Bose-Einstein Condensation in a Double-Well System

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ABSTRACT
The two-site Bose-Hubbard model is a simplified model of $N$ bosons confined to two quantum potential wells (Double Quantum Well, DQWs). We study systematically the behavior of low-temperature Bose-Einstein condensates (BECs) as a function of the strength of the repulsive interactions ($U$) between the bosons and the amount of quantum-mechanical tunneling between the wells ($t$), employing both analytical and computational methods.

In particular, we study the transition between a single BEC comprised of bosons in both wells, and a fragmented condensate consisting of two distinct BECs, one for each well.

Bose-Einstein Condensates
A Bose-Einstein condensate (BEC) is a “state of matter” associated with quantum behavior which manifests macroscopically:

Very nearly all the objects occupy the same quantum mechanical state, producing a “quantum condensed” system with many strange properties (superfluidity or superconductivity, for example).

We focus on the macroscopic quantum properties of the system near the absolute zero of temperature (experimental BEC systems have been produced with temperatures below one-millionth of a degree Kelvin above absolute zero).

Two-Site Bose-Hubbard Model

\[
\hat{H} = -t(a_1^\dagger a_2 + a_2^\dagger a_1) - \frac{V}{2}(a_1^2 a_1^\dagger - a_2^2 a_2^\dagger) + \frac{U}{2} \sum_{i,j=1}^2 a_i^\dagger a_i a_j^\dagger a_j
\]

Our analysis diagonalizes the Hamiltonian matrix to solve the Schrödinger equation for the DQW, obtaining the energies and states of the system.
- Our study assumes a repulsive inter-particle potential ($U > 0$).
- We typically generate results for the balanced case, without a bias or voltage ($V = 0$).

For a fixed number of particles, $N$, we study particles in various regimes: interested in systems in the Fock ($tN/2 << U$), Josephson ($2U/N << t << UN/2$), and Rabi ($U << 2t/N$).

Preliminary Results, Future Investigations
We observe that even very small amounts of tunneling lead to coherence (entanglement) between wells, producing a single condensate from a fragmented one. Future results will quantify this effect more carefully as a function of the tunneling.

In the future we will use the DQW to examine the amount of tunneling necessary to produce interlayer coherence in more complex systems, such as bilayer quantum Hall systems.

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