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A Professional Development Program to Improve Math Skills Among Preschool Children in Head Start

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Abstract

The purpose of this study was to examine the effect of providing early educators professional development experiences and activities to improve the mathematical skills of preschool children in Head Start around four domains of mathematics. Because of the need to provide necessary mathematical experiences to young children to improve their early understanding and skills and provide the foundation for future success in mathematics, we provided the treatment group of early educators with professional development and center-based activities to promote four critical areas in mathematics. By randomly selecting Head Start centers to participate as the treatment group or control group, we were able to examine the effects of the professional development and set of activities on preschool children's knowledge over a six-month period. We found children in the treatment group were more fluent and flexible with number concepts, were better at solving contextual problems, and had better measurement and spatial abilities than children in the control group.

Keywords: Early Childhood Mathematics; Professional Development; Student Achievement; Head Start

Introduction

Early literacy in mathematics is significantly increasing in importance in regards to preparing students to be more successful in later grades and later in life. Duncan et al. (2007) found early mathematics knowledge to be the best predictor for later mathematics achievement. The new Common Core State Standards for Mathematics (CCSSM) place quantifying number, measuring, and building spatial relationships as necessary constructs to be addressed in preschool curriculum (NGA, 2010).

When comparing students from different countries there are marked differences in what mathematical opportunities preschool children are given. And because of these different opportunities, children as early as age four already are shown to have different mathematical understanding, especially when comparing U.S. students to Asian students (Ginsburg, Lee, & Boyd, 2008). In the U.S., several researchers have demonstrated that students who complete preschool and kindergarten with an inadequate knowledge of basic mathematics concepts and skills will continue to experience difficulties with mathematics throughout their elementary and secondary years (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Morgan, Farkas, & Wu, 2009).

This research points to two critical areas that should be addressed in mathematics education in the early years. First, there is a need to improve the quality of mathematics instruction for pre-Kindergarten students. Secondly, students who are experiencing difficulties in mathematics must be identified early so instruction can be modified to provide students with specific opportunities to address these issues (Chernoff, Flanagan, McPhee, & Park, 2007; Ginsburg et al., 2008).

Opportunities Needed for Young Students

To begin addressing the concern that young children are lacking mathematical skills, we need to examine the literature on what type of mathematics skills are most important for future learning in mathematics and which opportunities are, then, necessary to be provided to them (Schwartz, 2005). The purpose of this study was (a) to determine these critical skill from the literature, (b) build a professional development model with activities for early childhood educators to use and (c) to determine the effects of these teacher and center activities on the four-year-olds' mathematical knowledge.

Most research in mathematics for early learners and primary level students have only focused on the area of number: recognition, sequencing, and magnitude (D. H. Clements & Sarama, 2008). However, recently the National Research Council (2009) called for better and more comprehensive quality instruments to diagnose students' level of competence in different areas of mathematics and asked which of the different areas of mathematics should be the central areas to highlight in preschool. Some early childhood researchers have proposed that preschool programs include more opportunities to address mathematical relationships, contextualized problems, and measurement and spatial tasks (Carpenter, Franke, & Levi, 2003; Clements & Sarama, 2007; Clements, Xiufeng, & Sarama, 2008; Elizabeth Fennema et al., 1996; Ginsburg & Baroody, 2003).

To assist with understanding which areas are most important the "Focus in Prekindergarten: Teaching with Curriculum Focal Points" was released by the National Council of Teachers of Mathematics (NCTM, 2010) and emphasized the learning progressions for young children. First, they describe key processes that should be the focus of early educators: unitizing, decomposing and composing, relating and ordering, and looking for patterns. Second, they discuss that young children need to experience multiple opportunities with core concepts in number, measurement, and space.

In more detail, Clements (2008) and Van de Walle (2007) describe how young children tend to build mathematical ideas. For number concepts, students begin by comparing and ordering (verbal counting and then counting strategies), then solve arithmetic and contextual problems with small number sets, followed by recognizing number and subitizing, and finally composing number. Within measurement, students learn what is shorter and longer and then begin to iterate and partition using paper strips and blocks. Discussing that zero is the starting position when measuring is also a necessary component. Finally, they describe the importance of geometry: identifying, comparing, representing, decomposing, and rotating shapes. Their views of geometry experiences that young children should have are markedly different than what is in the early-grade curriculum. Typically, these resources recommend focusing on students' invented definitions and descriptions prior to formal instruction in geometric terminology and definitions. This broader, property-based vision of geometry instruction places the topic at an equal level of importance alongside number concepts in the young child's mathematics learning.

Four Mathematical Domains

This review of the extant research on early childhood mathematics supports four key areas that predict students' future performance in mathematics: concepts of number, interpreting relationships, and reasoning within measurement and space (Clements, Wilson, & Sarama, 2004; Clements et al., 2008; Starkey, Klein, & Wakeley, 2004). We briefly highlight each area.

Number

Within the domain of number – number recognition, number sequencing, and fluency and flexibility – are described as important early number skills (Clements & Sarama, 2004; Clements et al., 2008; Ginsburg & Baroody, 2003; Y. S. Lee, Lembke, Moore, Ginsburg, & Pappas, 2007). Three mathematical skills: number knowledge, ordinality, and quantitative reasoning have been demonstrated to have an average effect size of 0.34 on later academic success (Duncan et al., 2007).

Fluency and flexibility are intimately linked. Students are *'fluent'* with whole numbers when they can solve fact problems, answer related questions, and extend patterns in a quick and efficient way (Baroody & Dowker, 2003; Griffin, 2003, 2004). By quickly recalling a basic addition fact, a student has demonstrated fluency. But *fluency* is often the byproduct of *flexibility* (Beishuizen & Anghileri, 1998; Thompson, 1997).

Flexibility is the ability to solve problems in a variety of ways, use information already known to solve unknown problems, and the capability to determine the most efficient method to use when confronted with a challenging problem (Star & Madnani, 2004). By thinking flexibly, students reduce the mental effort required to accomplish small steps associated with a task and can exert more effort toward completing the more challenging aspects of the task. Flexible mathematical thinkers have been shown to develop faster recall of basic facts and to be more successful in classroom settings (Beishuizen & Anghileri, 1998).

Interpreting Relationships

Understanding equality and the relationship of numbers and solving contextualized problems form the basis of algebraic understanding (Demby, 1997; L. Lee & Wheeler, 1989; Slavit, 1999; Van Amerom, 2003). Hiebert and Carpenter (1992) demonstrate that young students are capable of using properties of operations (such as the commutative, inverse, and identity properties) when solving arithmetical problems and naturally transfer informal knowledge of these operation properties to new situations. However, Demby (1997) and Lee and Wheeler (1989) provide evidence that by the time students reach high-school algebra they are reluctant or unable to use these operation properties when solving problems. Having realized this problem, other countries have built curricular opportunities to assist students in making the transition from solving contextualized problems and informal approaches to formalized symbolism and algebraic reasoning and notation (Anghileri, Beishuizen, & Van Putten, 2002; Van Amerom, 2003).

Accurately solving contextualized problems (e.g. word problems) is a key factor in early mathematics achievement and there is evidence that this skill is a characteristic found more often in academically successful students than in those with disabilities and low academic performance (Swanson & Jerman, 2006). When students solve word-problems, they are doing more than simply following computational steps. They are making mathematical sense of a realistic situation. This is not only important for students as they learn about mathematical operations, but also as a prerequisite ability for successfully applying algebra to the world outside of the classroom. Contextual problems lay the conceptual groundwork for a deeper understanding of mathematics than do rigid experiences with only arithmetical procedures absent of context.

Measurement

Measurement of length has a direct link to knowledge of fractions and decimals because measurements often do not use complete units (Cramer, Post, & del Mas, 2002; Lehrer, Jaslow, & Curtis, 2003; Watanabe, 2002). A table can be 3½ feet wide. Students must make sense of the 'part' of the unit left over after the 3 complete units are counted. This is different than just counting discrete objects like fingers or cubes (Kamii & Clark, 1997). When counting units of length, the student begins to develop a model for the continuous nature of rational numbers (e.g. fractions, decimals, percents). This knowledge supports the student in learning about fractions and ratios in later grades (Harel & Confrey, 1994). Many nations that use informal measurement and measurement estimation as a way to introduce fractions perform at a much higher level than the United States on rational number items found on standardized tests (Kamii, 1999; Watanabe, 2002). Students in these countries have an understanding of the meaning of rational numbers connected to measurement (Mullis et al., 1997). Measurement tasks also support the idea of proportional reasoning, which in turn helps develop a better sense of geometry, numeracy, and data analysis (NRC, 2001).

The key underlying principles of measurement are unit iteration, partitioning, comparative measurement, and the meaning of measurement. *Unit iteration* is the act of repeating a unit to measure an object's attributes. *Partitioning* is the act of either mentally, or physically, breaking an object into equal-sized measuring units (Lehrer et al., 2003). *Comparative measurement* is the process of using a known measurement from one part of an object to find an unknown measurement on either that same object or a different object. This is sometimes referred to as *transitivity*

(Kamii & Clark, 1997). For measurement, it is important that young children understand that many different attributes can be measured (e.g. weight, area, length, or time) and that we use units to measure attributes by making comparisons. These comparisons are sometimes qualitative in nature (e.g. longer, taller, or heavier), but can also be quantified using a unit. These quantifiable comparisons may utilize standard units of measure, but may also include the use of informal non-standard units producing comparisons such as 3 blocks longer, 2 inches taller, or 5 ounces heavier (D. H. Clements & Bright, 2003).

Spatial Reasoning

Researchers have demonstrated that spatial reasoning has a very high predictive value for mathematics achievement (Battista, 1981; Clements & Sarama, 2007; Gustafsson & Undheim, 1996; Lubinski & Dawis, 1992). There are three categories of spatial reasoning: spatial visualization, spatial orientation, and spatial relations (Lee, 2005). *Spatial visualization* includes the ability to visually or "mentally manipulate, rotate, twist, or invert pictorially presented stimuli" (Lee, 2005, 4). *Spatial orientation* is the ability to remain unconfused when the object's orientation changes (J. W. Lee, 2005; McGee, 1979). *Spatial relations* refer to the ability to recognize spatial patterns, to understand spatial hierarchies, and to imagine maps from verbal descriptions (Lee, 2005).

Because of the complexity of the items within spatial relations, when working with young students we focus on items within the first two categories: spatial visualization and spatial orientation. In order to examine the predictive validity of spatial relation items in the primary grades, it may take until middle school to observe the statistical relationship (Wolfgang, Stannard, & Ithel, 2001). Here is an example of the importance to future success in mathematics: To make sense of geometric formulas in the upper grades we note the usefulness of both spatial visualization and orientation. For instance, the formula for the area of a right triangle, A = ½ x base x height, is far easier to make sense of and remember when a student can mentally 'copy' the triangle and manipulate the copy to join it with the original triangle to create a rectangle.

Spatial reasoning also helps develop fluency with flexible operations in arithmetic and strengthens and supports students' ability in measurement (Battista, Wheatley, & Talsma, 1982; E. Fennema & Behr, 1980; Tartre, 1990). As with measurement, spatial reasoning builds concepts of proportional reasoning, which aids the student in areas as diverse as geometry and data analysis.

Professional Development and Activities

Mathematical knowledge originates from students' attempts to model situations, which can be represented in enactive, iconic, and symbolic forms (Bruner, 1964). As children build understanding of abstract topics (in this case mathematics), Bruner argues that they first "enact" or build physical models (typically with cubes) of the problem. Then, they should attempt to visualize or begin to draw or create an iconic model of the situation. It is only after children have a strong foundation with the enactment and iconic modeling do they begin using and understanding the symbolic representations. This implies that for preschool children, they need opportunities to enact and visualize before they build facts like "3 plus 2 is 5" (Doorman & Gravemeijer, 2009; Gravemeijer & van Galen, 2003).

There are three key features to using professional development to build this type of knowledge and to use materials that support this progression of ideas and models: building content knowledge, creating active learning, and demonstrating coherence with other learning (Garet, Porter, Desimone, Birman, & Yoon, 2001; Hawley & Valli, 2000).

Content knowledge of early educators needs to be well developed in areas of the structure of the mathematics and how young students learn mathematics (Ball, Hill, & Bass, 2005; Ma, 1999). When professional development focuses on specific content knowledge, not general, and is intertwined with how students learn both procedurally and conceptually, it has positive effects on student achievement (Kennedy, 1998).

Active learning is the second feature of quality professional development and includes engagement in tasks that improve teachers' own knowledge of the mathematics, covers the learning progressions, engages teachers in creating hypothetical learning trajectories, and encourages articulation and implementation of these ideas (Garet et al., 2001; Hawley & Valli, 2000). Tasks that improve teachers' knowledge must force them into cognitive dissonance and, then, allow them to integrate the new ideas in a way that makes sense mathematically and pedagogically. For example, an initial task might be to write two contextual problems for the number sentence 2 + 5 = □ and explain (a) the difference in how students will respond to each context and (b) what mathematical models (enactive and iconic) should be introduced in what order for pre-school students. In order to respond to this task, teachers must be able to compose or write a join and a part-whole context.

The third feature is coherence. If teachers do not perceive the professional development and intervention models to be a connected and an integral part of what the school is doing or encouraging, then the features being introduced will not persist over time (Garet et al., 2001). For coherence to be built the professional development activities should focus on the four mathematical domains described earlier.

Summary

Thus far, we have made the argument that there are international and national pressures to ensure that young children are being provided the necessary and appropriate opportunities to build a strong foundation in mathematics. The research describes that young students need opportunities within the domains of number, context, measurement and space. One vehicle to do this is through professional development that focuses on building early educators knowledge of these different mathematical domains, how students' knowledge progresses over time, and of the relevance of these topics for four-year-olds. We designed a study that included professional development with sets of activities that incorporated these four mathematical domains and that examined the change in children's knowledge in comparison to children who did not necessarily have these opportunities.

Method

Participants and Design

Six Head Start centers participated in the study. Using a random number generator, four of them were chosen to be in a treatment group and two of them chosen to serve as a control group. There were a total of 24 teachers in this study; 16 of them were part of the treatment group that received professional development and center activities and 8 were in the control group that received no professional development.

The teachers in both groups were between ages 22 and 40 and had taught on average 6.8 years (SD = 8.1). The teachers varied in level of education: 36% with a high-school degree, 17% with an associate's degree, 31% with a bachelor's degree and 14% with a master's degree.

The 16 treatment teachers taught 111 students who fully participated in the study (56 female and 55 male); the 8 control teachers taught 33 participating students (14 female and 19 male). The average age of the children in the treatment group when tested in the fall was 4.6 (SD = .34) and 5.1 (SD = .35) in the spring and all were eligible for Head Start. Of these children, 23% were English language learners (ELL). The control group was very similar. Their average age in the fall was 4.7 (SD = .52) and 5.2 (SD = .41) in the spring; there were also all eligible for Head Start. In the comparison group, there were 21% English language learners (ELL).

Student mathematics knowledge was tested prior to the professional development and again near the end of the year. Therefore, we used a 2 (treatment versus control) x 2 (pretest versus posttest) design.

Instrument: Prekindergarten – Primary Screener for Mathematics (PK-PSM)

In order to study children's knowledge of mathematics, we used the Pre-Kindergarten Primary Screener for Mathematics PK-PSM (Brendefur & Strother, 2010). This screener was chosen because it assesses children's mathematical skills in the four areas described in the literature review: fluency and flexibility (which included patterning and sequencing, basic addition and subtraction facts, numeracy, and reasoning about quantity), interpreting relationships (which included story problems and ideas of equality), measurement, and spatial reasoning. For analysis we labeled the constructs – Fluency and Flexibility, Contextual Reasoning, and Measurement and Spatial Reasoning (an aggregate of the measurement and spatial reasoning items).

The PK-PSM is composed of 26 items focusing on three related mathematical domains (see appendix). The internal consistency reliability of the overall instrument was good (Cronbach's $\alpha = .87$). Fourteen items measuring Fluency and Flexibility included object counting, sequencing, number order, number comparison, and addition and subtraction facts (Cronbach's $\alpha = .80$). Three items measured Contextual Reasoning – students' ability to solve problems within a context or in a story format (Cronbach's $\alpha = .71$). Nine items measured students' Measurement and Spatial Reasoning, which included unit iteration, unit comparison, shape composition, and shape rotation (Cronbach's $\alpha = .72$). See Appendix A for the description of the PK-PSM items.

The PK-PSM was administered individually to each child by one of four trained test administrators (the inter-rater reliability computed across pairs of administrators was performed ten times and was perfect, Kappa = 1.00). The assessment was given at the child's school in a quite testing area and took between 5 and 15 minutes to administer. The fall administration was completed in late September and early October and the spring administration was completed in late March and early April.

Treatment: Professional Development and Activities

Keeping the three professional development features in mind – pedagogical content knowledge, active learning, and coherence – workshop activities were created around the four foundation building areas of mathematics: number, context, measurement and spatial relationships. Eight hours of workshop activities were designed to (a) focus on introducing the mathematical topics, (b) provide the theoretical foundation for how young children learn mathematics and how to address early learning progressions, and (c) practice setting up the classroom activities and asking students questions and responding in ways to extend student understanding of the topic. As each of the four mathematical domains was introduced in the workshop, teachers were provided sets of classroom center-based activities with any needed student materials. There were eleven sets of activities (see Appendix). Early educators were given a few months to try out the classroom activities and then received follow-up professional development focusing on implementation. Here, teachers were able to re-learn some of the earlier learned ideas and ask questions on how to better implement the classroom activities.

Each of the classroom activities were designed to be implemented either as small-group activities that take 10 to 20 minutes with three to five students or were modified or extended into larger whole-group tasks. In some instances, activities could be implemented in either small group or whole-group settings without any variation (e.g. Collection Buckets or block play activities). The teachers were instructed to sit with the children, present the material and follow either scripted directions or a guiding activity template that provided stories, mathematical information, and questions. Teachers were encouraged to extend the lessons and follow-up with similar activities and questions either from related modules or readily available pre-school curricular materials that were similar in scope to the implementation ideas shared in the professional development sessions.

This professional development approach offered teachers an opportunity to reconsider their own preconceptions of mathematics, correct long-held mathematical misconceptions and to reflect on the experiences they might provide their students when teaching these mathematical activities (Schoenfeld, 1994). In some cases, participants received instruction regarding the facilitation of the modules that were specifically intended for pre-school age children (e.g. Story Mats, Ice Cream Shop, Dot Plates, etc).

Results

We conducted a 2 (treatment versus control) x 2 (pretest versus posttest) analysis of variance (ANOVA) to evaluate the effect of the professional development on students' mathematics knowledge—with the total score on the PK-

PSM as the dependent variable. There was a main effect for group, $F(1, 126) = 4.59$, $MSe = .03$, $p = .03$, *eta squared* = .04; and for time, $F(1, 126) = 112.07$, $MSe = .03$, $p < .001$, *eta squared* = .47; however, these main effects were mediated by a significant interaction, $F(1, 126) = 11.51$, $MSe = .03$, $p = .001$, *eta squared* = .08.

Test of simple effects showed that the groups did not differ on test performance at the pretest, $F(1, 126) < 1$, but differed significantly at the posttest, $F(1, 126) = 19.36$, $p < .001$, *eta squared* = .13. As seen in Figure 1, students in the treatment group improved more from pretest to posttest than did students in the control group.

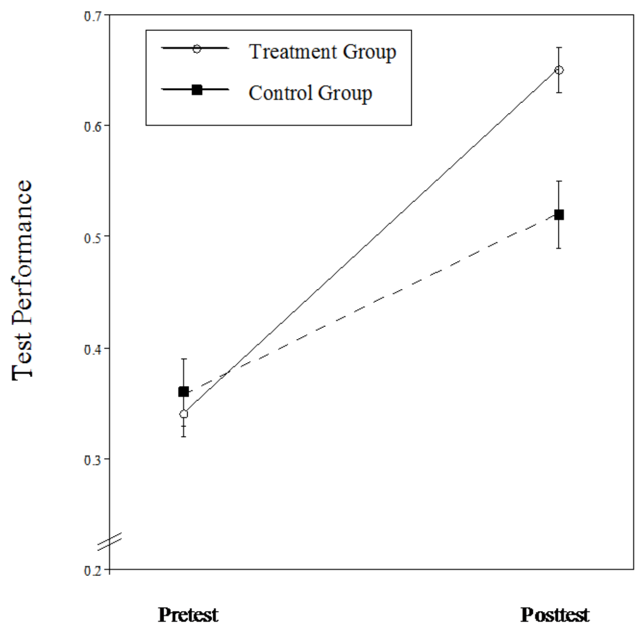

Figure 1. *Proportion correct on the overall test at pretest and posttest by group. Error bars are the standard error of the mean*

We also analyzed student performance by the different domains, which showed similar patterns of results as overall test performance. For Fluency and Flexibility, there was not a main effect for group, $F(1, 126) = 1.88$, $MSe = .04$, $p = .17$. There was a significant main effect for time, $F(1, 126) = 63.15$, $MSe = .03$, $p < .001$, *eta squared* = .33. The interaction was also significant, $F(1, 126) = 10.42$, $MSe = .03$, $p = .002$, *eta squared* = .08. Test of simple effects showed that the groups did not differ on test performance at the pretest, $F(1, 126) < 1$, but differed significantly at the posttest, $F(1, 126) = 14.67$, $p < .001$, *eta squared* = .10. As seen in top section of Table 1, students in the treatment group improved more from pretest to posttest than did students in the control group.

For Contextual Reasoning, there was not a main effect for group, $F(1, 126) = 1.12$, $MSe = .13$, $p = .29$. There was a significant main effect for time, $F(1, 126) = 43.76$, $MSe = .13$, $p < .001$, *eta squared* = .26. The interaction was not significant, $F(1, 126) = 3.45$, $MSe = .13$, $p = .07$. As seen in middle section of Table 1, students in both groups improved across time. The marginally significant interaction suggests that the treatment group may have improved more from pretest to posttest than did students in the control group, but given the interaction did not reach significance, more research is needed to confirm this finding.

For Measurement and Spatial Reasoning, there was a main effect for group, $F(1, 126) = 10.51$, $MSe = .04$, $p = .002$, *eta squared* = .08; and for time, $F(1, 126) = 72.47$, $MSe = .04$, $p < .001$, *eta squared* = .37; however, these main effects were mediated by a significant interaction, $F(1, 126) = 4.37$, $MSe = .04$, $p = .04$, *eta squared* = .03. Test of simple effects showed that the groups did not differ on test performance at the pretest, $F(1, 126) = 1.27$, but differed significantly at the posttest, $F(1, 126) = 16.57$, $p < .001$, *eta squared* = .12. As seen in the bottom section of Table 1, students in the treatment group improved more from pretest to posttest than did students in the control group.

The purpose of this study was to examine the effects on four-year-olds' knowledge of mathematics by introducing professional development and center-based mathematics activities around four mathematical domains to early educators' teaching in Head Start programs. Overall, we found that the professional development sessions coupled with activities to use in classrooms had a statistically significant effect on children's mathematical knowledge.
Much of the past research focused primarily on constructs within the domain of number to create professional development for early educators and to assess students' mathematical knowledge. We were able to demonstrate that by focusing professional development on developmental and conceptual learning progressions (Baroody & Dowker, 2003; Clements et al., 2004) and by focusing center-based activities within these progressions and including the use of enactive, iconic, and symbol models of representing the situations (Bruner, 1964), students were more fluent and flexible within number situations, increased their ability to solve contextual problems, and were better able to understand and solve more spatial situations.

The control group did improve in these areas over a six month period, but as Schwartz' (2005) claimed, given the opportunities the treatment group excelled. More specifically, providing children with specific types of mathematical activities and by asking specific follow-up questions, their knowledge increased dramatically. This is important as mentioned in the review of literature to begin providing more opportunities on more critical topics in mathematics to build a stronger foundation for young children as they enter elementary school.

One explanation for why students' knowledge for contextual situations did not increase as much over time as compared to the other constructs could be that the early educators found it more time-consuming and more difficult to have students solve these types of problems or it could be that students were just less interested in taking the time to solve these problems. The other – number, measurement, and spatial – activities could be set up as centers and seemed to engage students' interests quite readily without necessarily having the teacher oversee the activity.

Limitations

The professional development was limited in time to eight hours. With more professional development and more time in classrooms, teachers' mathematical practices could increase as would their students' mathematical abilities.

Implications for Future Studies

This study demonstrates that mathematics professional development focused on number, context, measurement and spatial activities have an impact on early educators' and students' mathematical knowledge. This initial bump in children's knowledge can possibly influence their mathematical abilities throughout their elementary years as proposed by other researchers (Duncan et al., 2007; Jordan et al., 2009; Morgan et al., 2009). Another study tracking the residual effects of these children's knowledge longitudinally would allow us to understand how important it is to provide young children specific mathematical opportunities.

To understand the effects of the professional development (Garet et al., 2001; Hawley & Valli, 2000) in more detail, we would like to provide the sets of classroom activities to a group of early educators to use throughout the year. This group would not receive any professional development. This would allow us to understand the degree of importance the professional development would have on teachers' practice and whether this effect with the modules would increase students' mathematical knowledge more so than just using the activities.

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Appendix
Description of PK-PSM items

	PK-PMA Item
1 – 5	Counting
6 – 8	Number fact
9 – 10	Numeral recognition or identification
11 – 14	Number comparison
15 – 17	Addition or Subtraction
18 – 19	Measurement
20 – 22	Shape Composing
23 – 26	Shape Rotation and Composing

Note: Number tasks: 1-14; arithmetic tasks: 15-17; measurement and space tasks: 18-26.

Pre-K mathematical content and examples of activities

Center	Mathematical content	Example
Dot Plates	Number: Counting a set; one-to-one correspondence, numeral recognition	Children are given a plate with a set of dots on one side and the numeral on the back.
Story Mats	Context: addition and subtraction of sets of objects	
Collection Buckets	Number: Counting a set; one-to-one correspondence, numeral recognition	Students must fill a bucket with the exact number of objects matching the number
Math Stories	Context; addition and subtraction of sets of objects	
Pattern Block Puzzles	Spatial sense: shape matching	Students were challenged to cover various picture mats with pattern blocks. The mats often did not have lines marking the shapes needed.
Pattern Block Trays	Spatial sense: decomposing and composing shape, shape matching	
I Spy Mats	Context and Number: addition and subtraction	Students reviewed pictures containing many quantities of different items (cars, birds, trees, etc.) and had to describe the events happening in the picture using numbers.
How Many Steps?	Measurement: comparison of length, measurement of length with nonstandard units	Students walked from one location in the room to a specified landmark, counting their steps and focusing on only counting when they had completed a step.
The Race Car Game	Measurement and Number: unit iteration, number identification, one-to-one correspondence	Students used dotted dice to move game pieces along a series of markings that form a track from one side of a large game board to another.
Small Blocks	Space and Number:	Building with various blocks to encourage children to make comparisons and geometric structures.
Big Blocks	Space and Number:	