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Anatomy of the 3D Innovation Production with the Cobb-Douglas Specification

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Abstract
This paper focuses on verifying the relevance of two theoretical propositions and related empirical investigation about the relationship between creativity and entrepreneurship. It draws upon a creativity process that considers three “dimensions” or “disciplines” (3D) critical for creative organizations—within discipline expertise, out of discipline knowledge, and a disciplined creative process. The paper first explores the Cobb-Douglas production function as a relevant tool for modeling the 3D creative process. The next part discusses the 3D process as a production function, which is modeled following the well-known Cobb-Douglas specification. Last, the paper offers implications for future research on disciplined creativity/innovation as a method of improving organizations’ creative performance. The modeling shows that labor and investment can readily enter into the 3D creativity process as inputs. These two inputs are meaningful in explaining where innovation outputs come from and how they can be measured, with a reasonable theoretical decomposition. It is not true that the more capital investments in the creativity process, the better the level of innovation production, but firm’s human resource management and expenditures should pay attention to optimal levels of capital and labor stocks, in a combination that helps reach highest possible innovation output.

Keywords
Organization of production, firm behavior, business economics, creativity/innovation processes, Cobb-Douglas production function

This paper focuses on verifying the relevance of previous theoretical discussions and empirical investigations (Napier 2010; Napier and Nilsson 2008; Napier and Vuong 2013; Napier, Dang, and Vuong 2012; Vuong, Napier, and Tran 2013) about three “dimensions” or “disciplines” (3D) critical for creative organizations, the creativity process of “serendipity”, the relationship between creativity and entrepreneurship and its link to a disciplined creativity process based on the useful information flow, filtering mechanism (Vuong and Napier 2012a). In essence, the paper examines whether creativity may possibly play a role in the production function and economic performance at the organizational level, with their production outcome being used by other departments and internal units.

INTRODUCTION, RESEARCH ISSUES, AND OBJECTIVES
This article focuses on the idea of learning how a creative process at the organizational level can
enhance managers’ understanding about economic principles of using labor force and investment for obtaining optimal results from such a production process. It is not obvious that one can see creative performance of an organization or departmental units as consumption of resources, which are limited and subject to further organizational constraints. That means, “creative power” should also be regarded as a limited resource subject to various economic laws at the organizational level, facing various issues that need to be sorted out, such as the “resource curse” problem and law of diminishing returns.

According to Vuong and Napier (2012b), the classic notion of “resource curse” has been discussed in terms of absence of creative performance, where over-reliance on both capital resource and physical asset endowments has led to inferior economic results for corporate firms. While successful companies clearly have to be able to activate sources of investment for future growth, the efficiency of investment must rest with innovation capacity, which needs to be modeled in some insightful way.

Naturally, this discussion has several key objectives as follows. First, the authors like addressing the question of whether or not one can consider creative performance, with its generally spoken about elusive nature, a process of putting production inputs together under a discipline. Second, a logical question should be whether any of the well-know production functions can play a role in describing the impact of each input in a way that helps enhance the managers’ understanding. Third, observing the results of such “experiment” should suggest management implications in terms of perceiving organization’s creative performance and suggestions toward making such “production process” better.

To this end, the paper has three main parts. First, an exploration of the Cobb-Douglas production function as a relevant tool for modeling such 3D creative process is made. The next part discusses the 3D process as a production function, which is modeled following the well-known Cobb-Douglas specification. The last part offers some further discussions and implications for future research on disciplined creativity/innovation as a method of improving organizations’ creative performance based on the concept of creative quantum and industrial disciplines.

### THE UNDERLYING RATIONALE FOR THE MODELING OF A 3D CREATIVE PROCESS USING THE COBB-DOUGLAS PRODUCTION FUNCTION

#### The Cobb-Douglas Function

The Cobb-Douglas production function was developed for the first time in 1927 by two scholars Charles W. Cobb and Paul H. Douglas, having its initial algebraic form of $Q = f(L, C) = bL^kC^{k'}$, following which they found $k = .802$ and $k' = .232$ for the US industrial production data from 1899 to 1922, using the least squares method (Cobb and Douglas 1928; Douglas 1976; Lovell 2004). In a typical economic model where Cobb-Douglas is plausible, $Q$ is aggregate output, while $L, C$ are total numbers of units of labor and capital employed by the production process for a period of time (e.g., a year), respectively.

This production function and also Leontief function are special cases of the CES (Constant Elasticity of Substitution) production function (Arrow et al. 1961). Another model by Solow (1957), also in the generic form of $Q = f(K, L; t)$, implies that the term “technical change” (or technological change) represents any kind of shift in the production function, and technology becomes part of the capital factor employed in a production process.

#### Why Modeling a 3D Process Following Cobb-Douglas Production Function Is Relevant

Despite its limitations as pointed out by several critics,
the Cobb-Douglas production function has still been a useful model, especially when it comes to describe small-scaled and simple “economy” such as the 3D innovation process. Albeit looking simple, the Cobb-Douglas production function is capable of modeling many scientific phenomena, and therefore can bring up useful insights while retaining the key characteristics learned from real world observations.

There are conditions that form the constraints for such a modeling effort, imposed by the economic nature, such as Inada conditions. About this aspect, Barelli and De Abreu Pessoa (2003) concluded that “for the Inada conditions to hold, a production function must be asymptotically Cobb-Douglas”. In fact, following Barelli and De Abreu Pessoa (2003), it can be seen that Cobb-Douglas was the limiting case of the CES production functional form of \( Y = A[aK^\alpha + (1-\alpha)L]^\gamma \) as \( \gamma \rightarrow 0 \).

Another useful linear function in logarithmic form can be written as: \( f(I) = \ln (Y) = a_0 + \sum a_i \ln (I_i) \), which bears similar meanings to standard form of the familiar production (and utility) function in economic discussions. Further discussion in relation to this specification can be found in Simon and Blume (2001: 175, 734).

Also, a 3D process can be viewed as an economy to produce innovative output, using inputs of “creative quantum” and resources in the form of industrial disciplines (Vuong and Napier 2012a). The analogy leads to the consideration of logic found in the Cobb-Douglas function that \( L \) can represent the “disciplined process” through which useful information and primitive insights about possible innovative solutions are employed and processed diligently, toward making innovative changes for a department or an organization as a whole. Such informational inputs can readily be considered as some kinds of “working capital” for the disciplined processes—together with any organizational machines serving the innovation goals—and can be somehow regarded as \( K \) in a specification of the Cobb-Douglas model.

## MODEL OF INNOVATION AS A PRODUCTION FUNCTION

This paper uses the concept of innovation provided in Adam and Farber (1994: 20-22), which is concerned with inventions, processes, and products (and services). These innovations could be considered “commercially realizable”, which was from Adam and Farber’s (1994) exact definition: “L’innovation est l’intégration des inventions disponibles dans de produits et procédés commercialement réalisables” (The innovation is the integration of inventions available in commercially feasible products and processes), by entrepreneurs and business managers with both outward and inward looking views.

Following the concept by Adam and Farber (1994), the innovation production in the Cobb-Douglas form is now written as:

\[
Q_l = F(L, K) = AL^\alpha K^\beta
\]

where \( 0 < \alpha, \beta < 1 \).

There are three cases where it is suggested if a company falls into the category of increasing innovation “return”, or constant or decreasing, it would be determined by:

\( \alpha + \beta > 1 \), \( \beta = 1-\alpha \), or \( \alpha + \beta < 1 \), respectively. In the general form, \( \alpha, \beta \) are technology-defined constants, which will later provide for some useful management implications.

The first attempt is now made to look at the first case, similar to Cobb and Douglass’s first look into the US economy in 1928 (Douglas 1976), where we solely consider the “corporate economy” exhibiting property of constant returns to scale:

\[
Q_l = F(L, K) = AL^\alpha K^\beta
\]

Equation (2) fits into the definition of an homogeneous function, by which a function \( f: \mathbb{R}^n \supseteq X \rightarrow \mathbb{R} \), where \( X \) is a cone, is homogeneous of degree \( k \) in \( X \) if

\[
f(\lambda x) = \lambda^k f(x), \forall \lambda > 0
\]
As shown in De la Fuente (2000: 189) following Euler theorem (p. 187), since \( f(x) = \prod_{i=1}^{n} x_i^{\alpha_i} \) is homogeneous of degree \( \sum_{i=1}^{n} \alpha_i \), the Cobb-Douglas production function for the 3D innovation process is in fact a linearly homogeneous function with continuous partial derivatives. This property is convenient to explore the behavior of the supposed 3D innovation production function.

Borrowing the concept of “technology and factor prices” advocated by economists in a neoclassical world, the specification in equation (1) refers to \( A \) as an indicator of “total factor productivity”. Businesswise, \( A \) is telling about the current state of technological level prevailing in the current business context.

The two parameters (which following proper regressions should become estimated coefficients), \( \alpha \) and \( \beta \), indicate elasticity measures of output to varying levels of stock of creative quantum (\( C \)) and investment in a typical 3D process (\( L \)). Economic theories have demonstrated that \( F(L,K) \) is a smooth and concave function that exhibits similar properties to a classic Cobb-Douglas function:

\[
\begin{align*}
Q_L, Q_K &> 0; \text{ and, } Q_{LL}, Q_{KK} < 0 \\
F_L &\to 0 \text{ as } L \to \infty; F_L &\to \infty \text{ as } L \to 0 \text{ and,} \\
F_C &\to 0 \text{ as } K \to \infty; F_L &\to \infty \text{ as } L \to 0
\end{align*}
\]

(4.a)

and

\[
\begin{align*}
F_L &\to 0 \text{ as } L \to \infty; F_L &\to \infty \text{ as } L \to 0
\end{align*}
\]

(4.b)

Clearly, equation (4.b) is a set of Inada (1963) conditions, while equation (4.a) simply states basic economic laws for increasing output function when each input (\( L \) or \( K \)) increases, \( ceteris paribus \), but with slower pace of incremental output, usually referred to as law of diminishing returns (Lovell 2004: 208-218).

For \( \lambda > 1 \), it implies that \( F(\lambda K, \lambda L) > \lambda F(K, L) \), which is said to show “increasing returns to scale”. In the case of Cobb-Douglas model, it is ready to see that:

\[
\begin{align*}
F(\lambda K, \lambda L) &= A(\lambda K)^{\alpha} (\lambda L)^{\beta} = \lambda^{\alpha+\beta} AK^{\alpha}L^{\beta} \\
&= \lambda^{\alpha+\beta} F(K, L)
\end{align*}
\]

This represents increasing returns only if \( \alpha + \beta > 1 \), and constant when \( \beta = 1-\alpha \).

The marginal product of labor is: \( \frac{\partial Q}{\partial L} = \alpha AL^{\alpha-1}K^{\beta} \), which can be simplified as \( \frac{\partial Q}{\partial L} = aQ/L \) (Lovell 2004).

Likewise, \( \frac{\partial Q}{\partial K} = bQ/K \) represents the marginal product of “creative quantum” as defined in Vuong and Napier (2012a). For the problem of maximizing profit from such Cobb-Douglas specification, the firm theory reaches the solution that determines maximal profit as:

\[
\frac{K}{L} = \left( \frac{\alpha}{\beta} \right) \frac{w}{r}
\]

(5)

Again, in the above ratio \( K/L \) of equation (5), \( L \) is “Labor” for creative discipline; \( K \) is “Capital” that can bring “creative quantum” into the innovation production process at the firm level. There are a few hints that are needed for a successful modeling of our 3D innovation process.

First of all, the function is considered as a special case where \( \alpha + \beta = 1 \), i.e., homogeneous of degree 1. Following the theory of the firm, homogeneous function of degree 1 implies that the technology this Cobb-Douglas function represents exhibits constant returns-to-scale. This Cobb-Douglas represents smooth substitution between goods or between inputs, which is different from Leontief production function.

The following graph (given in Figure 1) for a special case of Cobb-Douglas production function with \( \alpha + \beta = 1 \) is produced following the commands provided in the Appendix A.1 (also see Kendrick, Mercado, and Amman 2005; for a rich account of high-level computer packages dealing with computation economics problems).

Second, learning from the Consumer Theory (Lovell 2004; Simon and Blume 2001; Varian 2010), the maximizing of the 3D innovation production can be equivalent to the maximizing of a utility function of innovation, which can take a logarithmic form, without losing generality. The maximizing problem
Figure 1. Graph of a Cobb-Douglas Specification $\alpha + \beta = 1$.

Figure 2. Constraint of the Maximization Problem (6).

Figure 3. Graphical Presentation of the Maximization Problem (6).
has the form:
\[ \max u(K, L) = L^\alpha K^\beta \]  \hspace{1cm} (6)

where: \( m \) is total expenditure on innovation, and \( w, r \) labor unit cost (for instance, wage per hour per person) and cost of capital (interest rate for a loan used in the business process), respectively. This linear constraint can be observed graphically with numerical values \( w = .5, r = .25, m = 5 \) in Figure 2.

The maximization problem is now effectively becoming the problem of finding the optimal \((L^*, K^*)\) that makes \( Q \) maximal given the constraint \( m = wL + rK \), which should lie on the curve where the two surfaces (a plane in Figure 2 and a curvy surface in Figure 1) intersect, as shown in Figure 3.

The logarithmic transformation of \( u(K, L) \) gives us: \( \ln(u) = a \ln(L) + b \ln(K) \). To derive the system of equations known as the first order conditions (FOC) for finding maximum of the production, we follow the Lagrangian method by writing the following Lagrangian \( \mathcal{L} \) provided in equation (7):
\[ \mathcal{L} = \ln(u) + \lambda[m - (wL + rK)] = a \ln(L) + b \ln(K) + \lambda[m - (wL + rK)] \]  \hspace{1cm} (7)

where \( \lambda \) is a Lagrange multiplier.

The system of equations for FOC is derived from the above expansion by taking the first-order partial derivatives with respect to each of the variables \( L, K, \lambda \) of \( \mathcal{L} \) (for technical details, see De la Fuente 2000; Lovell 2004; Simon and Blume 2001; Varian 2010). And they are provided below:
\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial L} &= 0 = \alpha - w\lambda \\
\frac{\partial \mathcal{L}}{\partial K} &= 0 = \beta - r\lambda \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0 = m - rK - wL
\end{align*}
\]

These conditions represent necessary and sufficient conditions for the log function to have maximal value (for mathematical treatments and proofs in relation to this type of math problem, see De la Fuente 2000; Simon and Blume 2001; Varian 2010).

Therefore, the following solution set shows values where the system attains its maximum:
\[
\begin{align*}
\lambda^* &= \frac{\alpha + \beta}{m} \\
L^* &= \frac{m \alpha}{(\alpha + \beta)w} \\
K^* &= \frac{\beta m}{(\alpha + \beta)r}
\end{align*}
\]

The results can be analytically checked by using symbolic algebra computing package such as Mathematica® (see Appendix A.2 for ready-to-use interactive commands). Assigning numerical values \( \alpha = .8 \) and \( m = .5 \) enables us to produce the graph in Figure 4 showing the behavior of \( L \) with respect to \( w \) (see Appendix A.3). When wage is increasing, the consumption of labor stock reduces.

Then, a similar performance is done with respect to \( K \) and obtain a graph showing the corresponding behavior of \( K \) with respect to change in \( r \) in Figure 5 (see Appendix A.4). Similar to the labor factor, when cost of capital increases, the consumption of capital stock should decrease, too.

For a clear illustration, particular numerical values \( \alpha = .8, \beta = .2 \) and \( m = 1 \), optimal numerical values of \( L, K \) are \( \frac{8}{w} \) and \( \frac{2}{r} \), respectively, which when put together should yield a production level of:
\[
\left( \frac{8}{w} \right)^{\alpha} \left( \frac{2}{r} \right)^{\beta}
\]

**CONCLUSION AND MANAGEMENT IMPLICATIONS**

This section provides some conclusions about the above exercise, and then follows with implications at work for business managers.

**Overall Conclusions**

First, when innovation output can be measured in monetary terms, productive factors of labor work and capital expenditure can be modeled to reflect their
individual contribution under the Vuong-Napier's ideas of “creative quantum” and “3D process”. This modeling successfully clarifies where the value of creative performance comes from, basically work values. And to the hypothesis, these is exactly the nature “innovation” in industrial environments.

Second, the Cobb-Douglas function has shown its power in explaining contributions of labor and capital in a 3D creative process, which represent general input values in production. These are understandable and relevant to business managers, who are more familiar with the concept of “maximizing existing resources at hand for best business values”. The modeling satisfies this need of managers.

Third, observing the results of such modeling suggests managers about the “behaviors” of input factors which are determined by well-known laws of demand-supply with relevant business constraints. The principle of “resource scarcity” is reflected clearly in a business setting with preset goals and
given capital and physical resources.

**Some Key Management Implications**

The modeling of an innovation production following the Cobb-Douglas specification shows that \( L, K \) can enter into the 3D creativity disciplined process as inputs. As shown in the previous theoretical discussion and actual modeling, these two inputs are meaningful in explaining where innovation outputs come from and how they can be measured in terms of quantity, with a reasonable theoretical decomposition. Logically, this reinforces Vuong and Napier (2012a)’s concepts of “creative quantum” and “creative disciplined process”. To a certain extent, the concepts of “soft” and “permanent” banks in the said work can also reflect the “quantum” and “discipline” components in this discussion about a Cobb-Douglas specification.

Second, the useful meanings of separating novelty and appropriateness can be seen more clearly by decomposing the “value” of innovation process as a Cobb-Douglas function because the derived optimal value has a significant meaning since max innovation depends on: (1) technological level, given the business context; and (2) wage and borrowing rate in the financial marketplace. Clearly, it is not true that the more capital investments in the creativity process, the better the level of innovation production is.

This modeling also helps explore different typical cases where “returns-to-scale” are not just constant, but also increasing and decreasing. In fact, it is well-known that a company can be moderately creative in their performance, explosive or even not creative at all. With a feasible modeling, this exploratory exercise becomes both useful and ready with reasonable implications on management practices.

For business managers, their practices in human resource management and cost allocations should pay attention to appropriate levels of capital and labor stocks, in a combination that helps the organization reach optimal level of output, that is maximal innovation, as specified by such modeling, and not exceeding a budget constraint for input elements of their production process, such as what is discussed by equation (6).

Last but not least, this study shows that further empirical studies based on this modeling of creative disciplines following the Cobb-Douglas function in the real-world industries should provide for many important insights, which are ready for management applications, through the determining of numerical values for \( \alpha, \beta \), their empirical relationships to \( K, L \).

Such data sets, when obtained from real-world business samples, can also provide inputs for further discriminant analysis that distinctively classifies business populations into groups of creative performance without ambiguity. Previous observations following the result offered by Vuong et al. (2012) also suggest that such empirical investigations should even better model the difference between stages of business development in relation to firms’ creative performance.

**APPENDIX**

The following commands can readily work on Mathematica® interactive command window by copying and pasting each group of commands then pressing “Shift+Enter”. The computations were performed on Mathematica® version 5.2. A lucid presentation on practical usage of Mathematica® is provided in Gray (1997).

(1) A.1. For Figures 1, 2, and 3 (see Figure A1):

- Clear[L, K, a, b];
- \( a = 0.8 \)
- \( b = 0.2 \)
- Inno = L^a K^b;
- Constraint = \( m - (w \ L + r \ K) \);
- \( w = 0.5 \)
- \( r = 0.025 \)
\[ Q(L,K) = L^{0.8}K^{0.2} \]

\[
\begin{align*}
m &= 5 \\
P1 &= \text{Plot3D}[\text{Inno}, \{L, 0, 5\}, \{K, 0, 12\}, \\
&\quad \text{AxesLabel} \to \{"L", "K", "Q level"\}] \\
P2 &= \text{Plot3D}[\text{Constraint}, \{L, 0, 5\}, \{K, 0, 12\}] \\
&\quad \text{Show}[P1, P2, \text{DisplayFunction} \to \$\text{Display}\$\text{Function}] \\
\end{align*}
\]

(2) A.2. For algebraically solving for values of \( L, K, L \):

Clear[L, K, a, b, l, w, r];
lnu = a Log[L] + b Log[K];
budget = m - (w L + r K);
eqL = \text{Lagrangian} = \text{lnu} + l \text{budget};
foc1 = D[eqL, L]
foc2 = D[eqL, K]
foc3 = D[eqL, l]

Solving these FOCs using Mathematica

\[
\text{Solve}\{\text{foc1, foc2, foc3}, \{L, K, l\}\}
\]

should obtain the following results:

\[
\{ \{ l \to a + b \over m, \ L \to am \over (a + b)w, \ K \to bm \over (a + b)r \}\}
\]

(3) A.3. For Figure 4: In this computation, the transformation rules are: \( a \to .8 \), and \( m \to .5 \), which assign specific values to the parameters \( a \) (\( \alpha \)) and \( m \).

\[
w = a m / L; \\
\text{Plot}[w \ /. \{a \to .8, m \to .5\}, \{L, .01, .5\}, \\
&\quad \text{AxesLabel} \to \{"L", "w"\}, \text{PlotLabel} \to \text{"Demand for L"}]
\]

\[
r = b m / K; \\
\text{Plot}[r \ /. \{b \to .2, m \to .5\}, \{K, .01, .5\}, \\
&\quad \text{AxesLabel} \to \{"K", "r"\}, \text{PlotLabel} \to \text{"Demand for K"}]
\]

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References


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