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Put Another Way...

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## Skeptical Mathematics?

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**1** Ernst von Glasersfeld seems to say to social constructivists, “You attribute reality to society, but your society is just another construct, all you know is just the bits of light and shadow and color that your visual system provides to you.” Now, “society” is just a big word for “other people.” I can give to this text of von Glasersfeld’s either a short answer or a long one.

**2** Here’s the short answer. Von Glasersfeld is skeptical about whether I exist, but he is sure that he is skeptical about that. Moreover, he is sure that I should be skeptical about whether *he* exists, and that I ought to be quite sure about *that*.

**3** If that is too quick and easy, then here is a longer answer.

The problem for philosophy of mathematics is mathematical certainty. Everybody seems to be quite sure that, for example, the sum of the squares on the sides of a Euclidean right triangle equals the square on the side of the hypotenuse. But our ordinary everyday knowledge and our empirical scientific knowledge all seems to involve at least some slight tiny modicum of dubitability. How can we be so absolutely sure of mathematical truth? Formalists say, “These so-called math truths are just empty symbolism.” Platonists say, “They are perceptions of transcendental eternal inhuman realities.” Some, like me, say, “They are about concepts, which are socially held and historical. Therefore, questions about the nature of mathematical truth come down to questions about human thinking, at the social level, and have to be studied as empirical problems about thought and culture and even about the brain itself.”

**4** Now, it would be presumptuous of me to put words into the mouth of von Glasersfeld, so instead of imagining what he might say, let

me reason as a hypothetical “social constructivist” philosopher of mathematics who is actually convinced by the radical constructivism (really, Humean skepticism) of von Glasersfeld.

**5** “Aha! I have been trying to ground the reality of the Euclidean triangle in the reality of the text book, the classroom, and the consensus of the mathematical world. But now I see! I have no right to assume there *is* a mathematical world, or even a classroom, or even a text book. I only know these lights and darks and colors, that I hypothetically imagine may perhaps come from some conjectured reality that I myself have *constructed* into a text or a classroom or whatever! Instead of explaining the nature of mathematical knowledge, I now understand that mathematical knowledge itself is merely another construct! This may be of little or no use to me as a teacher, a student, or a researcher. But it will certainly be of great help if I am ready to give up any interest in teaching, studying, or researching.”

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### Preamble

**1** As Ernst von Glasersfeld has reminded us many times in many publications and presentations, what is written here “...does not purport to describe a real world, but merely proposes a model of *how one could imagine knowledge to be built up*” (§42, *emphasis in the original*). To bring up the point that what is written by a radical constructivist, such as von Glasersfeld, myself or others, is not an assertion of truth runs the risk that George Lakoff suggests in the title of his book *Don’t think of an elephant* (Lakoff 2004). To read the sentence is to do the very thing one is invited not to do. But, it seems, for realists, indeed anyone subjected to a life experience growing up in a realist society, one can hardly help associating what someone has written with attributions of truth on the part of the author. Since such associations are incessantly, but subtly and

implicitly, cultivated in the minds of all in cultures such as the one we live in, the only way to become independent of such reflexive associations is to wrestle with them explicitly.

**2** What follows is offered as an alternative explanation of the nature and status of the concept of society. This alternative is entirely compatible with radical constructivism as described elsewhere by von Glasersfeld (1995, 1999) and some others. This is not a claim of the truth-value of this alternative interpretation, but instead is no more and no less than a claim that there is an alternative explanation that fits and is useful. It is not even a claim of primacy or superiority in some comparative sense. It is merely the claim that this alternative explanation of the nature and status of the concept of society exists, is viable, and is compatible with radical constructivism: a radical constructivist’s explanation of “society.”

### The nature of knowledge of society

**3** My claim is that, at a certain level, asking “Where do our notions of society ‘come from?’”, “Where do our notions of love ‘come from?’” and “Where do our notions of force ‘come from?’” are all equivalent questions, in particular with regard to von Glasersfeld’s

claim to offer a theory of rational knowing (§46). In answering such questions, we encounter the nature and status of “society,” “love,” and “force.” My claim is that the nature and status of these three, as things we think about, are essentially the same.

**4** Science is one field that attempts to practice the construction of rational knowledge. In his book on concepts at the foundations of physics, Max Jammer (1957, p. 2) provides an intelligible account of what he calls the “objectives” of science: “its two major assignments are the description of certain phenomena in the world of experience and the establishment of general principles for their prediction and what might be called their ‘explanation.’”

**5** In this description Jammer seems to be saying that one can explain science as carrying out its program using two types of knowledge: experiential and explanatory. Scientists are trying to formulate rational, naturalistic explanations for specified sets of experiences, which meet certain conditions. This depiction in terms of experiential and explanatory knowledge is entirely compatible with radical constructivism as von Glasersfeld describes it, e.g., in Glasersfeld (1995, 1999).

**6** This enterprise is really not so unique to science. Human beings strive to formulate rational explanations for experience in many intellectual fields and in everyday experience. (This is not to claim construction of rational explanation is the whole of human cognition, but just that it is one of the activities we distinguish in what we call human cognition.) Differences may lay in qualifications placed on the set of experiences or the quality and rigor of the explanation, but the underlying enterprise is the same.

Jammer goes on to describe the “apparatus” used in science to carry out its “assignments.”

“For the efficient achievement of these two objectives science employs a conceptual apparatus, that is, a system of concepts and theories that represent or symbolize the data of sense experience, as pressures, colors, tones, odors, and their possible interrelations. This conceptual apparatus consists of two parts: (1) a system of concepts, definitions, axioms, and theorems, forming a hypothetico-deductive system, as exemplified in mathematics by Euclidean geometry; (2) a set of relations linking certain concepts of the hypothetico-deductive system with certain data of sensory experience” (Jammer 1957, p. 2).

**7** What does Jammer’s explanation of the nature of science have to do with our questions about society, love and force? I shall begin by asking, “How do we identify when two people are in love?” When pressed, most people answer with something like: “You can tell by the way the two act.” In other words, we draw the conclusion that love is present based on observable behavior. Do we see love or do we see the observable behavior? Where does the love come in? Love is our explanation for the observed behavior. The observable behavior is experiential knowledge. Love is the explanation that subsumes this observable behavior. In this sense can we see love? Not really. Love is a concept in explanatory knowledge and as such is “the free creation of the human mind.” (Einstein & Infeld 1938) As explanation, love is present in the human mind, not out there in the external world.

**8** The same applies to “force.” Do we measure force? Not really. We quantify effects that we explain are due to force. Using strain gauges, we quantify the change in electrical

properties of objects that deform. We explain that the deformation is due to force. We use the measured change in the electrical property of a strain gauge and our explanatory knowledge to compute the magnitude and direction of a force. Both the measured change in electrical property and our explanatory system for force are necessary for the computation. In other settings, we measure the motion of an object. We explain that the motion of objects has a certain relationship to force. With this explanation we can use the measured motion of an object to compute the magnitude and direction of a force. We do not measure or observe force itself. We compute the magnitude and direction of force from specified experiences based on our hypothetico-deductive explanatory system involving the concept, force, and links to the specified experiences.

**9** In this sense both love and force have the same nature and status. They are both concepts in explanatory knowledge. Each is used to explain a specified set of experiential knowledge. We select specific elements of experiential knowledge to link, in ways of our own choosing, with aspects of the explanation for these experiences. We cannot see or measure love or force because they exist *only* in our minds as explanations for things we *can* see or measure.

**10** As concepts, which exist only in our minds, that we use to explain to ourselves elements of our experience and to make predictions, love and force do not exist physically to be transmitted from one person to another. Hence, as a consequence, in the radical constructivist explanation, each of us can only construct our *own* concepts of love and force that we modify or refine in order to reach a better fit to the experiences they are intended to explain.

**11** So, how about society? Is it experiential knowledge or is it explanatory knowledge? Society is the explanation of a specific body of experience parsed from the manifold of experiences we have and believe others have. In the case of society, one inescapable aspect of the fit we strive to achieve is a kind of agreement with others with regard to the explanation. As von Glasersfeld put it:

“Put in the simplest way, to understand what someone has said or written means no less, but also no more, than to have

built up a conceptual structure that, in the given context, appears to be compatible with the structure the speaker had in mind. And this compatibility, as a rule, manifests itself in no other way than that the receiver says and does nothing that contravenes the speaker’s expectations” (§28).

**12** Society, as an explanation, is constructed by each of us in coordination with those others we distinguish out of the manifold of our experiences. By this radical constructivist line of reasoning, employing a distinction between experiential and explanatory knowledge inspired by Jammer, the concept, society, has the same status, nature and origins as the concepts, love and force. The concept, society, exists in minds and nowhere else.

**13** Is it possible to give a different explanation: one such as that of the social constructivists? Certainly. Can productive use be made of such an explanation? Certainly. But, from a radical constructivist’s point of view, a position in which society is taken as a given, mind-independent reality that can be known to an individual, is both not necessary and potentially misleading. Not necessary, as demonstrated in von Glasersfeld’s target article and the present commentary. Potentially misleading in that it omits any attention to the origins and genesis of explanatory knowledge of society. A radical constructivist would hold that even more productive use might be made of an explanation of society consistent with radical constructivism.

**14** For the radical constructivist, two more questions arise. If an element of a mind-independent reality can be known, in this case society, how does coming to know it happen and why only society? In the case of the former question, we have not seen a successful, convincing response to the challenge that the Skeptics issued millennia ago.

**15** The second question has little meaning if the first has not been answered. But, suspending that problem, if society is not an explanation constructed by individuals in concert with others, but exists as a mind-independent reality and can be known to people as such, then why not everything else? If everything else can be known as such, then why use the label “constructivist” if construction is not necessary to know the world, either one part or all of it?