about one's cognitive knowledge does not necessarily translate into higher academic performance in this academic context.

VI. Future Work.

Pressley and Ghatala's (1990) conclusion that higher metacognitive levels did not necessarily lead to higher academic achievement was borne out in this research. Because students who received a D in the differential equations course did not necessarily enter the course more prepared than the C students, it is obvious that there is much more going on with the students' thinking than was analyzed in this study. The questions raised from this study have led the author to wonder how D students are using the control aspect of metacognition in their learning. Are these students setting low goals for themselves, yet are not studying appropriately to master material even at that level (Thiede & Dunlosky, 1999)? Are they studying the easier topics and not getting to the harder topics (Metcalfe & Kornell, 2004)? Do these students spend little or no time planning their problem solving strategies (Schoenfeld, 1992; Stillman & Galbraith, 1998)? When they have solved problems incorrectly do they examine where things went wrong and learn from that (Schoenfeld, 1985)?

It would be of interest to identify how students in differential equations control their study, and perhaps be able to answer questions like those above. If questions of these types can be answered, at least to some degree, it would then be helpful to build into the differential equations course ways to introduce productive metacognitive tools that would help the students to be more successful.

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Appendix

Appendix 1. Metacognition Survey Used.

Instructions: Please put a mark on the number line that corresponds to how true or false you feel the statement is for you in a mathematics course. Zero implies completely false and 100 implies completely true.

1. I try to use strategies that have worked in the past. (PK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

2. I understand my intellectual strengths and weaknesses. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

3. I know what kind of information is most important to learn. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

4. I am good at organizing information. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

5. I have a specific purpose for each strategy I use. (PK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

6. I learn best when I know something about the topic. (CK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

7. I know what the teacher expects me to learn. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

8. I am good at remembering information. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

9. I use different learning strategies depending on the situation. (CK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

10. I have control over how well I learn. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

11. I can motivate myself to learn when I need to. (CK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

12. I am aware of what strategies I use when I study. (PK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

13. I use my intellectual strengths to compensate for my weaknesses. (CK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

14. I am a good judge of how well I understand something. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

15. I find myself using helpful learning strategies automatically. (PK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

16. I know when each strategy I use will be most effective. (CK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

17. I learn more when I am interested in the topic. (DK)

0---------10---------20---------30---------40---------50---------60---------70---------80---------90---------100

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